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THE
Young Mathematician's
 COMPANION,
 BEING A COMPLEAT
 TUTOR
 TO THE
 MATHEMATICKS;

Whereby the *Young Beginner* may be early Instructed ;
 those who have lost the Opportunity of learning in
 their Youth may with very little Pains, and in a short
 Time become Proficients in this delightful and in-
 structive Science, and such whose *Business* it is to teach,
 may receive much *Useful Assistance*.

CONTAINING,

- | | |
|---|--|
| <p>I. Vulgar and Decimal Arith-
 metic, Extraction of <i>Roots</i>
 by Natural Numbers, and
 by Logarithms.</p> | <p>III. Plain and Spherical Tri-
 gonometry , Astronomy ,
 Dyalling, and Surveying of
 Land.</p> |
| <p>II. Description and Use of the
 Sector, with the most useful
 Definitions, Theorems, and
 Problems in <i>Geometry</i>.</p> | <p>IV. Curious Discourses, calcu-
 lated to render a Practical
 Knowledge of the Mathema-
 ticks more easy and familiar.</p> |

The Whole Interspersed with delightful and useful Que-
 stions, and adorned with proper Schemes in order to
 excite the Curiosity, and form the Minds of Youth.

By *CHARLES LEADBETTER*,
Teacher of the Mathematicks

L O N D O N :

Printed for J. HODGES, at the *Looking-Glass* on *London-Bridge*,
 1739. Price Bound 2 s. 6 d.

Just Published,

Neatly printed in Octavo, Price Bound in Calf 5s.

With a large Folio Cut, on Royal Paper,

Curiously Engrav'd,

LOGARITHMOLOGIA: Or, The Whole Doctrine of Logarithms, Common and Logistical, in Theory and Practice, in Three Parts.

Part I. The Theory of Logarithms; shewing their Nature, Origin, Construction and Properties, demonstrated in various Methods, *viz.* 1. By Plain Arithmetic. 2. By the Logarithmic Curve. 3. By Dr *Halley's* Infinite Series. 4. By Fluxions. 5. By the Properties of the Hyperbola. 6. By the Equiangular Spiral. 7. By a Logarithmic inspectional Scale of Twenty-two Inches Length. With the Construction of the artificial Lines of Numbers, Sines and Tangents. Also the Nature and Construction of Logistical Logarithms. The Whole illustrated and made easy by many and suitable Examples.

Part II. The Praxis of Logarithms; wherein all the Rules and Operations of Logarithmical Arithmetic, both Common and Logistical, by Numbers and Instruments, are copiously exemplified. Together with the Application thereof to the several Branches of Mathematical Learning.

Part III. A Three-fold Canon of Logarithms; in a new and more compendious Method than any extant;

- Viz.* { 1. A Canon of Logarithms of Natural Numbers.
2. A Canon of Logarithms of Sines and Tangents.
3. A Table of Logistical Logarithms.

The Whole being a *Compleat System* of this most useful Art; and enrich'd with all the Improvements therein from it's Original to the Present Time.

By *BENJAMIN MARTIN,*

Author of the *Philological Library of Literary Arts and Sciences, &c.*

Printed for *J. Hodges,* at the *Looking-Glass on London-Bridge.*



THE P R E F A C E.



MATHEMATICKS receive their Name from *Matheſis*, which ſignifies *Discipline and Doctrin*e, and doth not only ſignify to learn and underſtand, but alſo to call to Remembrance the Knowledge of thoſe Things which are imprinted in the Mind, that is, a Remembrance raiſed from Appearance. But now 'tis properly that Science which teaches or contemplates whatever is capable of being Number'd or Meaſured.

And that Part of *Mathematicks* which relates to Number only, is called *Arithmetic*: That which relates to Meaſure is called *Geometry*.

Mathematic, alſo is divided into *Speculative*, which propoſes only the ſimple Knowledge of the Thing propoſed, and the bare Conſideration of Truth and Falſhood. And, *Practical* which teaches how to demonſtrate ſomething uſeful, or to perform ſomething that ſhall be propoſed for the Benefit and Advantage of Mankind. In all Ages and Countries where Learning hath prevailed, the *Mathematical Sciences* have been look'd upon as one of the chief and moſt conſiderable among them.

There is no Science in the World that doth improve the Mind of Man so much as this; by giving it a Habit of close and demonstrative Reasoning; by freeing it from Credulity, Prejudice and Superstition; by rendering it exact and capable of solving the greatest Difficulties; and lastly, by regulating the Imagination, and giving the Mind the greatest Extention and Capacity that Human Nature is capable to attain.

The Usefulness of this Science is almost infinite, there being scarce any Knowledge, Art, or Science, in the Universe, but may be assisted and advanced by it. And indeed it is to this Science we owe the vast Improvements of Natural Knowledge in these last Ages, and some of the most Noble Inventions of the World.

Who then would be ignorant in a Science so Excellent, so Useful, and Beneficial to Mankind? Who would not take some Pains to attain a Competent Knowledge of an Art so truly valuable? I am confident there is no thoughtful and contemplative Person but would find unspeakable Pleasure and Satisfaction in the Study of it. And to such as are so inclin'd, I have here set down the best Method; in Arithmetic the Reader will find many useful and pleasant Questions, not to be found in any other Treat.

The Second Chapter treats of Geometry, in which I have given every Thing that is Useful in Practical Mathematicks, having every where purposely omitted the Speculative Part, or Things that are uselefs, which to Beginners would rather prove a Stumbling Block than any ways improve the Mind.

The Third Chapter treats of the Mensuration of all manner of Superficies, in a plain and familiar Method fitted to the Capacity of Beginners.

The Fourth Chapter treats of the Mensuration of all manner of Solids, concluding with two useful Propositions, the one in Statuary, and the other in Staticks.

The PREFACE.

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The Fifth Chapter is, the Description and Use of the Sector, which Instrument being of such Universal Use, claims a Place in this Work.

The Sixth Chapter treats of Plain Trigonometry, in a new and concise Method, perform'd by Natural Numbers and by Logarithms, concluding the same with some useful Problems.

Chapter Seven contains Spheric Trigonometry, both right and oblique Angled, in a clear Manner, adapted to the Capacity of young Beginners.

Chapter Eight treats of Astronomy in general.

The Ninth of Dialling.

And the Tenth of Surveying of Land.

These are the Subjects of the ensuing Work, and herein I have shewn the great Use and Advantage that may accrew to Mankind from them, being but seriously perused by a thoughtful Mind, the Reader may come to attain to a competent Knowledge in these Useful Arts.

These Arts are called by some the Internal, or Liberal Arts, because they are attained by the Faculties of the Soul, which is a liberal or free Agent, and not by the Labour or Ministry of the Hands.

How commendable it is to see Youth give their Minds to the Study of these Sciences, it frees the Mind of ill Habits, and raises the Soul above the common Way of thinking; he takes a View of the Universe, and sees at one Glance how the Great Architect has fram'd this wonderful Fabric.

Sophia, or Philosophia, Wisdom, or the Love of Wisdom, is the Knowledge of all Arts and Sciences which any way do conduce to the Knowledge of God: And next to Divinity are the Mathematical Sciences.

Most People now in this Age have an Ambition of giving their Children the best Education that they can, and truly, if I might advise, when Youth has attain'd Reading and Writing good English, let him be put to the Mathematical School; but if the Ability of the Parent

rent be in so low Circumstances that they cannot afford it, then let them buy this Book, and the Boy by his own Genius may come at what will please the Mind, and not only so, but make him capable of making Satisfaction to the Parent for the Expence they have been at, for his own Improvement. The Usefulness of these Arts cannot be denied, and therefore my hopes are ; that Youth will bend their Minds to the Study of these useful Sciences, and by that Means become useful Instruments in their Generation, which that every one of my Readers may do, is the hearty Prayer of their very

Humble Servant,

Charles Leadbetter.



THE





THE
Young Mathematician's
COMPANION.

Of ARITHMETIC.

DEFINITION.



ARITHMETIC, is the Art of Numbering well; and in it we make use only of these Nine Figures, called Digits, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, and the Cypher 0. in which there are divers Rules which shall be treated of in the following Order. And First of

NUMERATION.

I. *Numeration*, is the true Distinction, Estimation, and Pronunciation of Numbers, or the Rule to read any Number tho' never so great, and to have a distinct Idea of each Place or Figure of it, which may easily be done, by getting the following Table by Heart. For the first Place to the Right Hand is the Place of Unites, that is, every Fi-

B

gure

gure possessing that Place is no more than the real Value, as if the Figure 1 stand there, it is no more than one if 2 'tis two, &c. but every Figure that stands in the Place of Tens, are as many Tens as the Figure is in Value; those in the third Place are Hundreds, the fourth Place are Thousands, the fifth, Tens of Thousands, the sixth Place are Hundreds of Thousands, the seventh, Millions, &c. there are several Ways of disposing of Figures to form a Table, but this which follows, I conceive the most easy for young Beginners.

										9	Nine.
									9	8	Eight.
								9	8	7	Seven.
							9	8	7	6	Six.
						9	8	7	6	5	Five.
					9	8	7	6	5	4	Four.
				9	8	7	6	5	4	3	Three.
			9	8	7	6	5	4	3	2	Two.
		9	8	7	6	5	4	3	2	1	One.
9	8	7	6	5	4	3	2	1	0		Cypher.
Thousands Millions.	C. Millions.	X. Millions.	Millions.	C. Thousands.	X. Thousands.	Thousands.	Hundreds.	Tens.	Unites.		

The lowermost Line of the Table is thus read, *viz.* Nine Thousand Eight Hundred Seventy Six Millions, Five Hundred Forty Three Thousand, Two Hundred and Ten: And so of the rest. These Ten Places are as far as any common Business will require, but for Curiosity's Sake, I will here shew how to read 66 Places or more at Pleasure, thus; begin at the Right Hand, and point or prick under the Seventh Place, and over it place the Figure 1, that is the Place of Millions, tell on Seven Places more, and over that place the Figure 2, the Number 2 stands in the Place of Million of Millions, which is better expressed by Billions, (*i. e.* Bi-Millions)

Millions) tell on again Seven Figures more, and there place the Figure 3 ; this is the Place of Trillions, and Seven Places more will be Quartrillions, the next Seven Quinquillions, &c. according as the Figures over every Seventh Place doth shew. So that the First Place to the Right Hand is Unites, the Seventh is Millions, the Thirteenth is Bi-Millions, the Nineteenth Place is Trimillions, the Twenty Fifth Place is Quartrillions, the Thirty First Place Quinquillions, the Thirty Seventh Place is Sexquillions, the Forty Third Place is Septillions, the Forty Ninth Place is Octillions, the Fifty Fifth Place is Nonillions, or the Place of a Million multiply'd Nine Times by itself ; as for instance, suppose I would Number this Train of Figures, viz.

789876543210⁹123456⁸789876⁷543210⁶123456⁵789876⁴
543210³123456²789876¹543210

Here are Sixty Six Places, and are thus read; supposing them to stand all in one Line, begin with the uppermost Line at the Left Hand, and by the Directions of the Table above, begin with the Cypher, over which the 9 standeth, and tell the Value of Places to the 7 at the End of the Line, and they are Seven Hundred Eighty Nine Thousand Eight Hundred Seventy Six Millions, Five Hundred Forty Three Thousand Two Hundred and Ten Nonillions, One Hundred Twenty Three Thousand Four Hundred Fifty Six Octillions, Seven Hundred Eighty Nine Thousand Eight Hundred Seventy Six Septillions, Five Hundred and Forty Three Thousand Two Hundred and Ten Sexquillions ; One Hundred Twenty Three Thousand Four Hundred Fifty Six Quinquillions, Seven Hundred Eighty Nine Thousand, Eight Hundred Seventy Six Quartrillions, Five Hundred and Forty Three Thousand Two Hundred and Ten Trimillions, One Hundred Twenty Three Thousand Four Hundred Fifty Six Bi-Millions, Seven Hundred Eighty Nine Thousand Eight Hundred Seventy Six Millions. Five Hundred Forty Three Thousand Two Hundred and Ten. And thus you see 'tis easy to read or pronounce any Number at Pleasure.

II. Of ADDITION.

ADDITION, in General, is the putting of two or more Quantities together, and that Quantity which arises or results from thence, is called the Sum, or Aggregate of those Quantities, as we shall make plain by Examples.

The PENCE TABLE.

s.	d.		d.	s.	d.
1	12		10	0	10
2	24		20	1	8
3	36		30	2	6 half a Cro.
4	48		40	3	4
5	60		50	4	2
6	72		60	5	0 Crown.
7	84		70	5	10
8	96		80	6	8 Noble.
9	108		90	7	6
10	120		100	8	4
11	132		110	9	2
12	144		120	10	0 Angel.
13	156		130	10	10
14	168		140	11	8
15	180		150	12	6
16	192		160	13	4 Mark.
17	204 Pistole.		170	14	2
18	216		180	15	0
19	228		190	15	10
20	240 Pound.		200	16	8
21	252 Guinea.		210	17	6
22	264		220	18	4
23	276		230	19	2
24	288		240	20	0 Pound.

This Table got by Heart, will be very ready for Tradesmen to cast up Things by the Head.

And

And First of one Denomination.

Set your Numbers down one under another, Unites under Unites, Tens under Tens, Hundreds under Hundreds, &c. Then begin at the Right Hand, and collect each Column singly into one Sum, and if the Unites in that Row are less than Ten, set them down under the Line, but if they are more than Ten, set down all above Ten, and carry the Tens to the next Row; in which set down under the Line the odd Tens, and carry the Hundreds to the next Row, and so continuing to set down the Unites, and carry the Tens to the next Column till all be done. Thus, Suppose I have 481 Yards of Velvet, 387 Yards of Plush, 245 Yards of Calicoe, 129 Yards of Camblet, how many Yards are there in all.

Velvet	481
Plush	387
Calicoe	245
Camblet	129

Total 1242 Yards, the Answer.

Thus, the Numbers being set down as above, begin and say 9 and 5 is 14 and 7 is 21 and 1 makes 22, set down the 2 Unites under the Line and carry the 2 Tens to the next Row; saying 2 that I carry and 2 is 4 and 4 is 8, and 8 is 16, and 8 is 24, set down 4 under the Line and carry 2 to the next Row, saying 2 that I carry and 1 is 3, and 2 is 5, and 3 is 8, and 4 makes 12, now because there are no more Figures to carry, I set down 12, and the Sum of all the given Numbers is 1242, as above.

The common Way of Proving Addition (as taught in Schools) is to cut off the Top Line, and cast the Sum up again, only leaving out the Top Line, then the Sum of these added to the Top Line gives the whole Total. But this seems to be invented more to hinder, than forward the Work, and is so unmasterly, that I shall not recommend it, but rather advise the Reader to cast up his Sum over again, and that's the best Way of proving his Work.

More Examples for Practice.

<i>Years.</i>	<i>Miles.</i>	<i>Bar.</i>	<i>Lib.</i>
17064	2706	4617	87069
8517	1042	3570	10952
321	875	2406	7403
298	263	1041	276
106	21	82	95
26306	4907	11716	105795

But if the Numbers be of different; or of several Denominations, then they must be added by summing up each Denomination by itself, and seeing how many of them will make one of the next Denomination, and carry them to the next Place. As for Example, suppose the following Sum of *English* Money were to be cast up, first consider that four Farthings make one Penny, twelve Pence make one Shilling, and twenty Shillings make one Pound.

Note. Over every Denomination is commonly placed *L. S. D. Q.*, signifying *Libra, Solidi, Denarii, Quadrantes*, that is Pounds, Shillings, Pence, Farthings.

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
10.	20.	12.	4.
4862	14	11	1
3721	18	10	2
2951	10	04	3
2563	19	01	2
1085	12	03	1
977	15	09	2
420	04	06	3
16583	15	11	2

Over

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Over every Denomination is placed the Denomitive Parts, that is, what you are to dot at, or shews how many of one Denomination makes one of next greater. As in the following Example of *Averdupoise Weight*, Sixteen Drams make one Ounce, Sixteen Ounces make one Pound, Twenty eight Pound make one Quarter of a Hundred, four Quarters make one Hundred Weight, and twenty Hundreds make one Ton.

10.	20.	4.	28	16.	16.
T.	C.	Q.	lib.	oz.	dr.
7843	14	2	20	12	10
6382	18	3	26	14	15
5671	11	1	18	10	07
4329	19	2	21	11	04
3573	17	3	25	13	13
2468	10	2	14	02	01
1073	16	1	17	06	04
Sum	31344.	09	3	05	07 06

Note, The *Troy* and *Averdupoise Weight*, bear Proportion one to the other according to

Mr *Boyle* found that $18\frac{1}{8}$ *lib. Troy*, are equal to $15\frac{1}{17}$ Pounds *Averdupoise*.

Mr *Wingate* found that one Pound *Averdupoise*, was equal to 14 oz. 12 *pw. Troy*. Here the *Troy Ounce* is more than the *Averdupoise Ounce*, but the *Averdupoise Pound* is greater than the Pound *Troy*.

Mr *Everard* found in the Year 1696, that fifteen Pounds *Averdupoise* were equal to 18 *lib* : 2 oz : 15 *pw. Troy*.

Mr *John Ward* found by a very nice Experiment, that 14 oz : 11 *pw* : $15\frac{1}{2}$ *gr. Troy*, was equal to one Pound *Averdupoise*.

Mr *Vincent Wing* says, the Proportion of the Pound *Averdupoise* to the Pound *Troy*, is as 60 to 73. And the Proportion of the Ounce *Averdupoise*, to the Ounce *Troy*, is as 80 to 73. By which the Pound *Averdupoise* is greater than the Pound *Troy* ; but the Ounce *Averdupoise*,

poise, is leſſer than the Ounce *Troy*, as is noted in *Win-
gate's* Experiment.

III. Of SUBTRACTION.

DEFINITION.

SUBTRACTION is taking a leſſer Quantity, or Number, from a greater, in order to find the Difference between them, which Difference is called the Remainder.

In order then to take one Number or Sum out of another, if they be Things of one Denomination or Name, ſet Unites under Unites, Tens under Tens, Hundreds under Hundreds, &c. under which two Numbers draw a Line, and remember that what you carried in *Addition*, you muſt here borrow in *Subtraction*, and in Things of one Name, you always carried one for every Ten they amounted to; ſo here when the Under Figure is greater than the Upper, you muſt in this Caſe add Ten to the Upper, and then take the Under out of it, then for this Ten which you muſt call One that you borrow'd, add One to the next Figure, and take it out of that which ſtands over it and ſet the Remainders under the Line, proceed thus 'till all the Differences be taken, and then the Work is done, as in theſe Examples.

	C.	L.	lib.
From	5026	2813	47015
Sub.	3578	1642	38148
Remains	1448	1171	8867

But in Numbers of different Denominations, you muſt always borrow one of the next greater Denomination and ſubtract, remembring always to pay the One you borrowed,

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rowed, and set the Remainder under the Line, as in these Examples of Money.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
Lent	32051	14	08	0
Paid	9447	16	10	1
Rem.	22603	17	09	3

P R O O F.

The Proof of this Rule, is to add together the Remainder, and the Number to be subtracted, and that Sum will be equal to the Number from which *Subtraction* was made. See an Example of *Troy Weight*.

	10 <i>lib.</i>	12 <i>oz.</i>	20 <i>pw.</i>	24 <i>gr.</i>
Bought	576	04	11	14
Sold	427	07	16	19
Rem.	148	08	14	19
Proof	576	04	11	14

} add

Q U E S T I O N.

In the Year of our Lord 508, *Terquin* was banish'd from *Rome*, for the Ravishing of *Lucretia*, I demand how many Years it is since, to this present Year 1739? Set the Numbers down thus.

Present Year	- - - - -	1739
<i>Lucretia</i> Ravished in	- - - - -	508
Answer		1231

IV. Of

IV. Of MULTIPLICATION.

DEFINITION.

MULTIPLICATION, is the taking or repeating of one Number or Quantity as often as there are supposed Unites in the other. The Number Multiplied is called the Multiplicand, the Number by which you multiply, is the Multiplier, or Multiplier, and that which is found or produced, is called the Product. Also for Brevity's Sake, the Multiplicand and Multiplier are called Factors, because between them they make up the Product. Multiplication serves instead of many Additions, and this is the Table:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The Table above is obvious to the meanest Capacity, for find one of your Factors on the Head, and the other
in

in the First Column on the Left Hand, and in the Common Angle, or Place of Meeting is the Product. Example, What is the Product of 6 multiply'd by 9? Now it matters not whether you seek for the 6 or the 9 on the Head, and the other on the Side, for 'tis all one, you'll find the Product to be 54, and so of any other.

The Ground of *Multiplication Table* may be demonstrated by this Cross in the Margin, for place the two *Digits* 6 and 9, (or any other two *Digits*) on the Left Side, and their Difference to Ten on the Right Hand, then take the Difference from the *Digit*, (not from it's own, but cross-wise) as Four out of Nine, or One out of Six, the Remainder will be Five, then the two Differences, *viz.* Four and One multiplied together make Four, which with the Five, place under the Line, as you see done, and it makes 54 for the Product.

$$\begin{array}{r} 9 \times 1 \\ 6 \times 4 \\ \hline 54 \end{array}$$

This may also be explained by the Fingers and Thumbs of your Hands, for as there are Nine Figures and the Cypher makes Ten, so have we Eight Fingers and Two Thumbs make also Ten; then if you would know how many six times Nine, or nine times Six is, (for it's all one) Gripe or Clench all your Fingers and Thumbs close, being thus, they are all Unites, but every one as you raise up is Ten, then raise the Thumb on the Right Hand, and call it Six, to answer the Six of the Multiplier, then raise the Thumb of the Left Hand, and call it Six, raise the First Finger, and call it Seven, raise the Second Finger and call it Eight, raise the Third Finger and call it Nine for your other Factor; now all the fingers and Thumbs that are up are Tens, and those down are Unites; add the Tens together, make Five, and multiply the Unites together, make Four, so I see that six times Nine make 54, as before.

Here

Here follows Examples for Practice.

Multiplicands, - -	52		365
Multipliers, - -	7		9
Products, - -	364		3285
Multiply me - -	1681	And - -	2561
By - -	12	By - -	28
	3362		20488
	1681		5122
Products, - -	20172		71708

Note, Ever remember that if your Multiplier consist of ever so many Places, that the first Figure of it's Product stand under the same Place that it's Multiplier possesseth, then add the particular Products together, gives the Product of the Whole.

Notes upon the Nine Digits.

I. The *Digit One*, hath a Property which no other Number hath, for it neither multiplieth nor divideth, but leaveth the Number to be multiplied or divided the same.

II. Multiplying any Number by Two, is the same with doubling of it; and that whether you add Two to itself, or multiply it in itself the Sum and Product are equal. And it is moreover worth taking Notice thereof, that no square Number, how large soever, can never terminate, or end with the *Digit 2*.

III. If you would multiply any given Number by 3, to that Number add the Double thereof, and the Sum shall be equal to the Product.

IV. If you would multiply any Number by Four, you must double the Dublication, and the Sum is the Product.

V.If

V. If you would multiply any Number by Five, add a Cypher to the given Number, and take half that Sum for the Product.

VI. If you would multiply any Number by Six, add a Cypher to the given Number, and take half thereof, to which add the given Number, the Sum shall be equal to the Product.

Note, Between these two last mentioned *Digits* Five and Six, there is a secret Property ; for if you multiply either of them in themselves, the Number produced by such Multiplications, shall terminate in themselves.

The Number Six hath another eminent Property, for all it's Aliquot Parts are equal to itself, as it's Half, it's Third, and it's Sixth, being all added together make Six. And of Numbers that have this Property, there are but ten to be found, between one, and one Million of Millions, which are these exhibited in this Table.

Numbers of Aliquot Parts. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$ }	6
	28
	486
	8128
	120816
	2096128
	33550336
	536854528
	8589869056
	137438691328

VII. If you would multiply any Number by Seven, add a Cypher to the given Number, and take half thereof, to which half add the double of the Number given, the Sum of them shall be the Product of the given Number, multiplied by Seven.

VIII. To multiply any Number by Eight, to the given Number add a Cypher, and from thence subtract the double of the Number given, the Remainder is the Product.

IX. To multiply by Nine, add a Cypher to the given Number, and from that Number subtract the given Number, the Remainder is the Product.

Observation.

This *Digit* Nine, hath a Privilege above all the other *Digits*; for if you take any Number, the Nines taken out of the gross Sum of that Number, or of all the Parts thereof severally, the remaining *Digit* will be still the same.

EXAMPLE,

In the Number 45 there are 5 Nines contained therein, so if you multiply Nine by Five; the Product is 45. In like manner, if you take the Nines out of this Number 476, it is all one as if you should take the Nines out of the single Figures 4, 7, 6, which do make Seventeen, from which Nine being taken, there will remain the *Digit* Eight, and also if you divide 476 by Nine, the Quotient will be Fifty two and Eight remaining.

Proof of Multiplication.

For hence proceeds that false Way of proving Multiplication by the Cross, or by throwing away the Nines out of the Factors and Product.

As for Instance, if 272802 be multiplied by 34125, the Product will be 9309432384.

Make a Cross as in the Margin, throw the Nines out of the Multiplicand 272802, and there rest 3 which I place on the Left Side the Cross. Secondly, throw the Nines out of the Multiplier 34125, and the Overplus is Six, which place at the Right Side the Cross: Thirdly, the Product of Three by Six is Eighteen, out of which throw the Nines, and there remains 0, which place at the Top of the Cross. Lastly, the Nines thrown out of the Product, Rest 0, which place at the Bottom of the Cross.



Hence

Hence, because the Top and Bottom Figures are the same, the Work is proved to be right : I say this is true, but suppose any of the Figures in the Product are transposed (as I have often known) I'll suppose thus, 9390423384, here the Figures being the same, when the Nines are thrown out, there will also remain 0, equal with the Figure at the Top of the Cross, which proves the Sum Right, tho' at the same Time it be most grossly false.

Compendiums in Multiplication.

Whenever your Multiplier has Cyphers in the Unites, Tens, or Hundreds Places, &c. place them to the Right Hand ; thus.

$\begin{array}{r} 487 \\ 10 \\ \hline 4870 \end{array}$	$\begin{array}{r} 392 \\ 20 \\ \hline 7840 \end{array}$	$\begin{array}{r} 726 \\ 500 \\ \hline 363000 \end{array}$
---	---	--

How to multiply by any Number under 20, 30, or 40, so as to bring the Whole Work into one Line.

First, Any Number under Twenty.

R U L E.

Multiply each Figure in the Multiplicand, by the Unites Figure in the Multiplier, adding to each it's back Figure, that stand on the Right Hand of that you multiplied, only mind that the first Figure you begin with, is the Thing itself, there being none to be added to it.

E X A M P L E.

Let 365 be multiplied by 12, and possess but one Line.

$$\begin{array}{r} 365 \\ 12 \\ \hline 4380 \\ C 2 \end{array}$$

First

First say 2 times 5 is 10, that is 0, and carry one, then 2 times 6 is 12, and 1 I carried is 13, and the 5 in the Unites Place of the Multiplicand makes 18, that is, set down 8 and carry one, then 2 times 3 is 6 and 1 I carried is 7 and the 6 in the Multiplicand makes 13, that is, 3 set down and carry 1, which is added to the 3 in the Multiplicand, and it makes 4 to set down, so the Product of 365 by 12, is = 4380.

More Examples.

$$\begin{array}{r} 764 \\ 15 \\ \hline 11460 \end{array}$$

$$\begin{array}{r} 6812 \\ 18 \\ \hline 122616 \end{array}$$

$$\begin{array}{r} 81263 \\ 19 \\ \hline 1543997 \end{array}$$

Secondly, Any Number between 20 and 30.

Multiply by the Unites of the Multiplicand, add to each Product the double of the Right Hand Figure, and at last add what you carried to the double of the last Figure in the Multiplicand, and that will be the true Product. See these Examples and mark them well.

$$\begin{array}{r} 7384 \\ 23 \\ \hline 169832 \end{array}$$

$$\begin{array}{r} 65981 \\ 25 \\ \hline 1649525 \end{array}$$

$$\begin{array}{r} 5486 \\ 28 \\ \hline 153608 \end{array}$$

In the first say, 3 times 4 is 12, that is, set down 2 and carry 1, then 3 times 8 is 24, and 1 I carried in Mind makes 25, and the double of 4 in the Multiplicand is 8 added to 25, makes 33, set down 3 and carry 3; then 3 times 3 is 9, and 3 I carried is 12, and 16 (the double of 8) makes 28, set down 8 and carry 2; then 3 times 7 is 21, and 2 I carried is 23, and 6 the double of 3 is 12, set down 2 and carry 2, which added to the double of 7, makes 16, which set down on the Left Hand of the others under the Line, and it makes the Whole Product 169832.

Thirdly,

Thirdly, Any Number between 30 and 40:

Multiply as before, and add the Triple of the Right Hand Figure in the Multiplicand to your Product, and at last, add the Triple of the last Figure in the Multiplicand, to what you carried, and set them down under the Line and then 'tis done.

See these Examples.

$$\begin{array}{r} 57806 \\ 34 \\ \hline 1965404 \end{array}$$

$$\begin{array}{r} 12948 \\ 36 \\ \hline 455328 \end{array}$$

$$\begin{array}{r} 705381 \\ 39 \\ \hline 27509859 \end{array}$$

Fourthly, By the first Rule, to multiply any Number compounded of any Number under 20, the Product is readily found, as by Examples.

$$\begin{array}{r} 46863 \\ 112 \\ \hline 562356 \\ 46863 \\ \hline 5248646 \end{array}$$

$$\begin{array}{r} 79062 \\ 112 \\ \hline 948744 \\ 79062 \\ \hline 8854944 \end{array}$$

Fifthly, To multiply by 1614, or 1517, or by 1716, &c.

First multiply by 14 in one Line, and by 16 in the other, and the other Sum by 17 in one Line, and by 15 in the other, &c. See the Work.

$$\begin{array}{r} 357948 \\ 1614 \\ \hline 5011272 \\ 5727168 \\ \hline 577728072 \end{array}$$

$$\begin{array}{r} 79062 \\ 1517 \\ \hline 486285 \\ 429075 \\ \hline 43393785 \end{array}$$

Ten Merry Men come to an Alehouse, and agree with the Landlord for a Guinea, *per Man*, are to have two Pints of

Drink, *per Piece, per Day*, for so long time as they all being placed together on one Form at every Drinking. There might be a divers Transmutation of Place; *Query*, How comes the Landlord off with his Guests, both as to Money and Time?

Sixthly, How to multiply by the Component Parts, instead of the Whole.

Example, - - 542, by 63? take 7 and 9.

$$\begin{array}{r} 7 \\ \hline 3794 \\ 9 \\ \hline \end{array}$$

Product - - 34146

If any odd Numbers are given which are not an even Product of any two of the Nine *Digits*. &c. then take two Figures whose Product comes nearest either, more or less than the given Multiplier, and add if you took a less Number, but subtract if you took more, as in these Examples.

Multiply 9684 by 35, and 738 by 29.

$$\begin{array}{r} 6 \\ \hline 58104 \\ 6 \\ \hline \end{array}$$

Too much
Subtract 348624
9684

Products 338940

$$\begin{array}{r} 7 \\ \hline 5166 \\ 4 \\ \hline \end{array}$$

Too little:
Add 20664
738

21402 True.

Seventhly, How to multiply several Denominations.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
How much is 7 times	58	12	9	2
				7
Answer	410	9	6	2

Of ARITHMETIC.

19

TROY WEIGHT.

		lb.	oz.	pw.	gr.
Multiply	=	476	8	14	17
By	=				29
Answer	=	13825	1	6	13

To multiply by *Logarithms*.

Logarithms were first invented by Lord *Neper*, a Scotch Baron, about the Year 1614, and after compleated by our Countryman, Mr *Henry Briggs*, Professor of Geometry at *Oxford*, and at *Gresham College* in *London*: They perform that by *Addition* and *Subtraction*, that Natural Numbers do by *Multiplication* and *Division*. And here

Note, That the Characteristick, or Index is always less by one, than the Number of Places of the absolute Number.

The RULE.

To multiply by *Logarithms*, find the *Logarithm* of two given Factors, and the Sum (rejecting *Radius*) is the *Logarithm* of the Product sought.

EXAMPLE.

Let the Product of 4736, by 28 be sought ?

OPERATION.

4736	-	-	-	3.6754117
28	-	-	=	1.4471580
132608	=	=		5.1225697

EXAMPLE.

EXAMPLE II.

Multiply me	375	2.5740313
By	15	<u>1.1760913</u>
Product	5625	3.7501226

VI. Of DIVISION.

DEFINITION.

DIVISION, signifies the distributing or parting of any Whole into Parts, and is only a Compendium of *Subtraction*; for the Divisor is so many Times contained in the Dividend, as there are Unites in the Quotient.

The Number to be divided is called the Dividend; that by which you divide the Divisor, and the Number of Times that the Dividend contains the Divisor is called the Quotient. And here Note, That *Multiplication* and *Division* interchangeably and infallibly prove each other, as will be shewn by and by.

*First ask how oft in Quotient, Answer make.
Then multiply, subtract, a new dividuall take.*

What's the Quotient of 354, divided by 6? Set the Numbers thus,

Divisor. Dividend. Quotient.

$$\begin{array}{r}
 6 \overline{) 354} \quad (59 \\
 \underline{54} \\
 \text{Rem.} \quad 0
 \end{array}$$

More

More Examples for Practice.

$$12) \begin{array}{r} 4170 \\ \dots \end{array} (347$$

$$\begin{array}{r} 57 \\ \hline 90 \\ \hline \text{Rem. } 6 \end{array}$$

$$19) \begin{array}{r} 17501 \\ \dots \end{array} (921$$

$$\begin{array}{r} 40 \\ \hline 21 \\ \hline \text{Rem. } 2 \end{array}$$

Compendium.

If the Divisor hath Cyphers in the Unites, Tens, Hundreds Place, &c. cut them off with a Line, and also cut as many Places off in the Dividend and divide with the remaining Figures, as you see in these Examples.

$$\begin{array}{r} 210) 1716 (8 \\ \text{Rem. } 16 \end{array}$$

$$46100) 1357100 (29$$

$$\begin{array}{r} 92 \\ \hline 437 \\ \hline 414 \\ \hline \text{Rem. } 2300 \end{array}$$

P R O O F of Division.

The surest Way of proving Division is by Multiplication, and of proving Multiplication is by Division.

For if the Product be divided by either Factor, the Quotient will be the other, and nothing will remain if the Sum is right. And in Division if you multiply the Divisor by the Quotient (and add the Remainder thereto, if any) the Product will be the Dividend. Example in Page 17, we have multiplied 357948, by 1614, and the Product is 577728072 : Now for Proof, if we divide 577728072 by

by 1614, the Quotient will be 357948, and nothing will remain, which proves the Work to be right.

Or, if we divide 577728072, by 357948, the Quotient will be 1614, and nothing will remain.

Short Division.

Short Division is done by subtracting as you go on, and only set down the Remainder, and not the Products, as you may perceive by this Work.

$$1614) 577728072 \quad (357948$$

.....
9382

12828

15300

7747

12912

Rem.

0

If you have any Thing to do by Division, and you would perform the same by Multiplication, you may find a Factor that will do it, that is, if you divide Unity by the Divisor, that Quotient will be a Multiplicator, by which if you multiply your Dividend, this Product will be equal to the Quotient.

As for Example.

Suppose I had 1740 to be divided by 4, what Number is that by which if I multiply 1740, the Product shall be the same as if I had divided it by 4) 1740 (435

14

20

0

4) 100 (.25 Factor.

$$\begin{array}{r} 1740 \\ .25 \\ \hline 8700 \\ 3480 \end{array}$$

Product 435 The same with the Quotient

above.

Note, When 1 is divided by 4, there are two Cyphers added, for which Reason I omit two Cyphers in the Product.

Division by Logarithms.

Division by *Logarithms* is performed by Subtraction, for subtract the *Logarithm* of the Divisor, from the *Logarithm* of the Dividend, and the Remainder is equal to the *Logarithm* of the Quotient.

EXAMPLE I.

Let 132608 be divided by 28, what's the Quotient?

See the Work.

Dividend	-	-	132608	-	-	5.1225697
Divisor	-	-	28	-	-	1.4471580
Quotient	-	-	4736	-	-	3.6754117

EXAMPLE II.

Dividend	-	-	5625	-	-	3.7501226
Divisor	-	-	15	-	-	1.1760913
Quotient	-	=	375	-	-	2.5740313

CHAP. II.

Of REDUCTION.

DEFINITION.

REDUCTION is no Rule properly of itself, but depends wholly on *Multiplication* and *Division*. It is either Ascending or Descending: Ascending is performed by *Division*, and is when you would Reduce, or bring Things of a less Name, to Things of a greater, as Farthings into Pounds, &c. Descending is when you would Reduce, or bring Things of a greater, to Things of a lesser Name, as Pounds into Shillings, Pence, or Farthings, and this is always done by *Multiplication*.

QUESTION.

A Gentleman dying, left 410625 Pounds, to be equally divided amongst Twenty Nine Persons; I demand, what each must have to his Share?

See the Work,

$$29) 410625 \quad (14159 \text{ l.}$$

....

$$\begin{array}{r} 29 \\ \hline 120 \\ 116 \\ \hline 46 \\ 29 \\ \hline 172 \\ 145 \\ \hline 275 \\ 261 \end{array}$$

Rem. 14 Pounds.

$$\begin{array}{r} 29) 280 \quad (9 \text{ Shillings.} \\ 261 \end{array}$$

$$\begin{array}{r} 19 \\ \hline \text{Pence in a Shilling} \quad 12 \end{array}$$

$$29) 228 \quad (7 \text{ Pence.}$$

$$\begin{array}{r} 203 \\ \hline \text{Rem.} \quad 25 \text{ Pence.} \end{array}$$

$$\begin{array}{r} 4 \\ \hline \text{Farthings in a Penny} \end{array}$$

$$29) 100 \quad (3\frac{13}{29} \text{ Farthings.}$$

$$\begin{array}{r} 87 \\ \hline \text{Rem.} \quad 13 \text{ Farthings.} \end{array}$$

Answer 14159 l. 9 s. 7 d. $3\frac{13}{29}$ q.

In reducing of great Things into lesser, multiply by the Denominate Parts, and add in the odd Parts if there be any, and the last Product is the Answer.

EXAMPLE.

In 95 l. 17 s. 2 d. 1 q. how many Farthings?

Answer 92025 Farthings.

D

In

In 92025 Farthings, how many Pence, Shillings and Pounds?

	12	2 0	l.	s.	d.	q.				
4)	92025	(	23006	(	19117	(	95	17	2	1
					1					
	1		2		17					

How many Pounds are there in 527 Moidores? Multiply by 27, and divide by 20, and the Answer will be 711 *l.* 9 *s.*

In 711 *l.* 9 *s.* how many Moidores? Multiply by 20, and divide by 27, and the Answer you will find to be 527 Moidores.

Quest. 3. In 627 *l.* 15 *s.* 6 *d.* how many Pieces of 16 *d.* of $14\frac{1}{2}$, of $12\frac{1}{4}$, of $10\frac{3}{4}$, of $8\frac{1}{2}$, of $5\frac{3}{4}$, and of $1\frac{1}{4}$, and of each an equal Number? Add all the Particulars together, and they make 69 Pence, and Reduce the Money into Pence, are 150666 for a Dividend, the Quotient is the Answer $2183\frac{3}{8}$ Pieces.

Note, In Things of this Nature, the Divisor and Dividend must always be of one name.

Quest. 4. In 1580 Ells *Flemish*, how many Ells *English*?

Operation.

	1580	
	3	
5)	4740	(948 Ells <i>English</i>
	...	

Quest. 5. How many Miles, Furlongs, Poles, Yards, Feet, Inches, and Barley Corns will reach round the World, allowing *Norwood's* Measure of $69\frac{1}{2}$ Miles, to one Degree of a great Circle.

Answer, Miles 25020, Furlongs 200160, Poles 8006400, Yards, 44035200, Feet 132105600, Inches 1585267200, Barley Corns 475580600.

Quest.

Quest 6. How many square Yards are there in four square Miles.

Answer, 49561600 square Yards.

Of TIME.

From the Creation of the World, to this present Year 1739, by the best of Prophane History it is reckoned 5688 Years, how many Minutes are there in the same ?

See the Work.

	<i>d.</i>	<i>b.</i>
One Year is	365	6
	24	
	1466	
	730	
Hours	8766	in one Year.
	60	
Minutes	525960	in one Year
Years	5688	
	4207680	
	4207680	
	3155760	
	2629800	
Answer	2991660480	Minutes.

Quest 7. How many times doth a Clock that strikes Hours, Halves and Quarters, strike in a Year ?

Hence, because it strikes Quarters, that is ten times in one Hour, exclusive of the Hours, amounts to 240 times in 24 Hours, and the times it strikes in 24 Hours exclusive of the Quarters, are 156 added to 240, makes 396 times the Clock strikes in 24 Hours one Day, which multiply'd by $365\frac{1}{4}$, will produce 144639 times that the Clock strikes in a Year, But $396 \times 365 = 144540 + 81 = 144621$ the true Answer.

Quest. 8, If the Difference of the Times of the Lunations in 19 Years be 1 b. 28' 15", how long will it be before it comes to make one Day?

Answer 310 $\frac{150}{5293}$ Years.

Quest. 9. I would put 100 Hoshheads of London Beer into 50 Wine Pipes, what must the Cask contain which receives the Difference?

Answer 4 $\frac{18}{34}$ Beer Measure.

CHAP. III.

Of PROGRESSION.

DEFINITION.

PROGRESSION is two fold, *viz.* Arithmetical, and Geometrical.

First, Arithmetical *Progression*, is when Numbers do proceed by equal Difference, either increasing or decreasing.

As $\begin{cases} 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. \\ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, \&c. \\ 40, 38, 36, 34, 32, 30, 28, 26, 24, 22, 20. \end{cases}$

In the First of these, is a continual increase of 1; in the Second of 3; in the Third a decrease of 2.

If never so many Quantities are so proportional (as above) the double of the Mean is equal to the Sum of the two Extreams, as 1, 2, 3, the double of the 2, the mean, or middle Term, is equal to the Sum of 1 and 3 the two Extreams, *viz.* 4.

To find the Sum.

The Sum of any Arithmetical Proportion, the Sum of all the Terms may be found if you multiply the Sum of the first and last by half the Number of Places, or the Number Places, by half the Sum of the first and last Terms,

Terms, the Product will be equal to the Sum of all the Terms.

So in the First Rank of Numbers, above the Sum of 1 and 12 is 13, multiply'd by 6 half Number of Places, the Product 78 is equal to the Sum of all the Terms. And so often doth a Clock strike in 12 Hours.

Four Numbers in an Arithmetical Proportion, the Sum of the two Means, is equal to the Sum of the two Extreams. So 2, 5, 8, 11, the Sum of 5 and 8 is 13, and the Sum of 2 and 11 is 13.

A QUESTION.

There was a Wager laid between a Taylor and a Gentleman's Servant, the Taylor was to gather 100 Stones laid a Yard assunder, and the other was to run two Miles and a half reckoned, from the Place where the Basket was to stand, and back again. (I was an Eye Witness of the Performance). The Taylor won the Wager. Now let us see how many Miles the Taylor Run, in gathering up the Stones.

*An Hundred Stones right in a Line,
Exact a Yard assunder,
How many Miles then doth he go,
That gathers up that Number.*

See the Work.

Sum of the first and last Terms	=	101
Half Number of Places	- - - -	50
Sum of all the Terms	- - - -	<u>5050</u>
		2

Doubled are the Number of Yards = 10100

The Place where the Race was, they have 8 Yards to the Pole, so that it amounted to of their Miles 3, Furlongs 7, Poles 22, and 4 Yards. But $5\frac{1}{2}$ Yards to the Pole, it amounted to 5 M. 5 F. 36 P. 2 Y.

D 3

Quest.

Quest. 2. By having the first Term and Common Excess, to find the last Term of any Progression, tho' never so large.

R U L E.

Multiply the Number of Places less by one, by the Common Excess, and to the Product add the first Term, this Sum is equal to the last Term sought.

E X A M P L E.

In a Series of Numbers, where the first Term is 2, the Common Excess 3, and the Number of Places 11, what's the last Term ?

Operation.

Number of Places less 1	=	10
Common Excess - - - - -		3
Product - - - - -		30
First Term add - - - - -		2
Last Term - - - - -		32

Quest. 3. The first and last Terms and Total given, to find the Number of Places.

R U L E.

Divide the Sum of all the Terms by the Sum of the first and last Terms, the Quotient is equal to half the Number of Places.

E X A M P L E.

Let the Sum of all the Terms be 187, and the Sum of the first and last Terms 34, what's the Number of Places.

Answer 11.

Quest.

Quest. 4. The first and last Terms, and Number of Places given, to find the common Excess.

R U L E.

From the last Number take the first, the Remainder is a Dividend, which divide by the Number of Places less by 1, the Quotient is the Common ?

E X A M P L E.

Let the last Term be 32, and the first 2, the Number of Places 11. What's the common Excess ?

Answer 3.

Quest. 2. Progression Geometrical, or Geometrical Proportion continued, is when Numbers proceed by equal Proportion, or Ratio's, that is, according to one common Multiplier, or Exponent of the common Ratio, whether increasing or decreasing.

As, 2, 6, 18, 54, 162, 486, &c.

Here 3 is the common Excess or Multiplier.

Three Numbers in a Geometrical Proportion, the Square of the Mean, is equal to the Rect-angle of the two Extreams. So 2, 4, 8, being given, the Square 4 is equal to 16, equal to the Product of 2 by 8.

Four Numbers in a Geometrical Proportion, the Rect-angle, or Product of the two Means, is equal to the Rect-angle, or Product of the two Extreams.

E X A M P L E.

Let 6, 24, 96, 384 be given, the Product of 24 by 96, is equal to the Product of 6 by 384. viz. 2304.

If over any Rank of Geometrical Numbers, you place a Series of Arithmetical ones, beginning with 0, the Addition and Subtraction of the Indexes, answer to the Multiplication and Division of the Numbers they stand over.

E X A M P L E.

EXAMPLE.

Indexes 0 1 2 3 4 5 6 7.

Numbers 1, 2, 4, 8, 16, 32, 64, 128.

For 2 added to 3 make 5, which is the Index of 32, equal to the Product of 4 by 8, also the Index of 2 and 4 is 6, and the Product of 4 by 16 is 64.

In this are two Cases, which we shall briefly set down for the Learner's Practice,

CASE I.

The common Excess and the Number of Places given, to find the last Number.

RULE.

Multiply the Number belonging to any one Place by itself, and that Product shall be the Number or Term belonging to twice so many Places wanting one Place, which Term multiply by the common Excess, gives the Term of the Progression that you seek.

EXAMPLE.

Let the Term or Number answering to the 33d Place, or Sum of all the Terms be sought, the common Excess being 2?

Here by observing well the Premises the following Work will be evident.

For First, You see the Farthings answering the 5th Nail are 16, which being squared are 256, the Farthings answering the 9th Nail, or Place, and so on, as you see here set down.

Farthings.

	Farthings.	Nails.
	1	1
	2	2
	4	3
	16	5
16 squared is equal to	256	9
	256	
	1536	
	1280	
	512	
256 squared is equal to	65536	17
	65536	
	393216	
	196608	
	237680	
	237680	
	393216	
65536 squared is	=	4195967296 33

Here you see that 65536, the Farthings answering the 17th Place, or Nail, being squared, gives the Farthings answering the 33d Place, and so on *ad infinitum*.

CASE II.

Given, the common Excess and the last Term, to find the Sum of all the Terms.

RULE.

Multiply the last Term by the common Excess, and from the Product take the first Term, divide the Remainder by the common Excess less by one, the Quotient is the Sum of all the Terms.

EXAMPLE.

What's the Sum of 1, 2, 4, 8, 16, 32.

Operation.

Operation.

Last Term	- - - -	32
Common Excess	- - - -	2
		64
First Term Subt.		1
		1) 63 (63

Answer 63.

By this Rule is solved the Question of buying a Horse having four Shoes, and in every Shoe eight Nails, at a Farthing the first Nail, and so doubling to the End of 32 Nails, by working according as has been taught above, you will find the Sum of all the Terms, or Farthings to be 4294967295, which reduced are 4473924 *l.* 5 *s.* 3 *d.* 3 *q.* and so much doth the Horse cost at that Price.

CHAP. IV.

The Single RULE of THREE
Direct.

DEFINITION.

THE Rule of Three, or the Rule of Proportion, is commonly called the *Golden Rule*, for it's excellent Use, it teaches to find a Fourth Number, which shall have the same Proportion to the Third, as the Second has to the First, or as the First is to the Second, so is the Third to the Fourth.

This Rule is known thus ; If more require more, or less require less, the Question belongs to this Rule :

How to State the Question.

The Demand must be always in the Third Place, the First must be of the same Name.

And

And the Answer to the Question will be of the same Name with th the Middle Term: The Question being thus Stated, multiply the Second and Third Terms together, and divide the Product by the First Term, the Quotient is the Answer.

EXAMPLE.

Quest. 1st. If 8 Yards of Cloth cost 16 *l.* 13 *s.* 4 *d.* what will 64 Yards cost at that Rate?

Note, Here the Demand lyeth in the 64, therefore it must possess the Third Place.

Yds.	<i>l.</i>	<i>s.</i>	<i>d.</i>	Yds.
If 8	— 16	13	4	— 64
	20			4000
	333			8)256000(32000
	12			
	4000			16
				0

12)	32000	(266	6	(133
				
	80	Rem.	6	<i>s.</i>
	80			
	80			
	Rem.	8	<i>d.</i>	

Answer 133 *l.* 6 *s.* 8 *d.*

Quest. 2. A Fruiterer bought 2001 Apples at three for a Penny, and he also buys 2001 more, at 2 for a Penny, which he mingles together, and sells them out at 5 for Two Pence; I demand, whether he gain'd or lost by the Bargain, and how much?

Answer he lost in the Whole 5 *s.* 6 *d.* 2 $\frac{1}{2}$ *q.*

Quest. 3. A Poulterer had 60 *l.* left him for a Legacy, which he laid out in Eggs on this wise, viz. he bought 30 *l.*

30 *l.* worth in at two for a Penny and sold them out at four for a Penny, he also laid out the other 30 *l.* in Eggs, and bought them in at four a Penny, and then out at two a Penny; now do I demand whether he gained or lost in the Whole, and how much?

Answer he gained 15 *l.*

Quest. 4. How many Ten Inch Tiles will serve to pave a Floor that is 18 Yards long, and 18 Yards broad?

Answer 4199 $\frac{4}{100}$.

Quest. 5. If one Pound of Nutmegs be in value to 50 Oranges, and 70 Oranges to 40 Lemons, and one Lemon worth three half Pence, what's the Price of one Pound of Nutmegs at that Rate?

Answer 3 *s.* 6 *d.* 1 $\frac{3}{4}$.

Quest. 6. If 6 Pounds of Pepper be worth 13 Pounds of Ginger, and 19 Pounds of Ginger be worth 4 $\frac{1}{4}$ of Cloves, and 10 Pounds of Cloves be worth 63 Pounds of Sugar, at 5 *d.* per Pound, what's the Value of 100 Weight of Pepper?

Operation.

Pepper 6 = 13 Ginger,

Ginger 19 = 4 $\frac{1}{4}$ Cloves.

Cloves 10 = 63 Sugar.

Sugar 1 = 5 Pence.

What Pence = 112 Pepper.

Answer 7 *l.* 2 *s.* 5 *d.* 3 $\frac{420}{1140} q.$

CHAP. V.

The Single RULE of THREE Indirect, or the backward RULE, commonly called Inverse.

DEFINITION.

IN this Rule there are always three Things given to find a fourth, that bears such proportion to the first, that the Second doth to the Third.

How

How to State the Question.

Here, the Demand must be in the third Place, the first and third of one Name, every Term brought into it's lowest Name, then the Product of the First and Second divided by the Third, gives the Answer, which is always of the same Name with the middle Term. And the Questions that belong to this Rule are always known thus ; if less require more, or more require less, the Question is indirect.

EXAMPLE.

Quest. 1. If for 20s. I have 7 C. 3 Q. 4 lib. carried 40 Miles, how much Weight may I have carried 30 Miles for the same Money ?

Here less requires more, that is, less Miles, requires more Weight.

Operation.

<i>M.</i>		<i>C.</i>	<i>Q.</i>	<i>lib.</i>		<i>M.</i>
If 40 :		7	3	4	::	30
		4				
		31				
		28				
		252				
		62				
		872				
		40	<i>lib.</i>		<i>C.</i>	<i>Q.</i>
		30)	34880	(1162 $\frac{20}{30}$	= 10	1
				14 $\frac{2}{3}$.		

Quest. 2. How many Yards of half Yard-wide will line a Cloak, which hath in it 5 Yards of 7 Quarters wide?
Answer 17 y. 2 q.

E

Quest.

Quest. 3. A Messenger makes a Journey in 8 Days when the Day is 10 Hours long, I would know in how many Days he will go the same when the Day is 15 Hours long? More requires Less.

Answer 5 D. 8 H.

CHAP. VI.

Of PLURAL PROPORTION, Or the Double Rule of Three Direct.

DEFINITION.

HERE are always five Numbers given to find a Sixth in proportion; in three of which there always lyeth a Supposition, and a Demand in the other two, and this may be done either by two single Rules of three, or else at one Operation by stating the whole five Numbers, as I shall shew in an Example.

Quest. 1. A Carrier receives 14 s. for the Carriage of one Hundred Weight 50 Miles, I demand what he ought to receive for the Carriage of 3 C. 1 Q. 18 lib. 20 Miles.

First, I shall work it at two Operations by the Single Rule of Three Direct: And here Note, it matters not whether I take the Weight or the Miles in the first Operation, for the Answer will be the same which ever be taken. First I shall take the Weight and work it thus,

	<i>lib.</i>	<i>s.</i>		<i>C.</i>	<i>Q.</i>	<i>lib.</i>
If	112	: 14 ::	3	1	18	
			4			
			13			
			28			
			112			
			27			
			382			
			14			
			1528			
			382			
	112)		5348	(47 s. 9 d.		

Secondly,

2dly, If 50 M. : 47 s. 9 d. :: 20 M.

$$\begin{array}{r} 12 \\ \hline 573 \\ 20 \end{array}$$

$$50 \overline{) 11460} (229 d. = 19 s. 1 d.$$

Answer, 19 s. 1 d. $0\frac{4}{5} q.$

Secondly, If we take the Miles first, then the fourth proportional Term will be $5\frac{3}{5} s.$ which must be the second Term in the second Work. Thus

$$\begin{array}{cccc} \text{lib.} & \text{s.} & \text{lib.} & \text{s.} & \text{d.} & \text{q.} \\ \text{If } 112 : 5\frac{3}{5} :: 382 : 19 & 1 & 0 \end{array}$$

Answer 19 s. 1 d. $0\frac{4\frac{3}{5}}{560} q.$

The last Question varied, which is a Proof of the Rule.

Every Question in this Rule may be varied, that is, it may be stated by putting what was before given, now in the Demand's Place. Thus,

If a Carrier receive 19 s. 1 d. $0\frac{4}{5} q.$ for the Carriage of 3 C. 1 Q. 18 lib. 20 Miles, what must he receive for the Carriage of one Hundred Weight 50 Miles?

Answer 14 s.

Both Questions are answered at one Operation as you see below.

$$\begin{array}{cccccc} \text{lib.} & \text{M.} & \text{s.} & \text{C.} & \text{Q.} & \text{lib.} & \text{M.} \\ \text{First, If } 112 : 50 : 14 :: 3 & 1 & : 18 : 20 \end{array}$$

Answer 19 s. 1 d. $0\frac{4}{5} q.$

$$\begin{array}{cccccc} \text{C.} & \text{Q.} & \text{lib.} & \text{M.} & \text{s.} & \text{d.} & \text{q.} & \text{lib.} & \text{M.} \\ \text{Secondly, If } 3 & 1 & 18 : 20 : 19 & 1 & 0\frac{4}{5} : 112 : 50 \end{array}$$

Answer 14 s.

Quest. 3. What's the Simple Interest of 2 Guineas for 76 Days, at 5 per Cent. per Annum?

$$\text{Answer } 5 d. 0 \frac{7216}{23000} \frac{00}{100} q.$$

E 2

CHAP.

C H A P. VII.

Of PLURAL PROPORTION,
or the Double RULE of THREE
Indirect.

D E F I N I T I O N.

HERE are always five Numbers given to find a Sixth in an inverted Proportion. And as in all the foregoing Rules, so here, all the Difficulty lyeth in stating the Question, and to know truly when a Question falls under this Rule, that they are such as require to find Men, Principal, Time, Weight, &c.

To state the Question.

This being known, you must observe that the three first Terms must be the Supposition, and the Demand in the two last Places, and that you put in the first Place the Work, the Price, the Interest, the Produce, &c. and the second Term must be Time, Principal, Weight, Miles, Men; the third Term must be of that Name you are seeking; the second and fourth of one, and the second and fifth of the same Name. These things being known, the Product of the second, third and fourth Terms, divided by the Product of the first and fifth Terms gives the Answer to the Question.

E X A M P L E,

Quest. 1. A Carrier receives 19 s. 1 d. $0\frac{4}{5}$ q. for the Carriage of 3 C. 1 Q. 18 lib. 20 Miles. I demand how much he must carry 50 Miles for 14 s.

State it thus,

s.	d.	q.	M.	C.	Q.	lib.	s.	M.
If 19	1	$0\frac{4}{5}$	: 20	: 3	1	18	: 14	: 50

Work

Of ARITHMETIC.

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Work according to the Directions, and the Answer will be 112 lib. = one Hundred Weight.

Note, Every Question in this Rule may be varied four several Ways, as you may perceive by what follows.

Quest. 2. A Carrier receives 14 s. for the Carriage of 112 lib. 50 Miles, I demand how much he must carry 20 Miles for 19 s. 1 d. $0\frac{4}{5}$ q.

State it thus,

s.	M.	lib.	s.	d.	q.	M.
If 14	50	112	19	1	$0\frac{4}{5}$	20
Answer 3 C. 1 Q. 18 lib.						

Quest. 3. A Carrier receives 19 s. 1 d. $0\frac{4}{5}$ q. for the Carriage of 3 C. 1 Q. 18 lib. 20 Miles, I demand how many Miles he may carry one Hundred Weight for 14 s.

State it thus,

s.	d.	q.	C.	Q.	lib.	M.	s.	lib.
If 19	1	$0\frac{4}{5}$	3	1	18	20	14	112.
Answer 50 Miles.								

Quest. 4. A Carrier receives 14 s. for the Carriage of one Hundred Weight 50 Miles, how many Miles must he carry 3 C. 1 Q. 18 lib. 20 Miles for 19 s. 1 d. $0\frac{4}{5}$ q.

State it thus,

s.	lib.	M.	s.	d.	q.	C.	Q.	lib.
If 14	112	50	19	1	$0\frac{4}{5}$	3	1	18
Answer 20 Miles.								

Quest. 5. Put out for 7 Months a Sum of Money, at 5 per Cent per Annum, and received for Interest 2 l. 17 s. $4\frac{1}{4}$ d. I demand what was the Sum lent ?

Answer 98 l. 6 s. 5 d. $0\frac{19200}{33600}$ q.
E 3

Quest.

Quest. 6. An Usurer receives 2 *l.* 17 *s.* 4 $\frac{1}{4}$ *d.* for the Interest of 98 *l.* 6 *s.* 5 *d.* 0 $\frac{19200}{36000}$ *q.* for 7 Months, I demand in what time 5 *l.* will gain 1 *C.* Principal?

Answer 12 Months.

I have given now the Ground and Foundation of Arithmetic through all Parts of the Golden Rule. I shall only touch upon the rest of the Rules in a few Questions, and be as brief as possible.

And first of Single Fellowship, or Fellowship without Time.

I. Single Fellowship.

THIS is a Rule amongst Merchants for Balancing Accompts.

The RULE,

As the whole Stock of all the Partners, is in Proportion to the whole Gain or Loss; so is each Man's particular Share in the Stock, to each Man's particular Share in the Gain or Loss.

Quest. 1. A, B and C bought a House, for 329 *l.* 15 *s.* 7 *d.* A paid one half so much as B, B paid one half so much as C, what did each Man pay?

Note, You may suppose paid any Sum at Pleasure. As suppose 12 *l.*

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
Answer	188	8	10	$3\frac{9}{21}$
	94	4	5	$1\frac{15}{21}$
	47	2	2	$2\frac{18}{21}$
Proof	329	15	7	0

Quest. 2. Suppose the Grandfather A, the Father B, the Son C, met together and spent 20 *s.* when the Reckoning came to be paid, the Grandfather would pay one Half, the Father one Third, the Son one Fourth, now I would know how much each must pay of the Reckoning?

	<i>s.</i>	<i>d.</i>	<i>q.</i>
Answer	A 9	2	$3\frac{2}{6}$
	B 6	1	$3\frac{10}{6}$
	C 4	7	$1\frac{14}{6}$
Proof	20	0	0

Quest.

Quest. 3. Four Gentlemen at the Tavern drinking till the Reckoning came to 20*s.* the Duke would pay one Third, the Earl one Fourth, the Knight one Fifth, and the 'Squire one Sixth Part, which Parts all together makes only 19*s.* now I would know what each must pay?

		<i>s.</i>	<i>d.</i>	<i>q.</i>
Answer	{ Duke	7	0	0 $\frac{192}{228}$
	{ Earl	5	3	0 $\frac{12}{19}$
	{ Knight	4	2	2 $\frac{2}{19}$
	{ 'Squire	3	6	0 $\frac{96}{228}$
Proof		20	0	0

II. Fellowship with Time, or the Double Rule.

R U L E.

MULTIPLY the particular Stocks of each Person by the Time of Continuance, and the Sum of the several Products, make the first Term in the Single Rule of Three Direct, the whole Gain or Loss the Second, and every Man's particular Stock, Multiplied by it's Time, the Third.

Quest. 1. Three Merchants, *A, B* and *C*, make a Stock of 2840*l.* of which *A* lays in 899*l.* for 4 Months, *B* 1681*l.* for 7 Months, and *C* 260 for 12 Months, with this they gain 1420*l.* what is each Man's Share?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
Answer	{ <i>A</i>	276	5	5 05940
	{ <i>B</i>	904	0	6 27122
	{ <i>C</i>	239	14	0 118483

Quest. 2. A Ship's Company take a Prize of 718*l.* 15*s.* which is to be divided amongst them according to their Pay and Time they have been on Board; the Officers and Midshipmen 6 Months, and the Sailors 4 Months, the Officers one with another 40*s.* per Month, the Midshipmen

Men 30 Shillings per Month, and the Sailors 24 Shillings per Month, there were 6 Officers, 13 Midshipmen, and 93 Sailors, what must each have to his Share?

		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
Answer	{ Officers	104	12	9	2 ¹²⁶⁴⁸
	{ Midshipm.	127	10	7	1 ⁶³⁴⁸
	{ Sailors	486	11	6	3 ⁷³⁸⁰ <u>13188</u>
Proof =		718	15	0	0 (2)

III. Of Alligation Medial.

DEFINITION.

TEACHES to find a mean Price of several Simples propounded.

Quest. 1. Admit there were melted and mixed together two kinds of Silver, one worth 5 *s.* 3 *d.* and the other 4 *s.* 9 *d.* per Ounce, and there were 8 Ounces of the Former and 17 of the Latter, what's the Value of one Ounce of that Mixture?

Answer 4 *s.* 10 *d.* 3¹⁷/₂₃ *q.*

Quest. 2. An Hostler mix'd Provender for his Horses, viz. 18 Bushels of Oates, at 2 *s.* 1 *d.* per Bushel, with 16 Bushels of Beans, at 4 *s.* 9 *d.* per Bushel, and 13 Bushels of Malt, at 3 *s.* 10 *d.* per Bushel, now do I demand what a Peck of this Mixture is worth?

Answer 10 *d.*

IV. Of Alligation Alternate.

DEFINITION.

THIS Rule hath it's Name from binding, tying, or uniting many Particulars into one Mass or Sum; for here are always given the particular Prices of several Simples,

Simples, and thereby we discover such Quantities of those Simples, as being mingled together, shall bear a certain Rate propounded.

Quest. A Vintner hath four Sorts of Wines of several Prices, viz. one of 4 s. per Gallon, another of 4 s. 6 d. another at 5 s. 2 d. and another of 6 s. per Gallon, of which Wines he would have a Mixture made of 60 Gallons, and to bear a Price of 5 s. per Gallon, I would know how much must be taken of each Sort?

Note, You must always be careful to link two together, whereof one is greater, and the other lesser than the common Price.

R U L E:

Then as the Sum of the Differences, is to the Sum proposed, so is each particular Difference, to each particular Quantity. And every Question may be linked several Ways as appears by these Examples.

60 d.	{	48 54 62 72	}	12 2 6 12	<i>Differences</i>
				Sum 32	

As 32 :	60 ::	{	12 : 22 $\frac{16}{32}$ 2 : 3 $\frac{24}{32}$ 6 : 11 $\frac{8}{32}$ 12 : 22 $\frac{16}{32}$	}
---------	-------	---	--	---

Proof = 60

A Second Way of Linking.

60 d.	{	48 54 62 72	}	12 2 12 6
				Sum 32

$$\text{As } 32 : 60 :: \left\{ \begin{array}{l} 12 : 22\frac{16}{32} \\ 2 : 3\frac{16}{32} \\ 12 : 22\frac{16}{32} \\ 6 : 11\frac{8}{32} \end{array} \right.$$

A Third Way of Linking.

$$\begin{array}{rcl} 60 d. & \left\{ \begin{array}{l} 48 \\ 54 \\ 62 \\ 72 \end{array} \right. & \begin{array}{r|l} 12 & 12 \\ 2, 12 & 14 \\ 6 & 6 \\ 6, 12 & 18 \\ \hline \text{Sum} & 50 \end{array} \end{array}$$

$$\text{As } 50 : 60 :: \left\{ \begin{array}{l} 12 : 14\frac{20}{50} \\ 14 : 16\frac{40}{50} \\ 6 : 7\frac{10}{50} \\ 18 : 21\frac{30}{50} \end{array} \right.$$

Sum 60 Proof.

By the Work above it is plain, that in the first Way of Linking, there must be $22\frac{1}{2}$ of 4s. *per* Gallon, in the Second Way there must be $22\frac{1}{2}$ of the same Sort; but in the Third Way, there must be of that Sort $14\frac{2}{5}$ Gallons, and so of the rest.

V. Of Vulgar FRACTIONS.

DEFINITION.

THE Word Fraction signifies a Breaking, or Breach of any entire Thing into Parts; now the Unite, or entire Number which is to be broken, may be any thing, as one Pound, one Yard, one Year, &c.

This Table will explain the several Names of Fractions, better than many Words.

Numerator

Numerator	$\frac{1}{4}$	and	$\frac{3}{5}$
Denominator	4		5

Proper Fractions are, $\frac{2}{3}$ or $\frac{5}{6}$ or $\frac{1}{2}$

Improper Fractions are, $\frac{3}{3}$ or $\frac{7}{5}$ or $\frac{14}{11}$

Single Fractions are $\frac{4}{5}$ or $\frac{11}{5}$ or $\frac{6}{11}$ or $\frac{3}{7}$

Compound Fractions are, $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{11}{13}$

Mixt Numbers are, $8\frac{2}{3}$ or $12\frac{1}{5}$ or $1\frac{1}{3}$ or $11\frac{1}{6}$

Now before we can proceed in the known Rules, we must first know how to reduce a Fraction into it's known Quantity, according to the Nature of the Question.

1. And if the Fraction is mixt, multiply the whole Number by the Denominator, and to the Product add the Numerator, that Product is a new Numerator, which place over the Denominator and 'tis done.

2. If you would reduce a whole Number into an Improper Fraction, multiply the given Number by the intended Denominator, and place the Product for a Numerator over the intended Denominator.

3. When you have an Improper Fraction to reduce into a Whole or mixt Number, divide the Numerator by the Denominator, the Quotient is the whole Number, and if any thing remain, place it for a Numerator over the Divisor, or Denominator.

4. If you would reduce Fractions into their lowest Term.

First, If they end in Cyphers cut them off, and take the half of the significant Figures as long as you can.

Secondly, If they both end in 5, or one 5 and the other in 0, then divide them severally by 5, and the two Quotients shall be equal to the Fraction given.

Thirdly, But the best Way is to find a Common Measure, by dividing the Denominator by the Numerator, continually till nothing remain, and this last Quotient will be a Common Measure that will divide both Numerator and Denominator without any Remainder, and reduce the Fraction in it's lowest Term.

So

So if $\frac{246}{864}$ were the Fraction given to be brought into it's lowest Term, the Work will stand thus,

$$\begin{array}{r}
 246) \quad 864 \quad (3 \\
 \underline{738} \\
 126) \quad 246 \quad (1 \\
 \underline{126} \\
 120) \quad 126 \quad (1 \\
 \underline{120} \\
 6) \quad 120 \quad (20 \\
 \underline{} \\
 0
 \end{array}$$

6 is the common Measure.

$$\begin{array}{r}
 6) \quad 246 \quad (41 \\
 \underline{} \\
 6) \quad 864 \quad (146 \\
 \underline{}
 \end{array}$$

Answer $\frac{41}{146}$ in it's lowest Term:

5. When you have a compound Fraction to bring into a single Fraction, multiply all the Numerators together for a new Numerator, and all the Denominators together for a new Denominator. So $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{6}$ will be $\frac{10}{144}$ single.

Quest. 6. When you would find the Value of a Fraction in any known Measure, &c. multiply the Numerator of the given Fraction, by the Parts of the next inferiour Denomination, and divide the Product by the Denominator, the Quotient is it's Value in the same Parts you multiplied by. So if $\frac{5}{7}$ of a Pound Sterling were given to know it's Value, you will find it to be 14 s. 3 d. $1\frac{5}{7}$ q.

Also $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of 4 Shillings is 1 s.

And $\frac{3}{4}$ of $\frac{5}{8}$ of a Mile, you will find to be 3 Furlongs, 30 Poles.

And $\frac{2}{3}$ of $\frac{4}{5}$ of a Yard, is 2 q. $0\frac{8}{15}$ n.

And $\frac{13}{14}$ of an Ell *English*, is 4 q. $2\frac{7}{14}$ n.

And $\frac{11}{13}$ of a Week, is 5 d. 3 b. 12 l.

7. When you have Fractions of unequal Denominations, first bring them into their lowest Terms, then multiply all the Denominators together, for a common Denominator, then multiply continually the Numerator of each Fraction by all the Denominators except it's own; and

and that will give new Numerators to be set over the common Denominator.

Reduce $\frac{2}{3}$ and $\frac{4}{7}$ to a common Denominator.

Answer $\frac{14}{21}$ and $\frac{12}{21}$.

8. When you have Fractions to be reduced from one Denomination to another, either ascending, or descending.

1. When a Fraction is brought from a lesser to a greater Denomination, make of it a Compound Fraction, by comparing it with the intermediate Denominations between it, and that you would have it reduced to, then by the 5 hereof, bring it into a single Fraction, and 'tis done.

EXAMPLE.

What Part of a Pound Sterling is $\frac{1}{4}$ of a Penny?

First make of it a Compound Fraction, thus $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$, and by the 5 hereof it is, $\frac{1}{960}$ of a Pound Sterling.

So $\frac{1}{2}$ of a Penny is $\frac{1}{24}$ of a Shilling.

What part of a Pound is 8 d?

Answer, $\frac{1}{30}$.

What part of a Crown is $\frac{3}{4}$ of a Penny?

Answer, $\frac{3}{240}$.

2. But if you would reduce a Fraction of a greater, to a Fraction of a lesser Name, multiply the Numerator by the Parts contained in the several Denominations betwixt it; and that you would reduce it to, and place the last Product over the Denominator of the given Fraction.

EXAMPLE.

Reduce the $\frac{1}{2}$ of a Pound to the Fraction of a Penny?

Answer, $\frac{240}{1}$.

VI. Of ADDITION.

BEFORE Fractions can be added they must be reduced to a common Denominator, if they have not such.

F

Quest.

Quest. 1. What's the Sum of $\frac{1}{2}$ and $\frac{2}{3}$ and $\frac{3}{4}$ and $\frac{1}{4}$ of a Shilling?

Answer, 2 s. 2 d.

Quest. 2. What's the Sum of $\frac{1}{4}$ of a Farthing, $\frac{3}{4}$ of a Penny, $\frac{1}{4}$ of a Shilling, and $\frac{3}{4}$ of a Pound?

First by reducing them to the Fraction of one Farthing.
 $\frac{1}{4} q.$ $\frac{3}{4} d.$ $\frac{1}{4} s.$ $\frac{3}{4} l.$

First. The Numerator 3 of the Pound is multiplied by the Farthings in a Pound, $960 = 2880$.

Secondly, The Numerator of the Shillings is multiplied by the Farthings in a Shilling $48 = 48 qrs.$

Lastly, The Numerator 3 of the Farthings, is multiplied by 4, the Farthings in a Penny, and it is $\frac{12}{4} qrs.$ $\frac{1}{4}$; and now because they have all one Denominator, the Work stands thus.

Numerator.	{	$\begin{array}{r} 2880 \\ 48 \\ 12 \\ 1 \end{array}$
		$\begin{array}{r} 4 \quad 12 \quad 3 \\ 4) 2941 \quad (735 \quad (183 \quad (15 \\ \underline{} \quad \underline{} \quad \underline{} \quad \underline{} \\ 1 \quad 3 \quad 3 \end{array}$

Answer 15 s. 3 d. $3\frac{1}{4} q.$

Secondly, I shall give the Solution by reducing them to the Fraction of a Pound, by making of them compound Fractions; thus,

$$\left. \begin{array}{l} \frac{3}{4} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} \text{ Single } \frac{3}{960} \\ \frac{1}{4} \text{ of } \frac{1}{20} \text{ Single } \frac{1}{80} \\ \frac{3}{4} \text{ is } = - - - \frac{3}{4} \end{array} \right\} \text{ Pound.}$$

Being reduced to a Common Denominator will be
 $\frac{360}{307200} \quad \frac{3840}{307200} \quad \frac{230400}{307200}$

Answer $\frac{235200}{307200} = 15 s. 2 d. 3\frac{1}{4} q.$

Quest. 3. What is the Sum of $\frac{2}{3}$ of a Minute, $\frac{4}{5}$ of an Hour, $\frac{1}{6}$ of a Day, $\frac{7}{8}$ of a Week, $\frac{3}{4}$ of a Month, and $\frac{8}{9}$ of a Year; supposing the Year to contain only 13 Months?

Answer 12 m. ow. 6 d. 13 b. 8' 40".

VII. Of SUBTRACTION.

IF the Fractions have unequal Denominators, they must be reduced to equal Denominators as has been taught above.

What's the Difference between $\frac{9}{16}$ and $\frac{5}{16}$?

Answer $\frac{4}{16} = \frac{1}{4}$.

What's the Difference between $\frac{3}{4}$ and $\frac{5}{7}$ of any thing?

Answer $\frac{1}{28}$.

What's the Difference between $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{1}{2}$ of $\frac{5}{8}$ of a Pound Sterling?

Answer $\frac{3}{16} = 3s. 9d.$

What's the Difference between 3 and $5\frac{1}{2}$?

Answer $2\frac{1}{2}$.

What's the Difference between 1 and $\frac{3}{4}$?

Answer $\frac{1}{4}$.

What's the Difference between $14\frac{3}{4}$ and $29\frac{7}{8}$?

Answer $14\frac{3}{4}$.

VIII. Of MULTIPLICATION.

IF the Fractions be single, multiply the Numerators together for a new Numerator, and the Denominators together for a new Denominator.

2 If the Fractions are mixt Numbers, reduce them to Improper Fractions.

3 If they are Compound, reduce 'em into single Fractions.

Quest. 1. What's the Product of $\frac{5}{7}$ by $\frac{9}{11}$?

Answer $1\frac{1}{4}$.

Quest. 2. What's the Product of $\frac{1}{2}$ by $2\frac{1}{2}$?

Answer $\frac{5}{2}$.

Quest. 3. What's the Product of $\frac{2}{7}$ by 12?

Answer $3\frac{2}{7}$.

Quest. 4. What's the Product of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{4}$ by $\frac{3}{4}$?

Answer $\frac{1}{16}$.

Quest. 5. What's the Product of $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ by $7\frac{3}{4}$?

Answer $\frac{31}{48}$.

Quest. 6. What's the Product of $5\frac{1}{2}$ by 3?

Answer $16\frac{1}{2}$.

Quest. 7. What's the Product of $\frac{2}{3}$ of a Pound multiplied by $\frac{3}{4}$ of a Pound?

Answer $\frac{1}{2} = 10s$.

Quest. 8. What's the Square of 3s. 11d. a Pound the Integer?

Answer 9d. $0\frac{18816000}{2304000}q$.

But if a Shilling be put for the Integer, the Answer will be 15s. 4d. $0\frac{48}{144}q$.

IX. Of Division of Vulgar Fractions.

WHAT has been said in the last Article of reducing of Fractions, must be observed here.

Quest. 1. What's the Quotient of $\frac{2}{3}$ divided by $\frac{1}{4}$?

R U L E.

Multiply the Numerator of the Divisor by the Denominator of the Dividend, and the Product is a new Denominator; multiply the Denominator of the Divisor by the Numerator of the Dividend, the Product is a new Numerator. See the Work.

$$\frac{2}{3})\frac{16}{15}=1\frac{1}{15}$$

Quest. 2. What's the Quotient of $\frac{5}{8}$ divided by $2\frac{2}{3}$?

$$\frac{12}{3})\frac{5}{8}(\frac{5}{24}$$

Quest. 3. What's the Quotient of $\frac{3}{4}$ divided by 1?

$$\frac{1}{1})\frac{3}{4}(\frac{3}{4}$$

Note, As in whole Numbers, so here you see Unity neither multiplies nor divides.

Quest. 4. What's the Quotient of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ divided by $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{5}{8}$?

$$\frac{120}{120})\frac{3}{2}(\frac{360}{960}=\frac{3}{8}$$

Quest.

Quest. 5. What's the Quotient of $\frac{2}{3}$ of $\frac{4}{5}$ divided by 7 ?

$$\frac{7}{33} \frac{20}{245} = \frac{27}{70}$$

Quest. 6. What's the Quotient of $\frac{3}{5}$ of $\frac{7}{8}$ of $\frac{5}{6}$ divided by $\frac{5}{8}$?

$$\frac{5}{216} \frac{45}{1050} = \frac{27}{70}$$

Quest. 7. What's the Quotient of $4 \frac{3}{4}$ divided by $3 \frac{3}{4}$?

$$\frac{11}{3} \frac{19}{44} = 1 \frac{1}{4}$$

Quest. 8. What's the Quotient of $8 \frac{1}{2}$ divided by $\frac{2}{3}$ of $\frac{1}{2}$?

$$\frac{6}{35} \frac{17}{2} \frac{395}{12} = 49 \frac{7}{2}$$

Quest. 9. What's the Quotient of $9 \frac{5}{11}$ divided by 15 ?

$$\frac{15}{11} \frac{104}{11} \frac{104}{105} = 21 \frac{1}{3}$$

Quest. 10. What's the Quotient of 16 divided by $\frac{3}{4}$?

$$\frac{3}{1} \frac{16}{3} = 21 \frac{1}{3}$$

Quest. 11. What's the Quotient of 20 divided by $\frac{2}{3}$ of $\frac{1}{6}$?

$$\frac{2}{18} \frac{20}{1} \frac{360}{2} = 180$$

Quest. 12. What's the Quotient of 30 divided by $5 \frac{1}{2}$?

$$\frac{11}{2} \frac{30}{11} \frac{60}{11} = 5 \frac{5}{11}$$

X. The Single Rule of Three Direct in Vulgar Fractions.

QUESTION I.

IF $12 \frac{1}{2}$ Yards of Cloth cost 15 s. 9 d. what will 48 Yards cost at that Rate? A Shilling the Integer.

State it thus,

$$\begin{array}{rclcl} \text{If } \frac{25}{2} & : & \frac{63}{4} & :: & \frac{193}{4} : \frac{24318}{400} \\ \text{Answer } 3 \text{ l. } 0 \text{ s. } 9 \text{ d. } & & 2 \frac{64}{400} & & \end{array}$$

F 3

The

The same Question answered by putting a Pound for the Integer.

State it thus,

$$\text{If } \frac{2}{3} : \frac{252}{320} :: \frac{193}{4} : \frac{97272}{32000}$$

Answer 3 *l.* 0 *s.* 9 *d.* 2 $\frac{5120}{32000}$ *q.*

Quest. 2. If $\frac{1}{4}$ of a Yard of Cloth cost $\frac{2}{3}$ of a Pound, what will $\frac{3}{5}$ of an *English* Ell cost?

Note, The Yard must be reduced to the Fraction of an Ell, or else the Ell to the Fraction of a Yard; by the first it will stand thus,

$\frac{3}{4}$ *y.* is $\frac{1}{4}$ of $\frac{3}{5}$ of an Ell. Single $\frac{4}{20}$ Ell.

$$\text{If } \frac{4}{20} : \frac{2}{3} :: \frac{3}{5} : \frac{120}{20} = 2$$

Answer 2 *l.*

Quest. 3. If an Ounce of Gold be worth 3 *l.* 18 *s.* 10 *d.* how much may I buy for $3\frac{13}{20}$ *l.*

Answer 18 *pw.* $10\frac{2264}{744}$ *gr.*

Quest. 4. If $1\frac{1}{2}$ Herring cost $1\frac{1}{2}$ *d.* how many may I buy for 11 *d.*

Answer 11.

For as the second Term is equal to the first, so will the fourth be to the third.

XI. Rules of PRACTICE.

DEFINITION.

PRACTICE is a Rule, which expeditiously answers Questions in the Rule of Three, when the first Term is Unity or 1: And for the Reader's better Improvement, he must be sure to get the following Table by Heart.

A Table of Aliquot Parts:

Even Parts of a Pound.	Fractional Parts.	Uneven Parts of a Pound.	Even Parts.	Fractional Parts.
s. d.				
10 0	$\frac{1}{2}$	19	10 5 4	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$
6 8	$\frac{1}{3}$ $\frac{1}{4}$	18	10 4 4	$\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{5}$
5 0	$\frac{1}{4}$	17	10 5 2	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{10}$
4 0	$\frac{1}{5}$ $\frac{1}{6}$	16	10 4 2	$\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{10}$
3 4	$\frac{1}{6}$	15	10 5 0	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$
2 6	$\frac{1}{8}$ $\frac{1}{10}$	14	10 4 0	$\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{5}$
2 0	$\frac{1}{10}$	13	4 4 5	$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{4}$
1 8	$\frac{1}{12}$ $\frac{1}{16}$	12	10 2 0	$\frac{1}{2}$ $\frac{1}{10}$ $\frac{1}{10}$
1 3	$\frac{1}{16}$	11	5 4 2	$\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$
1 0	$\frac{1}{20}$ $\frac{1}{24}$	9	5 4 0	$\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{5}$
0 10	$\frac{1}{24}$	8	4 4 0	$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{10}$
0 8	$\frac{1}{30}$ $\frac{1}{32}$	7	5 2 0	$\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{10}$
0 0	$\frac{1}{32}$	6	4 2 0	$\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{10}$
0 7 $\frac{1}{2}$	$\frac{1}{40}$	3	2 1 0	$\frac{1}{10}$ $\frac{1}{20}$
0 6				

Even Parts of a Shilling.	Even Parts of a Pound.	Even Parts of a Pound.	Even Parts of a Shilling.	Even Parts of a Penny.
d.		q.	q.	q.
6 is $\frac{1}{2}$	6 $\frac{1}{40}$	1 $\frac{1}{960}$	1 $\frac{1}{48}$	1 $\frac{1}{4}$
4 $\frac{1}{3}$	4 $\frac{1}{80}$	2 $\frac{1}{480}$	2 $\frac{1}{24}$	2 $\frac{1}{2}$
3 $\frac{1}{4}$	3 $\frac{1}{60}$	3 $\frac{1}{320}$	3 $\frac{1}{16}$	3 $\frac{3}{4}$
2 $\frac{1}{6}$	2 $\frac{1}{120}$			
1 $\frac{1}{8}$	1 $\frac{1}{80}$			
1 $\frac{1}{12}$	1 $\frac{1}{240}$			

Note, After Division, if any thing remain it is of the same Name with your Divisor, as in the first Example following, at 3 *qrs.* per Pound, we divide by 16, the Quotient is 53 Shillings, and there remains 6, that is 6*d.* 3 Farthings, or $4\frac{1}{2}d.$ the like is to be observed of any other.

$\begin{array}{r l} \frac{1}{16} & 854 \text{ at } 3q. \text{ per } lib. \\ \frac{3}{30} & 53 \text{ s.} \\ & l. \quad s. \quad d. \\ \hline \text{Answer} & 2 \quad 13 \quad 4\frac{1}{2} \end{array}$	$\begin{array}{r l} \frac{1}{48} & 90\frac{1}{2} \text{ at } 5\frac{1}{4}d. \text{ per oz.} \\ & 1 \quad 2 \quad 7 \quad 2 \\ & 15 \quad 1 \quad 0 \\ & 1 \quad 10 \quad 2\frac{1}{2} \\ \hline \text{Answer} & 1 \quad 19 \quad 7 \quad 0\frac{1}{2} \end{array}$
---	--

$\begin{array}{r l} \frac{1}{4} \text{ and } \frac{1}{24} & 750 \text{ at } 3\frac{1}{2} \text{ per } C. \\ & \frac{1}{14} \quad 187 \quad 6 \\ & \frac{1}{24} \quad 31 \quad 3 \\ & \frac{1}{20} \quad 218 \quad 9 \\ & l. \quad s. \quad d. \\ \hline \text{Answer} & 10 \quad 18 \quad 9 \end{array}$	$\begin{array}{r l} \frac{1}{4} & 414\frac{1}{2} \text{ at } 9\frac{1}{2}d. \text{ per } yd. \\ & \frac{1}{2} \quad 5 \quad 3 \quad 7\frac{1}{2} \\ & \frac{1}{24} \quad 10 \quad 7 \quad 3 \\ & 17 \quad 3\frac{1}{4} \\ \hline \text{Answer} & 16 \quad 8 \quad 1\frac{3}{4} \end{array}$
--	---

When the given Price is 2 *s.* double the Figure in the Unites Place for Shillings, and the rest are Pounds.

Quest. 5. 680 at $11\frac{1}{4}d.$ per Pound ?

Answer 3*l.* 17*s.* 6*d.*

Quest. 6. $576\frac{1}{2}$ at $13\frac{1}{4}d.$ per Yard ?

Answer 3*l.* 16*s.* 6*d.* $2\frac{1}{2}q.$

Quest. 7. $207\frac{1}{4}$ at 19*d.* per Day ?

Answer 16*l.* 8*s.* $1\frac{3}{4}d.$

Quest. 8. 863 at $22\frac{1}{2}d.$ per Gallon ?

Answer 80*l.* 18*s.* $1\frac{1}{2}d.$

Quest. 9. $845\frac{1}{2}$ at 2*s.* $3\frac{1}{4}d.$ per Ell ?

Answer 95*l.* 19*s.* 11*d.* $3\frac{1}{2}q.$

Quest. 10. $760\frac{3}{4}$ at 2*s.* $5\frac{1}{2}d.$ per Bushel ?

Answer 93*l.* 10*s.* 2*d.* $0\frac{1}{2}q.$

Quest. 11. 639 at 2*s.* $10\frac{3}{4}d.$ per Quart ?

Answer 92*l.* 10*s.* $5\frac{1}{2}d.$

Quest. 12. $707\frac{1}{2}$ at 3*s.* $2\frac{1}{4}d.$ per Foot ?

Answer 112*l.* 15*s.* 1*d.* $3\frac{1}{2}q.$

Here

*How to cast up Expences, the Price of Goods, &c. by
divers easy Rules.*

1. By knowing your weekly Expence, to know what
it comes to in the Year.

R U L E.

*The Farthings in each Week doth make appear,
The Shillings and the Pence spent in the Year.*

Quest. 1. If you pay 12qrs. or 3d. to the Poor, per
Week, what doth that come to in the Year?

Answer 12s. and 12d. = to 13s.

By knowing your daily Expence, to know what it
comes to in a Year.

R U L E.

*Number the Pennies of each Day's Expence,
So many Angels, Groats and Pence
You spend within the Year's Circumference.*

Quest. 2. If a Labouring Man Earn 9d. per Day,
what is that per Annum?

Solution.

			l.	s.	d.
9	{ Pounds	{	9	0	0
	{ Angels	{ is }	4	10	0
	{ Groats		0	3	0
	{ Pence		0	0	9
	Answer		13	13	9

Note, If the Daily Expence be Shillings and Pence,
reduce all into Pence, then Work as above.

3. By

3. By knowing your yearly Expence, to know what it costs per Day.

R U L E.

*The third Part of the Pounds you yearly have,
Yields Two Pence every Day to spend or save,*

Note, For every one that remains add Three Farthings; but this Rule will give a little too much in the Answer, however it may be of Service to some who do not understand Figures.

E X A M P L E.

If the yearly Rent of a House be 6*l*. what is that per Day?

Answer 4*d*. by the Rule, but in Truth 'tis only 3*d*. $3\frac{285}{63}$.

4. By knowing the Price of a Pound Weight of any Comodity in Farthings, to know the Price of the great Hundred, or 112*lib*.

R U L E.

*For every Farthing that one Pound doth cost,
Reckon Two Shillings and a Great.*

E X A M P L E.

At 7 *qrs*. per Pound, what 112 *lib*?

Answer 16*s*. 4*d*.

5. Another Rule by Half Pence, to know what 112 Pounds comes to.

R U L E.

*As many Half Pence that each Pound doth cost,
So many Crowns the Hundred, bating so many Groats.*
E X A M P L E.

EXAMPLE.

At $3\frac{1}{2}d.$ per Pound, what cost 112 *lib.*:

Solution.

$$3\frac{1}{2} = 7 \text{ Half-pence.}$$

5

$$35 s. = 1l. 15s. \text{ the Answer.}$$

6. Another Rule by Half-pence.

*Each Half-penny a Pound that's sold or bought,
Yields four times Shillings, twice as many Groats.*

EXAMPLE.

At $8\frac{1}{2}d.$ per Pound, what cost 112 *lib.*

Solution.

$$8\frac{1}{2}d. = 17 \text{ Half-pence}$$

17

4

2

68 Shillings

34 Groats = 11s. 4d.

Add

11 : 4d.

20) 79 (3l. 19s. 4d. the Answer.

7. By knowing the Price of 112 Pounds, in Pounds and Shillings to know what one Pound cost.

RULE.

Take half the Number of Shillings that 112 *lib.* cost, and as often as 7 is contained in that half, take so many Farthings from that half Sum, and what is left are Farthings, the Price of one Pound.

EXAMPLE.

EXAMPLE.

If 112 *lib.* cost 6*l.* 1*s.* what cost 1 *lib.*

Solution.

$$\begin{array}{r}
 6\text{ l. } 1\text{ s.} \\
 \underline{20} \\
 121 \\
 7 \overline{) 60} \text{ (8} \\
 \text{Quot. } 8 \\
 \text{Rem. } 52 \text{ Farthings} = 13\text{d}
 \end{array}$$

This generally gives too much.

XII. Of Barter.

DEFINITION.

BARTER among Merchants, is the Exchanging Wares for Wares, or one Comodity for another, and informs them so to proportion their Goods that neither may sustain Loss.

Quest. 1. Two Merchants, A and B Barter, A hath Nutmegs at 4*s.* 6*d.* per Pound, and B hath 246 Pounds of Pepper, at 2*s.* 4*d.* per Pound, I demand how much Nutmegs A must give B for his Pepper?

First find what the Pepper comes to thus,

$$\begin{array}{cccc}
 \text{lib.} & \text{d.} & \text{lib.} & \text{d.} \\
 \text{If } 1 & : 28 & :: 246 & : 6888
 \end{array}$$

$$\begin{array}{cccc}
 \text{d.} & \text{lib.} & \text{d.} & \text{lib.} \\
 \text{adly, If } 54 & : 1 & :: 6888 & : 127\frac{3}{4} = \frac{5}{8} \\
 \text{Answer } 127\frac{3}{8} \text{ lib. of Nutmegs.}
 \end{array}$$

Quest.

Of ARITHMETIC.

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Quest. 2. A hath $3\frac{1}{2}$ C. of Pepper, at $13\frac{1}{2}d.$ per Pound, B hath Ginger, at $15\frac{1}{4}d.$ per Pound, I demand how much Ginger must be delivered for the Pepper?

$$\begin{array}{cccccc} \text{lib.} & d. & C. & l. & s. & \\ \text{Say, If } 1 & : & 13\frac{1}{2} & :: & 3\frac{1}{2} & : 22 \quad 1 \end{array}$$

$$\begin{array}{cccccc} d. & lib. & l. & s. & lib. & \\ \text{zdly, } 15\frac{1}{4} & : & 1 & :: & 22 & 1 : 347\frac{1}{61} \\ \text{Answer } 347\frac{1}{61} \text{ lib.} \end{array}$$

Quest. 3. A hath 14 C. of Sugar at $6d.$ per Pound, for which B gives him 1 C. $3q.$ of Cinnamon, I demand how B rated the Cinnamon per Pound?

$$\begin{array}{cccccc} C. & q. & d. & C. & s. & \\ \text{If } 1 & 3 & : & 6 & :: & 14 : 4 \\ \text{Answer } 4 \text{ Shillings.} \end{array}$$

Quest. 4. A hath 4 Tuns of Brandy, worth $37l. 16s.$ per Tun, ready Money, but in Barter he hath $50l. 8s.$ per Tun, and B gives A 21 C. $2q. 11\frac{1}{3} \text{ lib.}$ of Ginger for his Brandy, I desire to know how B sold his Ginger in Barter, per Hundred, and how much it is worth in ready Money?

$$\begin{array}{cccccc} T. & l. & s. & T. & l. & s. \\ \text{If } 1 & : & 37 \quad 16 & :: & 4 & : 151 \quad 4 \\ \text{If } 1 & : & 50 \quad 8 & :: & 4 & : 201 \quad 12 \end{array}$$

$$\begin{array}{cccccc} C. & q. & lib. & l. & s. & C. & l. & s. & d. & q. \\ \text{If } 21 \quad 2 \quad 11\frac{1}{3} & : & 201 \quad 12 & :: & 1 & : & 9 \quad 6 \quad 7 \quad 3\frac{36}{7258} \end{array}$$

Lastly, To know how much it was worth in ready Money.

G

If

C. q. lib. l. s. C. l. s. d. q.
 If 21 2 11 $\frac{1}{3}$: 151 4 : : 1 : 6 19 11 3 $\frac{4}{7}$ $\frac{17}{58}$

Answer { It was worth } l. s. d. q.
 { in Barter. } 9 6 7 3 $\frac{36}{72}$ $\frac{74}{58}$
 { Worth ready }
 { Money. } 6 19 11 3 $\frac{45}{72}$ $\frac{70}{58}$

Quest. 5. A and B Barter, A hath 320 Dozen of Candles, at 4s. 6d. per Dozen, for which B gives him 30l. in Money, and the rest in Cotton, at 8d. per Pound, I demand how much Cotton he must give him more than the 30l.

D. d. D l. l. l.
 If 1 : 54 : : 320 : 72 — 30 = 42

Now say,

d. lib. l. C. q.
 If 8 : 1 : : 42 : 11 1 the Answer.

Quest. 6. A and B Barter, A hath 608 Yards of Broad Cloth, worth 14s. per Yard, for which B gives him 125l. 12s. ready Money, and 85C. 2q. 24lib. of Bees Wax, I do demand how he reckon'd his Wax per Hundred ?

y. s. y. l. s. l. s. l.
 If 1 : 14 : : 608 : 425 12 — 125 12 = 300

C. q. lib. l. C. l. s.
 Then if 85 2 24 : 300 : : 1 : 3 10

Quest. 7. A hath Sugar at 8d. per Pound, ready Money, but in Barter he requires 10d. per Pound, and B hath Thread at 2s. 3d. per Pound ready Money, at what Price ought B to deliver his Thread at per Pound, to be equal in Barter with A ?

If

d. d. d. s. a. q.
If 8 : 10 :: 27 : 2 9 3 the Answer.

Quest. 8. A hath Cheefe at 25s. per C. ready Money, but in Barter demands 32s. B delivered in Barter Linnen Cloth, at 18d. per Yard, what did the Cloth cost in ready Money?

s. s. d. s. d. q.
If 32 : 25 :: 18 : 1 2 0⁹⁶/₃₈₄

Quest. 9. A hath 6 Tuns of Wine, at 40l. per Tun, ready Money, and in Barter requires 44l. per Tun, besides he requires $\frac{1}{3}$ Part of his over Price in ready Money, B hath 50C. of Tobacco, at 3l. 1s. perC. ready Money, how ought he to sell it in Barter?

See the Work.

l.	l.	s.	d.		l.	s.	d.	l.	s.	d.
$\frac{1}{3}$ of 44	= 14	13	4	From	40	0	0	44	0	0
				Sub.	14	13	4	14	13	4
				Rem.	25	6	8	29	6	8

l.	s.	d.	l.	s.	d.	l.	s.	l.	s.	d.	q.
If 25	6	8	: 29	6	8	:: 3	10	: 4	1	0	3 ¹³⁶⁸ / ₈₆₇₂
Answer	4	1	0	3 ¹³⁶⁸ / ₈₆₇₂							per Hundred.

XIII. Of Rebate, or Discount.

DEFINITION.

REBATE, is the Difference between a certain Quantity of Money, due at a certain Day, and the present Value or Worth of it; or it is when one Man gives

gives another Man Credit for the Goods he sells, but the Debtor is willing to pay ready Money, with this Condition, that he may have lawful Interest for the Money he pays, and for so long Time as he pay it before it becomes due, and this amongst Tradesmen is called Rebate, or Discount.

How to State the Question.

First, See what the Interest of 100*l.* comes to for the Time demanded.

Secondly, Add that Interest to 100*l.* which must always be the First Term in the Rule of Three, and 100*l.* the Second, and the Sum to be rebated the Third.

Quest. 1. A sells to B a Watch for 20*l.* to be paid in 6 Months, but B is willing upon an after-agreement to pay present Money, upon Rebate, after 5*l.* per Cent. per Annum, Simple Interest, I demand the Sum paid and rebated?

$$\begin{array}{cccc} m. & l. & m. & l. & s. \\ \text{1st, If } 12 & : 5 & :: 6 & : 2 & 10 \end{array}$$

$$\begin{array}{ccccccc} l. & s. & l. & l. & s. & d. & q. \\ \text{2dly, If } 102 & 10 & : 100 & :: 20 & : 19 & 10 & 2 \quad 3\frac{1450}{1030} \end{array}$$

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
Sum agreed for	20	0	0	0
The present Money	19	10	2	$3\frac{1450}{1030}$
Sum rebated	0	9	9	$0\frac{600}{1030}$

Note, But if you would find the rebated Sum, say, As the Hundred Pounds with it's Interest is to the Interest of 100 for the given Time, so is the present Sum to the Rebate.

See the Work of the above Question

$$\begin{array}{ccccccc} l. & s. & l. & s. & l. & s. & d. & q. \\ \text{If } 102 & 10 & : 2 & 10 & :: 20 & : 9 & 9 & 0\frac{600}{1030} \\ \text{Answer } 9s. & 9d. & 0\frac{600}{1030}q. & \text{as above.} & & & & \end{array}$$

Quest.

Of ARITHMETIC.

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Quest. 2. Delivered a Bond to a Creditor of 264*l.* 13*s.* 4*d.* which will be due 8 Months hence, but upon Rebate at 5*l.* per Cent. per Annum, I am willing to make present Payment, how much is the present Payment, and also the Discount.

1st, If $12 : 5 :: 8 : 3 \quad 6 \quad 8$

2dly, If $103 \quad 6 \quad 8 : 100 :: 264 \quad 13 \quad 4 \quad 0$
 Present Money $256 \quad 2 \quad 6 \quad 3 \frac{16}{248}$
 Answer { Rebate $8 \quad 10 \quad 9 \quad 0 \frac{32}{248}$

Quest. 3. What's the Discount of one Pound for one Year, at 5*l.* per Cent. per Annum?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
Present Money	0	19	0	$2 \frac{30}{103}$
From	1	0	0	0
Rebate - -	0	0	11	$1 \frac{75}{103}$

By this it appears that one Pound in a Year is decreased to 19*s.* 0*d.* $2 \frac{30}{103}$ *q.* at 5*l.* per Cent.

Quest. 4. What's the present Money, and Discount of 58*l.* 16*s.* 3*d.* for 172 Days, at 5*l.* per Cent. per Annum, Simple Interest?

Answer { Pref. Sum $57 \quad 9 \quad 2 \quad 0 \frac{11904000}{35865800}$
 Discount $1 \quad 7 \quad 0 \quad 3 \frac{23961600}{35865800}$

Quest. 5. What's the present Money, and Discount of 76*l.* 8*s.* 4*d.* for 239 Days, at 5*l.* per Cent. per Annum Simple Interest?

Answer { Pref. Money $73 \quad 19 \quad 10 \quad 2 \frac{128152}{361872}$
 Rebate $2 \quad 8 \quad 5 \quad 1 \frac{233420}{361872}$

XIV. Of Profit and Loss.

DEFINITION.

THE Title itself is a sufficient Explanation, for in Trade it is only to know what you gain or lose, by Buying and Selling any Comodity. In which are four Varieties.

1. To know what is gain'd or lost *per Cent*.
2. To know what it shall be sold for, to gain or lose so much *per Cent*.
3. Having gain'd or lost so much *per Cent*. to know what it cost.
4. *Lastly*, There being so much gain'd *per Cent*. when sold at such a Price, to know what is gain'd *per Cent*. when sold for more, or what is lost *per Cent*. when sold for less.

Quest. 1. If 1 *lib.* of Tobacco cost 18*d.* and is sold for 21*d.* I demand what is gain'd *per Cent*.

$$\begin{array}{cccccc} d. & d. & l. & l. & s. & d. \\ \text{If } 18 & : & 3 & :: & 100 & : 16 \quad 13 \quad 4 \end{array}$$

That is; by laying out of 100*l.* you will gain that Sum, *viz.* 16*l.* 13*s.* 4*d.*

Quest. 2. If 90 *English* Ells of Cambrick cost 60*l.* for how much must one Yard be sold to gain 18*l.* *per Cent*.

First find what one Yard will come to at the given Price.

$$\begin{array}{r} \text{If } 90 \text{ E.} : 60 \text{ l.} :: 1 \text{ Y.} \\ \quad \quad \quad \underline{5} \quad \quad \quad \underline{4} \\ 450 : 60 :: 4 : 10 \text{ s. } 8 \text{ d.} \end{array}$$

Then

Then to - - 100l.
Add - - 18

If 100l. : 10s. 8d. :: 118 : 12s. 7d.

Quest. 3. If 276 Fother of Leads (each 19½) be sold for 256l. at 5 Months Credit, and I gain 11l. *per Cent. per Annum.* The Question is, how much the Whole cost ready Money?

See the Work.

To - - - - 100l.
Add - - - - 11

111

l. l. l. l. s. d. q.
If 111 : 256 :: 100 : 230 12 7 1½

Quest. 4. If Cloth sold at 12s. per Yard, be 10l. *per Cent.* Profit, what Gain or Loss *per Cent.* should I have had I sold the same for 7s. per Yard?

Operation.

s. l. s. l. s.
If 18 : 110 : 7 : 96 5 Sub.
100 0 from

Answer lost - - 3 15 *per Cent.*

A General Rule.

Where there is Gain *per Cent.* add the Gain *per Cent.* to 100l. but where there is Loss *per Cent.* subtract as much as you lose *per Cent.* from 100l. the Sum or Difference is the third Number in the Rule of Three.

XV. Of Exchange.

DEFINITION.

EXCHANGE is to pay Money in one Country, and receive the like Sum or Value in another, with Consideration of either Loss or Gain.

Note, The Par of Exchange is the fix'd and Standard Value of Foreign Coins, &c. expressed in Sterling Money of our own; 'tis so called, because in Exchange one equal Value for another is given.

The Course of Exchange is the Current Price of Exchange, always unsettled, being sometimes above, and sometimes below the Par, according to the various Circumstances and Accidents of Trade and Nations.

Before I give any Examples it will be proper to give the Value of Gold and Silver Coin, both of our own and other Nations: And first of our own as weigh'd in Air and Water. *Troy Weight.*

		Weigh'd in Air.		Water.	
		pw.	gr.	pw.	gr.
A Guinea	—	5	9	5	$1\frac{3}{4}$
A Crown	—	19	$8\frac{1}{2}$	17	$11\frac{1}{3}$
Half a Crown	—	9	$16\frac{1}{4}$	8	$17\frac{3}{4}$
A Shilling	—	3	$20\frac{9}{16}$	3	$11\frac{1}{2}$
Six Pence	—	1	$22\frac{1}{2}$	1	$17\frac{3}{4}$

		l.	s.	d.
One Pound of Gold is worth	—	48	0	0
One Ounce is worth	—	4	0	0
A Penny-weight is worth	—	0	4	0
One				

		<i>l.</i>	<i>s.</i>	<i>d.</i>
One Grain is worth	—	0	0	2
One Pound-weight of Silver	—	3	2	0
One Ounce is worth	—	0	5	2
One Penny-weight is worth	—	0	0	3: 0 $\frac{2}{3}$
One Grain is worth	—	0	0	0: 0 $\frac{1}{2}$

Scotch Money.

		<i>l.</i>	<i>s.</i>	<i>d.</i>
A Pound	—	0	1	8
A Mark	—	0	1	1 $\frac{1}{2}$
A Noble	—	0	0	6 $\frac{3}{4}$

Irish Money:

A Pound	—	0	15	0
A Harper	—	0	0	9
An Obb, or Cob	—	0	0	4 $\frac{1}{2}$

Note, 100l. in Gold, weighs 2 lib. 10 oz. Troy:

And 100l. of Silver weighs 26 lib. 4 oz. And 22d. in Copper weighs one Pound *Averdupoise*,

Dutch Money.

A Dutch Stiver	—	0	0	1 $\frac{1}{2}$
6 Stivers, or 1 Skilling <i>Flemish</i>	—	0	0	7 $\frac{1}{2}$
20 Stivers, 1 Gilder	—	0	2	0
A <i>Flemish</i> Pound,	33s. 4d. =	1	0	0
A Zealand, or common Dollar	—	0	3	0
A Duccatoon,	—	0	6	3 $\frac{1}{2}$
A Specie Dollar	—	0	5	0
A Cross Dollar	—	0	4	2
Old Philip's Dollar	—	0	5	0
Ferdinando's Dollar	—	0	4	3
Prince of Orange's Dollar	—	0	4	4
Leopald's Dollar	—	0	4	3
Rudolphus's Dollar	—	0	4	4
Maxamilian's Dollar	—	0	4	5
Holland Rider	—	1	5	9

German

German Money.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
A Rix Dollar of the Empire	0	4	3 $\frac{1}{4}$
A Gilder of <i>Noremberg</i>	0	7	1
A Mark Lubs	0	16	0

French Money.

A Denier	0	0	0 $\frac{3}{4}$
A Soulz = 12 Deniers	0	0	0 $\frac{1}{16}$
A Liver = 20 Soulz	0	1	6
A Crown = 3 Livers	0	4	6

Spanish Coins.

13 Malvadies =	0	0	0 $\frac{1}{2}$
A Rial =	0	0	6 $\frac{1}{4}$
8 Rials, or 1 Piece of Eight	0	4	6 $\frac{1}{4}$
A Piece of Eight Mexico	0	4	6
A Piece of Eight Peru	0	4	5
A Piece of Eight <i>Seville</i>	0	4	6
A Doubloon of <i>Castile</i>	0	5	6
A <i>Valentia</i> Ducat	0	5	3
<i>Saragoza</i> Ducat	0	5	6
<i>Barcelona</i> Ducat	0	6	0
<i>Barcelona</i> Crown	1	2	0
Pattavoon	0	4	8

Portugal Coin.

42 Rees	0	0	1
A Vintine,	0	0	3
A Rial	0	0	6
A Testoon	0	1	3
An old Crusado	0	5	0
A Mill Ree = 1000 Rees	0	6	8

Italian Coin.

A Liver at <i>Leghorn</i>	0	0	9
A Current <i>Florence</i> Crown	0	5	3
A Ducat de Banco <i>Venice</i>	0	4	4
A St Mark	0	2	10

A

Of ARITHMETIC.

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	<i>l.</i>	<i>s.</i>	<i>d.</i>
A Julio, or Julier	0	0	6
A Dollar	0	4	6
7 Livers = to 1 Ducat	0	5	9
The Ducat of <i>Rome</i>	0	7	6
The Crown at <i>Rome</i>	0	7	6
The Crown of <i>Placentia</i>	0	7	6
The <i>Naples</i> Ducat	0	5	0
The Crown Current	0	5	6
The Torri	0	1	0
A Bajocke	0	0	0 $\frac{1}{2}$

Poland and Russian Coin.

At Dantzick.

90 Grosz = 1 Rix Dollar	0	4	6
A Gilder	0	1	6
The Stiver or Dollar	0	2	3
Silver Mark or Gilder	0	1	6

Sweden Coin.

Silver Dollar	0	2	3
Silver Mark, or Gilder	0	1	6

Arabian and Persian Coin.

The Abaffis	0	1	4
A Mamada	0	1	0

At Bantam.

1000 Patees of Copper	0	5	0
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At Aleppo and Scanderoon.

16 Shekees = 1 Piece of Eight	0	4	6
14 Shekees = 1 Lyon Dollar	0	4	0
A Dina	1	10	0
A Soraphat Goa,	0	4	6

At Siam.

A Gold Coin worth	0	10	7
Silver Coin	0	2	5

A

At Surret.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
The Asper	0	0	$1\frac{1}{4}$
The Larin	0	4	0
The Saraphim	0	4	6
The Rupie	0	2	0
The Fenon, or Fanam	0	0	3
A Pagod	0	9	0
A Saltanin	0	8	0

Quest. 1. A Merchant at *Legborn* delivered 500 Livres to receive at *London* in Sterling Money, how much must he receive?

Liv. d. Liv. l. s.

If 1 : 9 :: 500 : 18 15

The Course of Exchange is published Weekly, which compared with the Par, will shew whether it is above or below at any time. In *December 1735*, the Course of Exchange stood thus.

	<i>s.</i>	<i>d.</i>
<i>Amsterdam</i>	35	10
<i>Ditto at Sight</i>	35	6
<i>Rotterdam</i> $2\frac{1}{2}$ Usance	36	
<i>Antwerp</i>	36	6
<i>Hambourgh</i> $2\frac{1}{2}$ Usance	36	
<i>Paris</i>	31	$\frac{7}{8}$
<i>Bourdeaux</i> $\frac{1}{2}$ Usance	30	$\frac{5}{8}$
<i>Cadiz</i>	40	$\frac{1}{2}$
<i>Madrid</i>	40	$\frac{1}{4}$
<i>Bilboa</i>	40	$\frac{1}{4}$
<i>Legborn</i>	50	$\frac{3}{8}$
<i>Genoa</i>	52	$\frac{1}{2}$
<i>Venice</i>	50	$\frac{3}{8}$
<i>Lisbon</i>	5	$6\frac{7}{8}$
<i>Porto</i>	5	$3\frac{3}{8}$
<i>Dublin</i>	12	$\frac{7}{8}$

The Reader must here take Notice, that the Course of Exchange is in a constant Flux, they continue not at a sett Standard as our Sterling Money doth, but are sometimes raised, and sometimes depressed.

Quest.

Quest. 2. A Merchant in *London* delivers 481*l.* Sterling, to receive the same at *Paris* in *French* Crowns, at 4*s.* 6*d.* I demand how many *French* Crowns he ought to receive?

<i>s.</i>	<i>d.</i>	<i>C.</i>	<i>L.</i>	<i>Crowns.</i>
If 4	6	:	1	:
			481	:: 2137 $\frac{2}{34}$

Quest. 3. If 180*d.* at *London*, be equal to 2 Ducats at *Rome*, and 5 Ducats at *Rome*, equal to 16 $\frac{18}{27}$ Dollars at *Danzick*, how many Pence at *London* are equal to 10 Dollars at *Danzick*?

State it as has been taught in the Rule of Three, p. 36.

Pence at *London* 180 = 2 Ducats at *Rome*.
 Ducats at *Rome* 5 = 16 $\frac{18}{27}$ Dollars at *Danzick*.
 London 10 = *d.* Dollars at *Danzick*.

900)243000(270

Answer 270*d.*

XVI. Of Single Position.

DEFINITION.

THIS Rule of False Position takes it's Name from working by any false or feign'd Number we find out a true Answer. And this is the Rule.

*As the Parts of the Position
 Is to the Position itself,
 So is the given Number by guess,
 To the true Number sought.*

H

EXAMPLE.

EXAMPLE.

Quest. 1. A certain Person being demanded how old he was, to avoid a direct Answer, said, If the Half, the Third, the Fourth, and the Sixth Parts of his Age, were added together they would make just $52\frac{1}{2}$, now how old was he?

Take any Position at Pleasure, that will be divided into the Parts above-mentioned, as suppose 18.

$$\text{Then } \left\{ \begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{6} \end{array} \right\} \text{ of } 18 \text{ is } = \left\{ \begin{array}{l} 9 \\ 6 \\ 4\frac{1}{2} \\ 3 \end{array} \right.$$

22 $\frac{1}{2}$ the Sum.

Now say,

$$\text{As } 22.5 : 18 :: 52.5 : 42.$$

P R O O F.

$$\left\{ \begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{6} \end{array} \right\} \text{ of } 42 \left\{ \begin{array}{l} 21 \\ 14 \\ 10.5 \\ 7. \end{array} \right.$$

Sum 52.5

Answer 42.

Quest. 2. Three Men, A, B, and C, make a Purchase of 3000/. B is to share double with A, and C to share triple with B, now what are their respective Shares?

$$\text{Answer } \left\{ \begin{array}{l} \text{A } 333 \quad 6 \quad 8 \\ \text{B } 666 \quad 13 \quad 4 \\ \text{C } 2000 \quad 00 \quad 0 \end{array} \right.$$

*Proof 3000 00 0

Quest. 3. One who had wasted $\frac{2}{3}$ and $\frac{1}{5}$ of his personal Estate had 36 Pounds remaining, now I would know what was his Estate?

Answer 270/.

XVII. Of Double Position.

DEFINITION.

THIS is when we have two false Positions given at Pleasure, to find a true Answer to a Question propounded.

When a Question is propounded, make such a Cross as this,



and make choice of any Number, which call your first Position, and place it at the Top of the Cross, where *a* stands: then work according to the Nature of the Question, and if the true Number come out, 'tis done, otherwise what it differs call the First Error, which place at *c*. Make choice of another Position, which place with *b*, work with this as if it were the true Number, and if it differ from the Truth, set the Difference at *d*, and call it the Second Error, which Errors mark with *Plus* +, and *Minus* —, and thus the Positions will stand at the Top of the Cross, and the Errors at the Bottom.

Then multiply the First Position by the Second Error, (that is, Cross-wise) and the Second Position by the First Error, and reserve the Products.

Then, if the Errors be both too much, or both too little, the Difference of the Products is the Dividend, and the Difference of their Errors is a Divisor, the Quotient arising from that Division is the Answer sought.

But if the Errors be one too much and the other too little, the Sum of the Products, is a Dividend, and the Sum of the Errors is a Divisor, the Quotient of this Division is the Answer.

EXAMPLE.

Quest. 1. Three Persons, A, B, and C, thus discoursed together concerning their Age, quoth A I am 18 Years
H 2 of

of Age, quoth B I am as old as A and half as old as C, and quoth C, I am as old as you both if your Years were added together, now I desire to know the Age of each Person ?

Solution.

Suppose the Age of C to be	40
Then B must be	20
A's Age add to B	18
For the Age of B	38
Add	18
Sum	56
Subtract	40
Error +	16

2dly, Suppose C	60
The Half	30
Add	18

40	60
+	+
16	6
60	40
960	240
240	

B - Sum	48
	18
	66
	60
Error +	6

10)720(72

Answer 72 Years, the Age of C.

36 is half of 72

Add 18 A.

Sum 54 B.

Quest. 2. Two Men, viz. A and B discoursed thus together, A said to B, I think you have this Year 20 Geese, no said B to him again, that I have not, but if I had as many as I have, and half so many, and two Geese and

and a half, then I should have 20, now I demand how many Geese he had ?

Answer 7.

Quest. 3. A Man going to an Alehouse, said to the Host, if you'll give me as much Money as I have in my Purse I will spend my Sixpence, the Host did so, and he spent his Sixpence ; he went to a Second Alehouse, and said, if you'll give me as much Money as I have in my Purse, I will spend my Sixpence, the Host did so, and he spent his Sixpence ; he went to a Third Alehouse and did likewise, and when he had spent his Sixpence he had nothing left, now the Question is, what the Tipler had in his Purse at first ?

Answer $5\frac{1}{4}d.$

Quest. 4. There is a Cistern with Four Cocks, which holds 8 Barrels of Water, the First Cock will run it all out in 6 Hours, the Second in 4 Hours, the Third in 3 Hours, and the Fourth in 2 Hours, now I demand in what Time they will all run it out, if they were all set a running at once ?

Answer in 48 Minutes.

Quest. 5. A Gentleman hired a Workman for 20 Days, at 18d. per Day for every Day he worked ; but for every Day he played he was to bate 12d. At the End of 20 Days he received 8 Shillings, which was his full Due. Now I demand how many Days he wrought, and how many Days he played ?

Answer 11d. 2b. 48'

And play'd 8 19 12'

Quest. 6. A and B receive 100l. for the Ranfom of a Prisoner, fall at Variance in parting the Money, and after each had snatch'd what he could (upon Agreement) B gave A $\frac{1}{3}$ of what he had snatch'd from him, and A gave B $\frac{1}{3}$ of what he had snatch'd from him, which done, each of them had 50l. now how much Money did each Man snatch up ?

		l.	s.
Answer {	A	37	10
	B	62	10

Sum 100 Proof.

Quest. 7. A Thief breaking into an Orchard, Stole from thence a certain Number of Pears, and at his coming

ing forth he met with Three Men, one after another, who threatned to accuse him of Theft; and for to appease them, he gave unto the First Man, half the Pears that he had Stole, who return'd him back 12 of them; then he gave unto the Second half of them he had remaining, who return'd him back 7 of them; and unto the Third Person he gave half the Residue, who return'd him back 4. and in the End he had still remaining 20 Pears, now the Question is, how many Pears he had stole in all?

Answer 76.

Quest. 8. A Gentleman hired a Servant for a Year, at 6*l.* Sterling and a Livery Cloak, valued at a certain Rate, but when $\frac{7}{12}$ of the Year was expired, they fell at variance, and the Gentleman put away his Servant, gave him the Cloak together with the 5*s.* of the Money agreed for, which was the Servant's full Due for the Time of his Service, the Question is, to find what the Cloak was valued at?

Answer 2*l.* 8*s.*

Quest. 9. If to my Age you add,
The Four Fifths of Three Fourths, and Two Fifths more,
The Number Sixty Eight will then be had,
What was my Age in Years above a Score?

Answer 34 = 14 above 20.

XVIII. Of Compound Interest.

DEFINITION.

AS for Simple Interest, it is sufficiently explained in the Golden Rule of Five Numbers. Compound Interest, is when a Sum of Money is put out to Interest, and the Interest thereof becoming due, is still continued in the Hand of the Debtor, so as to become Part of the Principal, Interest being reckoned for it, from the Time it became due, for which Reason it is called Interest upon Interest. And as Simple Interest increaseth by a Series of Arithmetical Proportionals continued, so doth Compound Interest.

Interest increase by a Rank or Series of Geometrical Proportionals.

And *Note*, That the Simple Interest of One Hundred Pounds being known, the Compound Interest of any other Sum for any Number of Years may be likewise found out by so many single Rules of Three, as there are Years. For as 100*l.* is to it's Interest for one Year, so is any other Sum, to it's Interest for any other Time.

And so is the First Year's Interest to the Second, and the Second Year's Increase to the Third, &c.

EXAMPLE.

What doth 10*l.* amount unto at the End of 7 Year's Compound-Interest at 5*l. per Cent.* Say,

$$\begin{array}{ccccccc} l. & l. & l. & l. & & & \\ \text{As } 100 & : & 105 & :: & 10 & : & 10.5 \end{array}$$

Here I have given the Fourth Term in a Decimal mix'd Number, which is 10*l.* 10*s.* which is the best Method of answering Questions in Compound Interest; for by what follows you may easily conceive, that it is only the Principal Sum, multiply'd into it's Increase for one Year, so often as the Time requireth. As in the Example 105, being what 100*l.* is increased to for one Year, is multiplied 7 Times into the Principal Sum, 10*l.* &c. and the last Product is the Answer to the Question.

100 <i>l.</i> is increased in one Year to	-	-	-	105
It's Principal Sum	-	-	-	10
First Year's Increase	-	-	-	10.5
				105
Second Year's Increase	-	-	-	11.025
				105
				55.125
				11025
Third Year's Increase	-	-	-	11.57625
				105
				5788125

		5788125
		<u>1157625</u>
Fourth Year's Increase	- -	12.1550625
		<u>105</u>
		607753125
		<u>121550625</u>
Fifth Year's Increase	- -	12.762815625
		<u>105</u>
		63814078125
		<u>12762815625</u>
Sixth Year's Increase	- -	13.40095640625
		<u>105</u>
		6700478203125
		<u>1340095640625</u>
Seventh Year's Increase	- -	14.0710042265625

The Value of this Decimal of a Pound Sterling, is 1s. 5d. 0.16405759. so that by letting out 10l for 7 Year, without taking up any Interest, it will grow in that Time to 14l. 1s. 5d. 0.1640575 Parts of a Farthing Interest.

But how expeditiously this may be done by Logarithms, will best appear by the following Work. For to the Logarithm of the Increase of one Hundred Pounds for one Year, viz. 105, you add the Logarithm of the Principal Sum 10l. and from the Sum of those two Logarithms Subtract the Logarithm of 100l. the Remainder is the Logarithm of the First Year's Increase 10l. 10s. which Logarithm of one Year's Increase multiply by 7, the Number of Years, omitting the Index, gives the Logarithm of the Increase of 10l. at the End of 7 Years, Compound Interest.

See the Work.

100l. in one Year is increased to		105l. 2.0211893
Principal Sum	- -	<u>10. 1.0000000</u>
Sum of the Logarithms	- -	3.0211893
100l.	- -	<u>2.0000000</u>
First Year's increase	- -	10.5. 1.0211893
Number of Years	- -	- - - 7
Answer	- -	<u>14.071..1.1483251</u>

You

Of the Square Root.

81

You see we have got the three first Decimal Places of a Pound, which is near enough the Truth.

EXAMPLE II.

What will 4*l.* amount unto in 10 Years, at 5*l.* per Cent.

Operation by the Logarithms.

$$\begin{array}{rcl} \text{Logarithm } \S & 105 & = 2.0211893 \\ \text{of } \S & 100 & = 2.0000000 \end{array}$$

$$\text{Remains Logarithm} \quad 0.0211893$$

$$\text{Years multiply} \quad - \quad - \quad - \quad 10$$

$$\underline{0.2118930}$$

$$\text{Principal Sum } 4\textit{l.} = \text{Logar. } .6020600$$

$$\text{Answer } 6.5155785 \text{ Logar. } .8139530$$

The Value of the Fraction is 10*s.* 3*d.* 2.95536*q.* So that we see 4*l.* being put out 10 Years at Compound Interest, it will be increased to 6*l.* 10*s.* 3*d.* 2.95536*q.*

XIX. Of the Square Root.

DEFINITION.

THE Square of any Number is, when it is multiplied into itself, as the Square of 4 is 16, the Square of 5 = 25, &c. therefore having any Square Number given to find it's Root, do thus, *viz.* begin at the Unites Place, and there make a Dot, and so over every other Figure ; now as many Dots as there are, so many Places or Figures you will have in the Root. By pointing of it thus, the given Square is divided into Periods, then see what is the nearest Less Square of the first Period to the Left Hand, and place it's Root in the Quotient, which Square of that Root place under the first Period, and subtract, set the Remainder under the Line, and thereto bring

bring down the next Period, then double the Root, and place it on the Left Hand a crooked Line for a Divisor. See how often that Divisor is contained in the Dividend last brought down and placed under the Line, always making allowance of one Place in the Dividend, because the same Figure you place in the Root, the same must be placed in the Unites Place in the Divisor. Repeat this Work as often as there are Periods or Pairs of Figures to bring down: And at last, if any Thing remain, add two Cyphers, and for every two so added, you will have one Decimal place in the Root.

The following Examples will make it plain to the diligent Reader.

What's the Square Root of 2401 ?

$$\begin{array}{r}
 \text{2401 (49 Root.} \\
 \underline{16} \\
 89) \text{801} \\
 \underline{801} \\
 \text{Rem. } 0
 \end{array}$$

P R O O F.

To prove the Work, multiply the Root in itself, and to the Product add the Remainder, if any, this last Sum will be equal to the given Square.

More Examples for Practice.

$$\begin{array}{r}
 \text{459684 (678 Root.} \\
 \underline{36} \\
 127) \text{996} \\
 \underline{889} \\
 1348) \text{10784} \\
 \underline{10784} \\
 \text{Rem. } 0
 \end{array}$$

814602573

814602573 (28541.24 Root. A mixt Number.

$$\begin{array}{r}
 4 \\
 \hline
 48 \overline{) 414} \\
 \underline{384} \\
 565 \overline{) 3060} \\
 \underline{2825} \\
 5704 \overline{) 23525} \\
 \underline{22816} \\
 57081 \overline{) 70973} \\
 \underline{57081} \\
 570822 \overline{) 1389200} \text{ Rem.} \\
 \underline{1141644} \\
 5708244 \overline{) 24755600} \\
 \underline{22836976} \\
 \text{Rem. } 1918624
 \end{array}$$

Note, The Square Root is of excellent Use in solving Arithmetical and Geometrical Questions, which I shall exhibit as follows.

1. To find a mean Proportional between any two Numbers.

R U L E.

Multiply the two given Numbers together, and extract the Square Root of the Product, gives the Answer.

E X A M P L E.

Quest. 1. What's the Geometrical Mean, between 4 and 6?

Answer 4.89.

Quest. 2. What's the Geometrical Mean between 20 and 30?

Answer 24.49.

2. To find the Diameter of a Circle that shall be equal in Area to an Ellipsis whose Transverse and Conjugate Diameter are given.

R U L E.

R U L E.

Multiply the Two Diameters of the Ellipsis together, and extract the Square Root of the Product will give the Diameter of a Circle equal.

E X A M P L E.

Quest. 3. Let the Transverse Diameter be 36, and the Conjugate 23.5 Inches, what is the Diameter of a Circle equal?

Answer 29.08 Inches.

3. Any two Sides of a right lin'd plain Triangle being known, the Third is easily found.

For in the 47th Proposition of the First Book of *Euclid's Elements* it is Demonstrated, that the Square of the Hypothenuse, is equal to the Square of the other Two Sides.

E X A M P L E.

Admit three Towns, as *London*, *Landaff*, and *Boston*; *Boston* lieth from *London* directly North 89 Miles, and *Landaff* lieth directly West from *London* 131 Miles, now I would know how far *Landaff* is from *Boston*?

Answer 158.37 Miles.

4. Admit two Ships set Sail from one Port, the one sails directly North 89 Miles, and the other directly West, and at the End of their sailing they were distant 158.37 Miles, I demand how far the Second Ship sailed?

Answer 131 Miles.

5. Two Men Travel from the same Place, A Travels West 131 Miles, B goes directly North 'till he be distant from A 158.37 Miles, I demand how far B travell'd?

Answer 89 Miles.

6. The Area of a plain Superficies given, to find the Side of the Square that shall be equal thereto?

R U L E.

The Square Root of the Area is the Answer.

E X A M P L E.

EXAMPLE.

Let the Area of a plain Superficies be 576 Inches, what's the Side of the Square that bounds it?

Answer 24 Inches.

In like Manner, if you extract the Square Root of the Content or Area of a Circle, *Pentagon*, *Hexagon*, &c. whether the Figure be regular or irregular, it will give the Side of a Square equal to that Superficies.

7. Given, the Side of a Square, to make other Squares, Two, Three, Four Times, more or less, greater or lesser, than that Square proposed.

RULE.

Square the Side of the given Square, and it gives the Area thereof, which multiply by 2, 3, 4, &c. the Square of the Product will give the Side of whose Area shall be 2, 3, 4, &c. times greater than the given Square.

EXAMPLE.

Let the Side of a Square be 12 Inches, and let it be required to make three other Squares, to contain Double, Triple, and Quadruple the given Square?

Given Square's Area is 144 144 144
 Multiply by 2 and 3 and 4

 288(16.97 432(20.78 576(24
 Answer Side Square { 16.97 Double. } in Area.
 { 20.78 Triple. }
 { 24. Quadruple. }

The like is to be observed of Circles.

8. In a plain right lined Oblique Angled Triangle having the Sides given severally, to find where the Perpendicular, let fall from the obtuse Angle, will fall upon the Base.

9. The Height of a Mountain being known, to find how far it shall be seen at Sea, or on plain Ground.

10. Given the Semidiameter of the Earth 3984.58 Miles, with the Distance of a Mountain seen at Sea, to find the Height of that Mountain, this is the Converse of the 9th.

11. The Diameter of a Pipe of a Cock given, to find the Diameter of another Pipe that shall fill the same in half the Time, more or less.

EXAMPLE.

There is a Pipe whose Diameter is $1\frac{1}{2}$ Inch, will fill a Cistern in 2 Hours, I demand the Diameter of another Pipe that will fill it in 1 Hour?

Answer 2.12 Inches.

12. The Circumference of a Cable being $7\frac{1}{4}$ Inches, and that one Fathom thereof doth weigh $16\frac{1}{4}$ Pounds, I demand how many Pound that Rope doth weigh whose Circumference is $11\frac{3}{4}$ Inches?

Operation.

square pounds square pounds.

As 52.5625 : 16.25 :: 138.0625 : 42.6828

Answer 42 *lib.* 10 *oz.* 14.7958. *dr.*

13. If the Length of a Cask be 40 Inches, Bung 28, and Head 24, what's the Diagonal Line?

Answer 32.8 Inches.

14. In a right lin'd Triangle, let one Side be 40 Inches, the next 26, and the least 22, what's the Area?

Answer 264 Inches.

15. A Company of Men drinking 'till the Reckoning came to 6*s.* 0 $\frac{1}{4}$ *d.* I demand how many Persons there were in Company, and what they paid a piece?

Answer 17 Persons paid 4 $\frac{1}{4}$ *d.* a piece.

16. A Company of Men Drinking, 'till the Reckoning came to 30*s.* 1*d.* I demand how many there were in Company, and what they paid a piece?

Answer 19 Men, paid 19*d.* a piece.

17. A Company of Men drinking 'till the Reckoning came to 17*s.* 6 $\frac{1}{4}$ *d.* I demand how many there were in Company, and what they paid a piece?

Answer 29 Men paid 7 $\frac{1}{4}$ *d.* a piece.

18. A Company of Men Drinking, 'till the Reckoning came to 6*l.* 16*s.* 8 $\frac{1}{4}$ *d.* I demand how many there were in Company, and what they paid a piece?

Answer 81 Men, paid 1*s.* 8 $\frac{1}{4}$ *d.* a piece.

19. In an Army of 75625 Soldiers, how to place them in square Battalia?

$$\begin{array}{r}
 \overline{75625} \text{ (275 Men in Rank and File.} \\
 \underline{4} \\
 47) \overline{356} \\
 \underline{329} \\
 545) \overline{2725} \\
 \underline{2725} \\
 \text{Rem. } 0
 \end{array}$$

20. To place any Number of Men, so that the Number of Men in Rank may be double to those in File.

$$\begin{array}{r}
 \text{Suppose } 35912 \text{ Men} \\
 \text{Half } \overline{17956} \text{ (134 File} \\
 \quad \quad \quad \underline{2} \\
 \quad \quad \quad 1 \quad \quad \underline{268 \text{ Rank.}} \\
 23) \overline{79} \\
 \underline{69} \\
 264) \overline{1056} \\
 \underline{1056} \\
 0
 \end{array}$$

For Proof, $268 \times 134 = 35912$.

21. Suppose 25000 Soldiers were to be martialled in a square Battalia of Ground, in such Sort that their distance in File should be 8 Feet, and the Rank 3 Feet, how many Men in Rank, and how many in File ?

$$\text{As } 8 : 3 :: 25000 : 9375 \text{ (96 File) } 25000 \text{ (260$$

Answer $\left\{ \begin{array}{l} 96 \text{ File} \\ 260 \text{ Rank} \end{array} \right\}$ besides 40 Men over and above.

22. If 47748 be the Number of Men to be martial'd in Battel Array in such Sort the Number of Men in File to those in Rank, should be as 8 to 20 ?

Answer 345 Rank, 135 File, besides 138 over.

XX. *Of the Cube Root.*

DEFINITION.

A Cube is a Geometrical Figure ; and as a Square has only Length and Breath, so a Cube has Length, Breadth and Depth : And in order to a right Understanding of this Work, it will be necessary for the Learner to get by Heart this Table of the Squares and Cubes of the Nine Digits.

Cubes.	1	8	27	64	125	216	343	512	729
Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

1. Point out the given Cube Number, beginning with the Unites Place, and so over every third Figure towards the Left Hand ; and as many Points or Periods as there are, so many Figures there will be in the Root.

2. Find the nearest lesser Cube in the first Period towards the Left Hand, which found, place in the Root on the Right Hand the crooked Line, cube this Figure (placed in the Root) and set it under the first Period, draw a Line and subtract, place the Remainder under the Line, to which bring down the next Period, and call this the Resolvend.

3. To find a Divisor, Triple the Root, and set it under the Line drawn under the Resolvend ; then square the Root, and triple the Square, set this Product down as in common Multiplication, *viz.* Unites under the Place of Tens, draw a Line and add these two Numbers together for the Divisor.

4. To find how often the Divisor is contained in the Resolvend, you must set aside the Figure in the Unites Place of the Resolvend, that as often as you can have the Divisor

Divisor in that Part of the Resolvend, set it in the Root, and observe these Steps.

1. Cube the Figure placed last in the Root.
2. Square the same, and multiply that Square by the Triple Root, (which Triple Root was found when you found the Divisor) placing Unites under Tens as in common Multiplication.
3. *Lastly*, Multiply the Figure last placed in the Root, by the Triple Square of the last Root, placing Unites under Tens as before; draw a Line under them and add these three last Lines of Figure into one Sum, and subtract it from the Resolvend; to the Remainder bring down the next Period of three Figures, or Cube, to which find a Divisor, and proceed in all the Steps as above directed, until all the Cubes are brought down and wrought, if any thing remain at last, add three Cyphers, and you will have a Decimal in the Root. The following Examples (being well minded) will make all plain.

EXAMPLE I.

Let the Cube Root of 1728 be required ?

$$\begin{array}{r}
 \begin{array}{r}
 \cdot \\
 \cdot \\
 1728 \text{ (12 Root:)} \\
 \hline
 1 \quad \text{Cube of 1} \\
 \hline
 728 \text{ Resolvend.} \\
 \hline
 3 \quad \text{Triple Root 1.} \\
 3 \quad \text{Triple } \square \text{ of Root 1.} \\
 \hline
 33 \text{ Divisor.} \\
 \hline
 8 \quad \text{Cube of Root 2:} \\
 12 \quad \text{Square of 2 by Triple 1.} \\
 6 \quad \text{Triple } \square \text{ of Root 1.} \\
 \hline
 \text{Sum } 728 \text{ Subtract from a Resolvend.} \\
 \hline
 \text{Rem. } 0
 \end{array}
 \end{array}$$

The Use of the Cube Root.

EXAMPLE II.

$$\begin{array}{r}
 45499293 \text{ (357)} \\
 27 \text{ Cube of 3.} \\
 \hline
 1849 \text{ Resolvend.} \\
 9 \text{ Triple of the Root 3:} \\
 27 \text{ Triple } \square \text{ Root 3.} \\
 \hline
 279 \text{ Divisor.} \\
 125 \text{ Cube of the Root 5.} \\
 225 \square \text{ of the Root 5, by Triple Root 3.} \\
 135 \text{ Root 5, by Triple } \square \text{ Root 3.} \\
 \hline
 \text{Sum } 15875 \text{ Sub.} \\
 2624293 \text{ Resolvend.} \\
 105 \text{ Triple Root 35.} \\
 3675 \text{ Triple } \square \text{ Root 35.} \\
 \hline
 36855 \text{ Divisor.} \\
 343 \text{ Cube of 7.} \\
 5145 \square \text{ Root 7, by Triple Root 35.} \\
 25725 \text{ Triple } \square \text{ Root 35, by Root 7, viz. 3675} \\
 \hline
 2624297 \text{ Subt. from Resolvend.} \\
 \hline
 \text{Rem. } 0
 \end{array}$$

In the like Manner the Cube Root of 9876543210 will be found to be 2145. and of 1603393328631365.740 to be 117043.1 *ferè*.

The Use of Cube Root.

Note, All Solids are Proportional, one to another, as the Cube of their Diameters.

Given, the Side of a Cube, to make another Cube, 2, 3 or 4 times greater or lesser.

R U L E.

1. To make it greater than the given Cube, multiply the Cube of the Side by the Quantity you intend it to be, and the Cube Root of the Product is the Side of the Cube required.

2. To

2. To make one lesser than the given Cube, divide the Cube of the Side by any Quantity, and the Cube Root of the Quotient is the Side of the Cube sought.

EXAMPLE.

Admit the Side of the Cube be 12 Inches, the Solidity of which is 1728, I would have three other Cubes that shall contain 2, 3, and 6 times greater and lesser than the given Cube?

Operation.

Side 12 Inches.

$$\begin{array}{r} 12 \\ \hline 144 \\ 12 \\ \hline 1728 \\ 2 \\ \hline \end{array}$$

3456(15.1 Cube Root.

$$\begin{array}{r} 1728 \\ 3 \\ \hline \end{array}$$

5184(17.3

$$\begin{array}{r} 1728 \\ 6 \\ \hline \end{array}$$

10368(21.8

Answer the Sides of the greater Cubes are

$$\left\{ \begin{array}{l} 15.1 \\ 17.3 \\ 21.8 \end{array} \right\} \text{ Inches.}$$

3. Also the Sides of a Cube lesser by 2, 3, and 6 times than the given one.

$$\text{The Sides will be } \left\{ \begin{array}{l} 9.5 \\ 8.3 \\ 6.6 \end{array} \right\} \text{ Inches.}$$

4. To three Numbers given, to find a Fourth in a Duplicate Proportion.

DEFINITION.

The Nature of this Proposition, is to discover the Proportion of Lines to Superficies, and Superficies to Lines, for like Planes are in a duplicate Ratio; that is as the Quadrat of their Homologal Sides.

RULE.

R U L E.

Multiply the Square of the Third Term by the Second Term, and divide the Product by the Square of the First Term, the Quotient is the Answer.

E X A M P L E.

Suppose 3, 4, and 5 be given, to find a Fourth in a Duplicate Ratio?

$$\begin{array}{r} 3 : 4 : 5 \\ 3 \quad \quad 5 \\ \hline 9 \quad \quad 25 \\ \quad \quad 4 \end{array}$$

$$9) 100(11\frac{1}{9})$$

5. Three Numbers given, to find a Fourth in a Triplicate Proportion.

D E F I N I T I O N.

The Nature of this Proposition is to discover the Proportion of Lines to Solids, and Solids to Lines: For like Solids are in a Triplicate Ratio, that is, to the Cubes of their Homologal Sides.

R U L E.

Multiply the Cube of the Third Term by the Second Term, divide the Product by the Cube of the First Term, the Quotient is the Answer; or which is all one, As the Cube of the Diameter of the given Bullet or Sphere, is to it's Weight or Solidity, so is the Cube of the Diameter of another Bullet, &c. to the Weight or Solidity thereof.

E X A M P L E.

Admit an Iron Bullet, whose Diameter is $6\frac{1}{2}$ Inches, do weigh $9\frac{1}{2}$ Pounds, what shall an Iron Bullet weigh, whose Diameter is $15\frac{1}{2}$ Inches?

See

See the Work Decimally performed.

$$\begin{array}{r}
 6.25 \\
 6.25 \\
 \hline
 3125 \\
 1250 \\
 \hline
 3750 \\
 39.0625 \\
 6.25 \\
 \hline
 1953125 \\
 781250 \\
 \hline
 7343750
 \end{array}$$

$$\begin{array}{r}
 15.5 \\
 15.5 \\
 \hline
 775 \\
 775 \\
 \hline
 155 \\
 240.25 \\
 15.5 \\
 \hline
 120125 \\
 120125 \\
 \hline
 24025
 \end{array}$$

As 244.140625 : 9.75 lib. :: 3723.875
9.75

$$\begin{array}{r}
 18619375 \\
 26067125 \\
 \hline
 33514875
 \end{array}$$

244.140625) 36307.78125 (148.71

Answer 148 lib : 11 oz. :: 5.76 dr.

6 Admit the solid Content of a Globe be 428307596 Inches, what is the Side of a Cube which shall be equal in Content?

Answer the Cube Root of the Globe 973.7 Inches is = to the Content.

7. Admit one of the Sides of a Cube, or the Diameter of a Globe to be $8\frac{3}{4}$ Inches, and the Side of another Cube, or Diameter of another Globe be $11\frac{1}{4}$ Inches, I demand the Side of another Cube, &c. that shall be equal in Content to them both?

R U L E.

The Cube Root of the Sum of the Two Cubes is the Side of another Cube equal to them both, which in this Example will be found to be 12.79 Inches.

8. Given the Diameter of the Concaves of two Guns, and the Quantity of Powder that will Charge one of them
to

to find the Quantity of Powder that will suffice to Charge the other.

R U L E.

As the Cube of the Diameter of the Gun given, is to the Quantity of Powder that will Load the same, so is the Cube of any other Gun's Diameter, to the Quantity of Powder that will Load the same.

E X A M P L E.

Admit a Cannon Royal whose Diameter at the Bore is 8 Inches, require 26 Pound of Powder to Charge it, what Quantity of Powder will charge a Piece whose Diameter is $1\frac{1}{2}$ Inch ?

Cube of 8 = 512, and Cube 1.5 = 3.375

Cube. lib. Cube. lib. oz. dr.

As 512 : 26 :: 3.375 : .171 = 2 11.776

9. Given, the Diameter of a Granado Shell, to find the Weight of the Metal, and the Content of the Concavity of the Shell in Cubic Inches.

R U L E.

First, Find the whole solid Content of the Shell, as if it was really so, which is done by multiplying the Cube of the Shell's Diameter by 11, and dividing the Product by 21, the Quotient is the solid Content in Cubic Inches of the Whole Shell; and also find the solid Content of the Concavity, which subtracted from the whole Content, gives the Content of the Metal, which divide by 4.422979 the Weight of a Cubic Inch of Common Iron *Averdupoise*, the Quotient are the Ounces, which divide by 16 gives Pounds.

E X A M P L E.

Admit a Granado Shell, whose Diameter from outside to outside is 12 Inches, and the Diameter within $7\frac{1}{2}$ Inches, so the thickness of the Metal is 2.25 Inches, I demand

demand the Content of the Metal in solid Inches, and also it's Weight in Pound *Averdupoise*.

Operation.

Cube of 12 = 1728.

Cube of 7.5 = 421.875

$$\begin{array}{r} 11 \\ 21 \overline{) 19008} \end{array} \quad \begin{array}{r} 11 \\ 21 \overline{) 4640.625} \end{array}$$

4.422979)684.161000000(154.683

16) 154.683 (9.667

Answer { 684.161 Solid Inches.
154.683 Ounces.
9.667 Pounds Weight.

10. Given the Mould and Burthen of one Ship, to build another of the same Mould of any assign'd Burthen greater or lesser.

R U L E.

First, The Dimensions taken in a Ship are, the Length of the Keel, the Breadth on the Midship Beam, and the Depth in the Hold : Therefore if the Cubes of these several Dimensions be multiply'd by 2, 3, 4, &c. the Cube Roots of these several Products will be the Dimensions of another Ship, whose Burthen will be 2, 3, 4, &c. times greater than the Ship given.

E X A M P L E.

Admit a Merchant's Ship, whose Burthen is 72 Tuns, the Length of the Keel be 45 Feet, the Breadth on the Midship Beam 17 Feet, and the Depth in the Hold 9 Feet, and it is required to build another Ship whose Burthen shall be five Times greater than the other Ship, I demand what must be the Dimensions of the Ship required ?

Answer { Keel 76.9 } Feet.
Breadth 29.0
Depth 15.3

XXI. To Extract the Square Root of a Fraction, either Vulgar or Decimal.

1. **F**IRST, If the Fraction propounded be not in it's least Terms, reduce it into it's least, and then by the Rules foregoing, Extract the Square Root of the Numerator, and also of the Denominator, so shall this new Fraction be the Square Root of the Fraction propounded, so the Square Root of $\frac{49}{8}$ is $= \frac{7}{2}$.

Note, But many times the Numerator and Denominator of a Vulgar Fraction hath not a perfect Square Root, to find which infinitely near, reduce it to a Decimal, by adding Cyphers to the Numerator, and dividing by the Denominator, and always be sure to let the Decimal have an even Number of Places, as 2, 4, 6, &c. and then extract it's Square Root as if it were a whole Number.

EXAMPLE.

Let $\frac{14}{5}$ be given, it is reduced to this Decimal .56 and by adding five Pair of Cyphers to it, I find it's Square to be .748331.

2. If your Vulgar Fraction be a mixt Number, reduce it into an improper Fraction, then work as before. But if the Fractional Part be not an exact Square Number, it is best to reduce it into a Decimal mixt Number.

As for Example.

If the Square Root of $6\frac{1}{4}$ be required, it will be $2\frac{1}{2}$ and it's Root $\frac{5}{2} = 2\frac{1}{2}$, which is not exact; therefore it will be this Decimal, 6.75, and it's Root 2.598, nearer the Truth.

XXII. *The Extraction of the Cube Root of a Vulgar Fraction.*

EXTRACT the Cube Root of the Numerator for a new Numerator, and the Cube Root of the Denominator for a new Denominator, and this new Fraction shall be the Cube Root of the given Fraction. So $\frac{27}{343}$ being given, it's Root will be found $\frac{3}{7}$.

If you want the Cube Root of a mixt Number, it is best to reduce it to a Decimal. As suppose $8\frac{2}{3}$ = Decimal 8.8 it's Cube Root is 2.08.

XXIII. *The Extraction of Roots by Logarithms.*

1. **T**O Extract the Square Root by Logarithms, is no more than to take the Logarithm of the given Number, and the Half thereof shall be the Logarithm of the Square Root sought.

E X A M P L E.

Suppose 576 were a Square to be extracted, it's Logarithm is 2.7604225, the Half of which is 1.3802112 the Logarithm of 24, the Square Root sought.

The Extraction of the Cube Root, by the Logarithms.

R U L E.

Seek the Logarithm of the given Cube, and divide it by 3, and the Quotient is the Logarithm of the Cube Root sought.

E X A M P L E.

Admit it were required to extract the Cube Root of 421875, it's Logarithm is 5.6251839, one third of which is 1.8750613 the Logarithm of 75, the Cube Root sought.

And after this Manner it is easy to extract the Biquadrate Root, the Sur-solid, the Squared Cube, the Seventh, Eighth, or any other Power by the Logarithms, as I will shew by Example by and by.

That the Extracting all Sorts of Roots is sometimes necessary, and that no Method is so easy as that by Logarithms, are Truths beyond all possibility of denial.

K

The

98 *The Extraction of Roots by Logarithms.*

The only Thing that allays their Excellency, is that the largest Canons (as those of *Sherwin's*) are so scanty, that without some other farther Invention, it will not furnish us with the Logarithm of the Cube of $343 = 40353607$, but by the following Method the Root of any Power may be extracted infinitely near.

R U L E.

Seek the Logarithm of 7 or 8 Places to the Left Hand of the Number given, and prefix to that Logarithm it's proper Index or Characteristic, answerable to the Number of Places in the whole given Number, that is, less by one, than the Number of Places of the absolute Number, and divide it by the Index or Exponent of the Power whose Root you seek, the Quotient is the Logarithm of the Root sought. That is, if you seek the Square Root, divide by 2; if the Cube Root, by 3; if the Biquadrate Root, divide by 4, &c. I shall here begin with the Root 2, and proceed to the Ninth Power, prefixing their Logarithms, by which the Learner may see by understanding this easy Process, how to extract a greater Root, with as much facility as he can wish.

Given the Root	2	} Logar.	0.3010300
Multiply by	2		
2d Square	4	-	0.6020600
	2		
3. Cube	8	- -	0.9030900
	2		
4. Biquadrate	16	- -	1.2041200
	2		
5. Surfsolid	32	- -	1.5051500
	2		
6. Square cubed	64		1.8061800
	2		
7th Power	128	-	2.1072100
	2		
8th Power	256	.	2.4082400
	2		
9th Power	512	-	2.7092700

And so on *ad infinitum*.

Here

The Extraction of Roots by Logarithms. 99

Here you see if the Logarithm of $2 = 0.3010300$ be multiply'd by the 9th Power, it will produce the Logarithm of $512 = 2.7092700$, consequently if we divide the Logarithm of 512 by it's Exponent or 9th Power, we shall gain the Logarithm of it's Root 2 .

EXAMPLE I.

Mr *Ward* has extracted the Biquadrate Root of 4857532416 , and finds it to be 264 , let us see how this agrees with our Method?

Hence, because the absolute Number has ten Places, the Characteristic of it's Logarithm must be 9 . Then in *Sherwin's* Logarithm, I find the Logarithm of the five leading Figures next to the Left Hand, *viz.* 48575 to be 4.6864128 with the Difference 89.4 . which multiplied by the remaining Figures of the absolute Number, *viz.* 32416 , the Product 28.979404 , then, because of the great Fraction I call 28 , 29 , and add it to the Logarithm of $48575 = 4.6864128$ makes $.6844157$ to which prefix it's proper Characteristic 9 ; I find the Logarithm of 4857532416 to be 9.6864157 which divided by it's Exponent, or Power 4 , gives 2.4216039 the Logarithm of 264 , the Root sought.

After this Manner let us extract the Root of the 9th Power of this Number 4722366482869645213696 ? Here being twenty two Places, the Characteristic must be 21 , and the Logarithm of it is 21.6741597 , which divided by the Power, or Exponent 9 , gives the Logarithm 2.4082399 , whose absolute Root is 256 , the Root sought.

But if you would have the Square Root of this vast Number of 22 Places, you cannot go to it immediately, for the Logarithm 21.6741597 being halfed, gives 10.83707985 for the Logarithm of the Square Root: And this having for it's Index 10 , shews that the Number to which it belongs must needs consist of 11 Places, which no Canon of Logarithms can come near.

But this seeming Difficulty is soon removed: For if you seek the Root of any higher Power, compound of such a Power as in this Case is the Square squared, squared Cube, or squaredly squared Square (See Geometrical Pro-

gression, page 33) the Root primarily intended, is by Multiplication soon gained.

As for Example.

If you divide the Logarithm 21.6741597 by 4, the Exponent of the squared Square, or 4th Power, it gives 5.4185399 the Logarithm of 262144, and that being the squared Square Root, or the Square Root of the Square Root. Therefore square this Root 262144 (that is multiply it in itself) it will produce 68719476736 the Square Root of 4722366482869645213696. Or if the said Logarithm 21.674159 had been divided by 6, the Index or Exponent, the Quotient would have been 3.61235995, the Logarithm of 4096, which being cubed, will be 68719476736, the Square Root, because the squared Cube Root, is but the Cubic Root of the Square Root.

XXIV.

I Shall here add three useful Problems concerning Interest and Annuities in the Logarithmetical Way.

PROBLEM I.

To find the Time wherein a given Sum of Money will be increased to any Sum required at any Rate of Compound Interest.

The RULE.

Subtract the Logarithm of the less Sum, out of the Logarithm of the greatest, and from this Difference subtract the Logarithm of one Pound and it's Interest for a Year, if any thing now remain add Cyphers, and you will have the Quotient in Years, and Decimal Parts of a Year.

EXAMPLE.

Logarithm 1000	-	3.0000000
Logarithm 600	-	2.7781513
Logarithm 1.05 = .0211893	)	0.2218487 (104.7
		.. 12

Month	5.64
	30

Days	19.20
------	-------

Answer 10 y. : 5 m. : 19.2 d.

PROBLEM III.

To find the Time wherein a yearly Payment will pay off a Debt,* at any Rate of Compound Interest.

* with interest upon it during the period

The RULE.

As in the former Problem, find the Principal, answering the yearly Rent at the Interest proposed, and subtract the Debt out of the Correspondent Principal, and the Logarithm of the Difference, from the Logarithm of the Correspondent Principal, this last difference, divide by the Logarithm of the Rate, shews the Time.

EXAMPLE.

In what Time will a yearly Rent of 58*l.* pay off a Debt of 740*l.* at 5*l.* per Cent. Compound Interest?

Operation.

If 5 <i>l.</i> : 100 <i>l.</i> :: 58 <i>l.</i> :	1160 <i>l.</i>
Debt Subtract	- 740

		420
Logar. of 1160	=	3.0644580
Logar. of 420	=	2.6232493
Logar. of 1.05 = .0211893	)	0.4412087 (20.82
		.. 12

9.84
30

25.20

Answer 20 y. : 9 m. : 25.2 d.

XXV. *Ceres and Virginum.*

I Shall conclude this Chapter with the Rule called *Ceres and Virginum*, which teaches only how to resolve Merry or Sporting Questions, to exercise young Practitioners in Numbers.

Quest. 1. A Lady's Caterer bought 10 Birds of two Sorts, *viz.* Turkeys and Geese, for 24s. the Turkeys cost 4s. and the Geese 2s. a Piece, how many did he buy of each?

R U L E.

Multiply the whole Price by the least Price, subtract the Product from the whole Price, the Remainder divide by the Difference between the two Prices, the Quotient is the Number of Turkeys, *viz.* $2 \times 4 = 8s.$ and 8 Geese $\times 2s. = 16 + 8 = 24s.$

Quest. 2. Suppose 21 Persons in Company, Men, Women, and Children, and amongst them they spent 26s. and so that every Man spent 2s. every Woman 1s. and every Child 6d. how many were there of each?

Answer {	8 Men, at 2s. each	16s.
	7 Women 1	7
	6 Children 6d.	3
	21	26

Or they may be

9 Men, at 2s. each	18s.
4 Women 1	4
8 Children 6d.	4
21	26

Or

Or they may be

7 Men, at 2s. each	14s.
10 Women 1	10
4 Children 6d.	2
21	26

Lastly,

$7\frac{3}{4}$ Men, at 2s. each	15s. 6d.
$7\frac{3}{4}$ Women 1	7 9
$5\frac{1}{2}$ Children 6d.	2 9
21	26 0

Quest. 3. Admit 900l. to be distributed according to the Will of a deceased Friend to thirty several Men, A, B, and C, so that each A may have 60l. each B 40l. and each C 20l. how many Men must there be of each Sort ?

Answer $\left\{ \begin{array}{l} A \\ B \\ C \end{array} \right\}$ may be either $\left\{ \begin{array}{l} 7, 6, 5, 4, 3. \\ 1, 3, 5, 7, 9. \\ 22, 21, 20, 19, 18. \end{array} \right.$

30

Or $\left\{ \begin{array}{l} 2 \text{ A at } 60l. \text{ each } 120 \\ 11 \text{ B at } 40 \quad \quad 440 \\ 17 \text{ C at } 20 \quad \quad 340 \end{array} \right.$

Sum 900

Quest. 4. Suppose 10 Persons of several Countries, *English, Dutch, French, and Spaniards*, to pay a Debt of 1000l. so that every *Englishman* pays 50l. every *Dutchman* 130l. every *Frenchman* 70l. and every *Spaniard* 150l. how many are there of each Country ?

Answer $\left\{ \begin{array}{l} 4 \text{ Spanish, at } 150l. \text{ each } 600l. \\ 1 \text{ Dutch, at } 130 \quad \quad 130 \\ 1 \text{ French, at } 70 \quad \quad 70 \\ 4 \text{ English, at } 50 \quad \quad 200 \end{array} \right.$

10 1000

Or thus,

2 Spanish, at 150l. each	300l.
3 Dutch, at 130	390
3 French, at 70	210
2 English, at 50	100

10

1000

Quest. 5. Suppose you buy 12 Loaves of Bread for 12d. so that some might be Two Penny, some Penny, some Half Penny, and some Farthing Loaves, I demand how many there must be of each Sort?

Now, because of 12 Loaves for 12d. the Mean Price is one, but one of the Particulars being also one, there should be no Penny Loaves, because there is no difference betwixt the Mean Price and one Penny. But it may be thus, viz. either,

4 Two Penny Loaves	=	8	} Pence.
2 Penny Loaves	=	2	
2 Half-penny Loaves	=	1	
4 Farthing Loaves	=	1	

12

12

Or else it may be,

3 Two Penny Loaves	=	6d. 0q.
4 Penny Loaves	=	4 0
3 Half-penny Loaves	=	1 2
2 Farthing Loaves	=	0 2

12

12 0

XXVI. Of Decimal Arithmetic.

DEFINITION.

THE Word *Decimal* is derived from *Decem*, Ten ; this kind of Numbering supposeth the Integer to be divided into Ten equal Parts : As suppose a Pound, a Mile, a Year, or any other thing whatsoever ; and as whole Numbers increase from the Right Hand towards the

the Left, in a Decuple Proportion from Unity, so Decimals decrease from Unity in the same Proportion from the Left Hands towards the Right, as is more evident from this Table.

Unites	1.	Unites, one Integer.
Primes	.1	One tenth Part.
Seconds	.01	One Hundred Part.
Thirds	.001	One Thousand Part.
Fourths	.0001	One ten Thousand Part.
Fifths	.00001	One hundred Thousand Part.
Sixths	.000001	One Million Part.
Sevenths	.0000001	One ten Million Part.
Eighths	.00000001	One hundred Million Part.
Ninths	.000000001	One Thousand Million Part.

Here you see Decimal Fractions are of several Denominations, as Primes, Seconds, Thirds, &c. and the Numerator is always wrote alone, and the Denominator of every Decimal is so many Cyphers with Unites annexed, as there are Decimal Places, thus, $\frac{25}{100}$, or $\frac{123}{1000}$, &c. and is produced by parting of an Unite into ten equal Parts, as this Table shews.

One	$\left\{ \begin{array}{l} \text{Unites} \\ \text{Primes} \\ \text{Seconds} \\ \text{Thirds} \\ \text{Fourths} \\ \text{Fifths} \\ \text{Sixths} \\ \text{Sevenths} \\ \text{Eighth} \\ \text{Ninth} \end{array} \right\}$	makes 10	$\left\{ \begin{array}{l} \text{Primes.} \\ \text{Seconds.} \\ \text{Thirds.} \\ \text{Fourths.} \\ \text{Fifths.} \\ \text{Sixths.} \\ \text{Sevenths.} \\ \text{Eighths.} \\ \text{Ninths.} \\ \text{Tenths.} \end{array} \right\}$
-----	--	----------	--

I. Of Reduction of Decimal Fractions.

To reduce a Vulgar Fraction to a Decimal.

R U L E.

Add to the Numerator of the Vulgar Fraction a competent Number of Cyphers, and divide that Sum by the Denominator

Denominator of the same Vulgar Fraction, the Quotient is the Decimal equivalent to the given Vulgar Fraction.

For as the Denominator of the given Vulgar Fraction is to it's Numerator, so is an Unite with so many Cyphers annexed as you intend your Decimal shall have Places to the Decimal required.

But here Note, That in every Quotient that arises in finding a Decimal Fraction, there must be as many decimal Places as you added Cyphers to the Numerator of the given Vulgar Fraction; for what the Work falls short of that, must be supplied by placing Cyphers to the left Hand of the Decimal, as you will the better perceive by the following Examples.

Reduce $\frac{1}{4}$ to a Decimal ?

$$4)100(.25$$

Reduce $\frac{1}{2}$ of any thing to a Decimal ?

$$2)10(.5$$

Reduce $\frac{3}{4}$ to a Decimal ?

$$4)300(.75$$

Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a Decimal ?

First it is reduced to this single Fraction $\frac{1}{4}$. See page 48.

Answer .25

Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ to a Decimal ?

It is this single Fraction $\frac{3}{32}$

$$32)300000(.09375$$

In finding the Decimal of Money, Weight, Measure, &c. you must reduce what you are seeking the Decimal of into the lowest Name mentioned, and divide by what you design for the Integer reduced into the same Name, the Quotient thence arising is the Decimal sought.

What's the Decimal of 1s. one Pound the Integer ?

$$20)100(.05$$

What's the Decimal of 6d. one Pound the Integer ?

It is this Vulgar Fraction $\frac{6}{12} = \frac{1}{2}$ of $\frac{1}{20} = \frac{1}{40}$

$$40)1000(.025$$

What's

What's the Decimal of 11 *d.* one Pound the Integer ?

It is this Vulgar Fraction $\frac{11}{20}$ of $\frac{1}{20} = \frac{11}{200}$

240)11.00000000(.045833333

.....

What's the Decimal of 3 *qrs.* one Pound the Integer ?

It is this Compound Vulgar Fraction $\frac{3}{4}$ of $\frac{1}{20}$ of $\frac{1}{20}$.

960)3.000000(.003125

.....

After this Manner are the Decimal Tables calculated in Books treating of Decimals.

To find the Value of a Decimal Fraction in the known Parts of Money, Weight, Measure, Time, &c.

R U L E.

Multiply the given Decimal by the Denominative Parts of it, and cut off to the Right Hand of the Product, so many Places as the given Decimal has places, and those on the Left Hand are the Value of the given Decimal.

Examples following.

What's the Value of this Decimal .748 of a Pound Sterling ?

	.748
	20
Shillings	14.960
	12
Pence	11.52
	4
Farthings	2.08

Note, Cyphers on the Right Hand of any Decimal are of no Use.

What's the Value of .74415513 of a Year

Answer 271 *d.* : 19 *b.* : 15 *s.* : 50 *d.*

A TABLE

A TABLE of the Decimal Parts of a Foot.

Incr.	Decimal.	Incr.	Decimal.	Incr.	Decimal.
$\frac{1}{4}$	.0208333	$\frac{1}{4}$	.3541666	$\frac{1}{4}$	.6875
$\frac{1}{2}$	.0416666	$\frac{1}{2}$	.375	$\frac{1}{2}$	.708333
$\frac{3}{4}$	.0625	$\frac{3}{4}$	.3958333	$\frac{3}{4}$	.7291666
1	.0833333	5	.4166666	9	.75
$\frac{1}{4}$	.104166	$\frac{1}{4}$	.4375	$\frac{1}{4}$	.7508333
$\frac{1}{2}$	.125	$\frac{1}{2}$	.458333	$\frac{1}{2}$	.7916666
$\frac{3}{4}$	.145833	$\frac{3}{4}$	.479166	$\frac{3}{4}$	.8125
2	.166666	6	.5	10	.833333
$\frac{1}{4}$	.1875	$\frac{1}{4}$	.520833	$\frac{1}{4}$	.8541666
$\frac{1}{2}$	.208333	$\frac{1}{2}$	.541666	$\frac{1}{2}$	.875
$\frac{3}{4}$	.229166	$\frac{3}{4}$	.5625	$\frac{3}{4}$	.895933
3	.25	7	.583333	11	.916666
$\frac{1}{4}$	.270833	$\frac{1}{4}$	.6041666	$\frac{1}{4}$	.9375
$\frac{1}{2}$	.291666	$\frac{1}{2}$	.625	$\frac{1}{2}$	.958333
$\frac{3}{4}$	.3125	$\frac{3}{4}$	.6158333	$\frac{3}{4}$	.979166
4	.333333	8	.6666666	12	.

ADDITION.

This is the very same with Addition of whole Numbers, in which you must observe to place Primes under Primes, Seconds under Seconds, Thirds under Thirds, &c. and cast the Sum up all one as if it were Whole. And when you have done, separate from the Total so many Decimals to the Right Hand, as the greatest Decimal hath Places. As these Examples will better inform you.

Of Decimal Arithmetic.

Fractions to be added.

.473	.870461
.3506	.725
.30751	.604318
.29	.5
.1806	.327

Sum 1.60171

3.026779

Mixt Numbers to be added.

<i>l.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
7.8104	7	15	2	1.984
6104	6	2	0	3.84
5.0081	5	0	1	3.776
4.9102	4	18	2	1.792
3.20759	3	4	1	3.2864
2.071	2	1	5	0.16
1.8	1	16	0	0.

Sum 30.91129

30 18 2 2.8384

20

Shill. 18.22580

12

Pence 2.7096

4

Farthings 2.8384 The same as on the Right Hand.

Of SUBTRACTION.

This is the very same with that of whole Numbers, only remember to separate to the Right Hand of the Remainder, so many Figures for Fractions, as there are Decimal Places in the greatest Number, and to place Primes under Primes, &c. as in these Examples.

Fractions.

Of Decimal Arithmetic.

III

Fractions.

l.

.57

.379

.191

Mixt Numbers.

Ells.

641.

.3765

640.6235

In subtracting Integers and Decimals, observe the following Order.

	<i>l.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
Lent	1681.176	-	1681	3	6	0.96
Paid	946.754	-	946	15	0	3.84
Pounds	734.422		734	8	5	112
	20					
Shillings	8.44					
	12					
Pence	5.28					
	4					
Farthings	1.12					

Multiplication of Decimals.

This in all Respects is the same with whole Numbers, only when the Work is done, observe to cut off to the Right Hand as many Decimal Places as are in both the Factors, that is in the Multiplicand and Multiplier, and if the Product doth not produce so many Places, then you must supply that defect by placing Cyphers to the Left Hand of the Product. Here are three Cases, which mark well.

Case 1. A Decimal Fraction multiply'd by a Decimal Fraction

L 2

Multiplicand

Multiplication of Decimals.

Multiplicand	.75	- - -	.685
Multiplier	.25		.125
	375		3425
	150		1370
Product	.1875		685
			.085625

Case 2. A Decimal Fraction multiply'd by a mix'd Number.

Multiply me	.943	and	- -	.6104
By	7.35	by	- -	29.18
	4715			48832
	2829			6104
	6601			54936
Product	6.93105			12208
				17.811472

Case 3. A decimal Fraction multiply'd by a whole Number.

Multiply me	.7306
By	5062.
	14612
	43836
	36530
Product	3698.2972

Contractions.

Let it be required to multiply 7.03215 by 4.06032, and to have only four decimal Places in the Product in the First, three in the Second, two in the Third, and one in the Fourth.

Note, The Multiplier must be transposed in the Way of Working, and the Unit's Place of the Multiplier is set under

under that Place of the Multiplicand, which you design to retain in the Product.

See the Operations.

I.

$$\begin{array}{r} 7.03215 \\ 230604 \\ \hline 281286 \\ 4219 \\ 21 \\ \hline 28.5526 \end{array}$$

III.

$$\begin{array}{r} 7.03215 \\ 230604 \\ \hline 2813 \\ 42 \\ \hline 28.55 \end{array}$$

II.

$$\begin{array}{r} 7.03215 \\ 230604 \\ \hline 28128 \\ 421 \\ \hline 28.549 \end{array}$$

IV.

$$\begin{array}{r} 7.03215 \\ 230604 \\ \hline 28.1 \end{array}$$

Note, You must always have Regard to the Increase of the two next Figures on the right Hand.

Questions in Multiplication of Decimals.

What will 3 s. 11 d. produce being squared, a Pound the Integer?

Answer 9 d. 0.81654 q.

Quest. 2. What's the Square of 3 s. 11 d. a Shilling the Integer?

Answer 15 s. 4 d. 0.33 q.

Quest. 3. What's the Square of 2 l. 11 s. 7 d. $\frac{3}{4}$ q, a Pound the Integer?

Answer 6 l. 13 s. 4 d. 1 q. 497.

Quest 4. What will 1 s. 6 d. produce being squared, Pound the Integer?

Answer 1 d. 1 q. 4.

But if a Shilling be made the Integer, the Answer will be 2 s. 3 d.

Note, Questions of this Nature admit of infinite Answers according to what you make the Integer.

Division of Decimals.

You are to proceed in this in every Respect as if they were whole Numbers, only when the Work is done you are to find out the Value of the Quotient, by subtracting the Number of decimal Places in the Divisor from the Number of Places in the Dividend, and the Remainder are the Number of Decimal Places to be prick'd off in the Quotient. Here are Nine Cases, which take in the following Order.

Case 1. A whole Number divided by a whole Number.

$$283 \overline{) 154605} (546.307$$

Note, If any Thing remain, add a Cypher, and so on, for every Cypher you add, there will be a Decimal in the Quotient. Here are three Cyphers added.

Case 2. A whole Number divided by a mixt Number.

$$28.3 \overline{) 154605.000} (5463.07$$

Case 3. A whole Number divided by a decimal Fraction.

$$.283 \overline{) 154605.000} (5463.07.$$

Case 4. A mixt Number divided by a whole Number.

$$283. \overline{) 1546.05} (5.46$$

Case 5. A mixt Number divided by a mixt Number.

$$28.3 \overline{) 1546.05} (54.6$$

Case 6. A mixt Number divided by a decimal Fraction.

$$.283 \overline{) 1546.050} (5463.$$

Case

Case 7. A decimal Fraction divided by a whole Numb.
 $283.) .154605(.000546$

Case 8. A decimal Fraction divided by a mixt Number.
 $28.3.) .154605(.00546$

Case 9. A decimal Fraction divided by a decimal Fract.
 $.283.) .154605(.546$

Contractions.

Because the Way of dividing Decimals often proves tedious, when it is required to continue the Division till the Value of the Remainder be small; for this Reason I say, the following Method was invented.

As many Decimals as you intend your Quotient shall contain, retain so many in your Dividend, and then begin to divide with the first Figure as in the common Way, and prick off one Place in the Divisor to the right Hand, and begin with the next Figure, in the Quotient to multiply it by the Figure in the Divisor which stand over the Dot, but do not set it down, but carry it's Increase to the next Figure, and set it down for the first Place of Work under the Dividend, proceed thus, by dotting under the Divisor, by which Means you drop a Place every time, which will much shorten your Work.

E X A M P L E.

Let it be required to divide 31.41592 by 579.268 and to retain five decimal Places in the Dividend.

$$579.268)31.41592(.0542338$$

$$\begin{array}{r} \dots\dots 2896340 \\ \hline 245252 \\ 231707 \\ \hline 13545 \\ 11585 \\ \hline 1960 \\ 1738 \\ \hline 222 \\ 173 \\ \hline 49 \\ 45 \\ \hline 4 \end{array}$$

Example

EXAMPLE II.

Let it be required to divide 406.371082 by 617.26 and to retain three decimal Places in the Dividend ?

$$\begin{array}{r}
 617.26 \overline{) 406.371082} \quad .65835 \\
 \underline{370356} \\
 36015 \\
 \underline{30863} \\
 5152 \\
 \underline{4937} \\
 215 \\
 \underline{185} \\
 30 \\
 \underline{30} \\
 0
 \end{array}$$

When you cannot have the Divisor in the Dividend, drop a Figure in the Divisor by dotting under it, and carry it's Increase to the next Product of the Divisor, as for Example, Divide 21.83 by 9.706 here two Places of Decimals are retained.

$$\begin{array}{r}
 9.706 \overline{) 21.83} \quad .225 \\
 \underline{1941} \\
 242 \\
 \underline{194} \\
 48 \\
 \underline{48} \\
 0
 \end{array}$$

Again,

$$97.06 \overline{) 218.3} \quad 2.25$$

$$\begin{array}{r}
 1941 \\
 \underline{242} \\
 194 \\
 \underline{48} \\
 48 \\
 \underline{0}
 \end{array}$$

The

The Single Rule of Three Direct in Decimals.

OF Ordering the Numbers and stating the Question is in all Respects as in whole Numbers, which we have taught in Chap. IV. therefore we shall proceed to give some Examples.

Quest. 1. If $20\frac{1}{2}$ Yards, of Cloth cost 36*l.* 15*s.* what will $64\frac{1}{4}$ Yards cost.

In Decimals it will stand thus, a Pound the Integer.

$$\begin{array}{r}
 \text{If } \begin{array}{c} y. \\ 20.5 \end{array} : \begin{array}{c} l. \\ 36.75 \end{array} :: \begin{array}{c} y. \\ 64.25 \end{array} \\
 \begin{array}{r}
 36.75 \\
 \hline
 32125 \\
 44975 \\
 38350 \\
 19275 \\
 \hline
 20.5 \overline{) 2361.1875} (115.179 \\
 \quad \cdot \cdot \cdot \cdot \cdot \\
 \quad 205 \\
 \quad \hline
 \quad 311 \\
 \quad 205 \\
 \quad \hline
 \quad 1061 \\
 \quad 1025 \\
 \quad \hline
 \quad 368 \\
 \quad 205 \\
 \quad \hline
 \quad 1637 \\
 \quad 1435 \\
 \quad \hline
 \quad 2025 \\
 \quad 1845 \\
 \quad \hline
 \quad 180
 \end{array}
 \end{array}$$

Remainder 180, to which add 3 Cyphers, and then there will remain 10. and the Quotient 115.179878*l.* = 115*l.* 3*s.* 7*d.* 0.68288*y.*

Quest.

Quest. 2. If 64.25 Yards of Cloth cost 115 *l.* 3*s.* 7*d.* 0.68288*q.* what will 20.5 Yards cost.

Answer 36*l.* 15*s.*

Quest. 3. If for 36 *l.* 15*s.* I buy 20.5 Yards of Cloth, how many Yards may I buy for 115 *l.* 3*s.* 7*d.* 0.68288*q.*

Answer 64.25 Yards.

Quest. 4. If 115 *l.* 3*s.* 7*d.* 0.68288*q.* will buy 64.25 Yards of Cloth, how many Yards may I buy for 36.75 *l.*

Answer 20.5 Yards.

And thus you see how every Question may be varied four several Ways, and one is a sure Proof of another.

Quest. 5. If a Labourer earn 2*s.* 1½*d.* per Day, what doth that come to in the Year? a Pound the Integer.

State it thus.

D.	L.	D.	l.	s.	d.
If 1.	: .1062499	:: 365	: 38	15	7½

But make a Shilling the Integer it will stand thus.

If 1*d.* : 2.125*s.* :: 365*d.*

Of Duodecimal Arithmetic.

THIS is what is called Cross Multiplication amongst Workmen and Artificers; they generally cast up all their measured Work by this Way, and therefore it claims a Place in this Treatise.

First then, Feet multiply'd by Feet produce Feet.

2. Feet by Inches, and divided by 12 are Feet.

3. Feet by Parts, and divided by 12 are Inches.

4. Inches by Feet, and divided by 12 are Feet.

5. Inches by Inches, and divided by 12 are Inches.

6. Inches by Parts, and divided by 12 are Parts.

7. Parts by Feet, and divided by 12 are Inches.

8. Parts by Inches, and divided by 12 are Parts.

9. Parts

9. Parts by Parts, and divided by 12 are Seconds.
That is a Name next less than Parts : As in this Table.

Factors.	Feet.	Inches.	Parts.
Feet.	Feet	are Inches $\div$ by 12 are Feet.	are Parts $\div$ by 12 are Inches.
Inches.	are Inches $\div$ by 12 are Feet.	are Parts $\div$ by 12 are Inches.	are Secon. $\div$ by 12 are Parts.
Parts.	are Parts $\div$ by 12 are Inches.	are Secon. $\div$ by 12 are Parts.	are Thirds $\div$ by 12 are Secon.

EXAMPLE,

Let 5 Feet, 3 Inches and 6 Parts, be multiplied by
2 Feet, 4 Inches and 6 Parts, what will be produced ?

See the Work.

	F.	I.	P.
Multiply	5	3	6
By	2	4	6
	10	6	0
	0	1	0
	1	8	0
	0	1	2
	0	2	6
		1	6
			3
Product	12	6	9 9

In adding the several Products together, you must set down all above 12, and carry the Twelves to the next Denomination.

More

More Examples.

	F.	I.	P.		F.	I.	P.
Multiply	7	8	0		0	11	9
By	0	9	3		0	5	0
	5	3	0		0	4	7
		6	0		0	0	3
		1	9	Prod.	0	4	10
			2				9
Product	5	10	11				

	F.	I.	P.		F.	I.	P.
Multiply me	49	11	9				
By	28	5	3				
	1372	0	0				
	25	8	0				
	1	9	0				
	20	5	0				
		4	7				
			3	9			
			3	0			
	1	0	3	0			
			2	9			
			2	3			
Product	1421	3	4	8	3		

The same performed decimally from the Table,
Page 109.

49.97916
28.4375
24989580
34985412
14993748
19991664
39983328
9995832
Feet 1421.282362500

Feet

Feet	1421.282362500
	<u>12</u>
Inches	3.3883500
	<u>12</u>
Parts	4.66020
	<u>12</u>
Seconds	7.9224
	<u>12</u>
Thirds	11.0688

This Kind of Multiplication may be rationally performed by beginning to multiply by the least Denomination towards the right Hand, and setting down all above 12, carry the Twelves to the next Figure, and add them all up by 12 except the Feet, which must be done by Tens, as in this Example.

		F.	I.	P.	S.	
Multiply me By		9	10	3	9	
		8	11	9	6	
		4	11	1	10	6
	7	4	8	9	9	
	9 0	5	5	3		
7	8 10	6	0			

Answer 88 : 6 : 9 : 1 : 2 : 7 : 6

That is 88 Feet, 6 Inches and 9 Parts, &c. of an Inch, the other Figures being of so small a Value that they are not worth mentioning.

CHAP. II.

Of GEOMETRY.

DEFINITION.

GEOMETRY Originally signifies the Art of measuring the Earth, or any Distances or Dimensions on or within it. But 'tis now used for the Science of Quantity, or Extension, Magnitude, abstractly considered, without any regard to Matter.

Geometry very probably had it's first rise in *Egypt*, where the *Nile* annually overflowing the Country, and covering it with Mud, obliged Men to distinguish their Lands one from another, by the Consideration of their Figure; and to be able also to measure the Quantity of it, and to know how to Plot it, and to lay it out again in it's just Dimensions, Figure and Proportion: After which 'tis likely a farther Contemplation of those Draughts and Figures, help'd them to discover many excellent and wonderful Properties belonging to them, which Speculation continually was improving, and is at this very Day. Before I proceed, I shall first explain some Terms.

1. *Axiom*, is a Principal in any Art, so evident, that it needs nothing but the Light of Reason to demonstrate it.

2. *Construction*, is the drawing of Lines and framing of Figures, on preparing the Proposition for a Demonstration.

3. *Corollary*, is a consequent Truth gained from preceding Demonstration.

4. *Definition*, is the unfolding, or explicating of the Nature and Affection of a Thing in a few Words.

5. *Demonstration*, is the proving of a Thing by Definitions and Axioms, and so from several Arguments drawing a Conclusion, that it has that Affection the Proposition did assert.

6. *Hypo-*

6. *Hypothesis*, is when a Thing is supposed, or given to be so.
 7. *Lemma*, is the Demonstration of some Premise, in order to shorten a following Demonstration.
 8. *Problem*, is when something is proposed to be done.
 9. *Proposition*, is used promiscuously, either for a Problem or Theorem.
 10. *Postulata*, is a grantable Request, or such a Demand as reasonably cannot be denied.
 11. *Scholium*, is a short critical Exposition, gained from a former Demonstration, or a Corollary, wanting an Explanation.
 12. *Theorem*, is when something is proposed to be demonstrated.
-

II. Of Geometrical Definitions.

1. **A** Point is that which hath no Parts, and is the very Beginning of Magnitude.
2. A Line is generated by the Motion of a Point, and is supposed to have Length without Breadth or Thickness.
3. A plain Superficies is generated by the Motion of a right Line, and has Length and Breadth only.
4. *Convex Superficies*, is generated by the Motion of a Curve, or crooked Line, and is well represented by the outside of a Bowl, or Bason.
5. A *Concave Superficies*, is also generated by the Motion of a Curve, or crooked Line, and truly represented by the Concavity of the Heavens, or by the inside of a Bowl.
6. A *Wavy Superficies*, may be represented by the Surface of the Sea in Rough Weather.
7. An Arch, or part of a Circle, flows from a Point described on some particular Centre, and is used in measuring Angles.
8. A plain or right lin'd Angle, is made by the meeting of two right Lines in a Point; and if one of the Lines be continued beyond the Point where the other meets it, there will then be two Angles formed: But if both the

Lines are continued beyond the Point, where they first touched they will then form four Angles.

9. Angles may also be either Acute Right, or Obtuse; that is in Quantity less than 90, just 90, or more than 90 Degrees.

10. A Triangle is a Figure made by three Lines, and if right Lines, 'tis called a right lin'd Triangle: And in respect of the Angles, may be either Acute, Right, or Obtuse.

11. Every plain Triangle may be considered, with relation either to it's Sides, or to it's Angles; as to it's Sides, it may be either Equilateral, Isosceles, or Scalénous.

12. An Equilateral Triangle, is that which has all it's Sides equal to one another, every Angle of which is 60 Degrees.

13. Isosceles, Equicural, or equal leg'd Triangle, is that which hath only two Sides equal: The Angle at the Base are equal.

14. A Scalénous Triangle is that which has no two Sides equal.

15. A Spheric Angle is made by the Interfection of two Curve Lines, or by the meeting of two Arches of two great Circles of the Sphere.

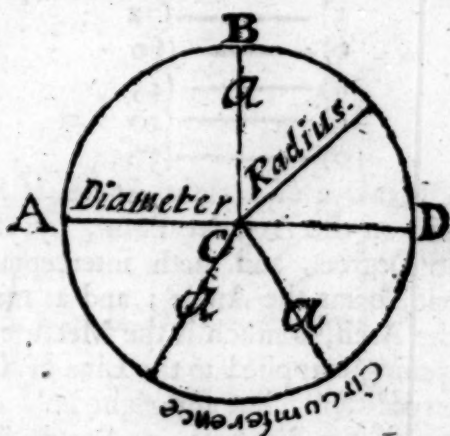
16. An Oblique Spheric Angle is made by the meeting of two Arches of two great Circles of the Sphere, and contains more than 90°, or a Quadrant.

17. A mix'd Angle is when one Leg is streight and the other curved, or crooked, and this you often meet with in the Stereographic Projection of the Sphere, that is, when the right Line cuts the Concave Side of the Arch. But if the Angle be made on the Convex Side of the Arch, being a Tangent to that Arch, then it may be called the Angle of Contact, which is less equal and greater than the Truth, as *Euclid* demonstrates in the 16th Proposition of his third Book; and consequently the Angle of Contact is not any known Quantity.

18. A Spheric Triangle is made by the Interfection of three great Circles of the Sphere, and the Sum of the three Angles of a Spheric Triangle is greater than two right Angles.

19. A Circle hath one Line which encloses it's Space, which is made by the Motion of a Point round some other Point,

Point, at some certain distance, and this curved Line when moved round is called the Circumference, or Periphery, and the Point exactly in the middle on which it was described is called the Centre, from whence all Lines drawn to the Circumference are equal, and are called Radius, whole Sine or Semidiameter, whose double is the Diameter or Chord of 180° . Consequently all Circles have in them 360 Degrees.



20. A Semicircle, is half the Whole Circle, being contained betwixt the Diameter, and that Part of the Circumference cut off by it. From which an Instrument in Surveying takes it's Name. It has in it 180 Degrees, as A B D.

21. A Quadrant is exact one Quarter of a Circle, or 90 Degrees, being bounded by a fourth Part of the Circumference, and two Semidiameters cutting each other at a right Angle in the Center as A C B.

22. A Segment of a Circle is a Part cut off by a right Line, (less than the Diameter) drawn within the Circle, represented by the Chord or right Line A d in the following Figure, and this Segment may be greater than a Semicircle, as A I d, or lesser, as A b c d.

23. A Sector is formed by Part of a Circle, and two Semidiameters drawn from the Center to the Circumference, it's Area is equal to the Superficial Content of a right Cone.

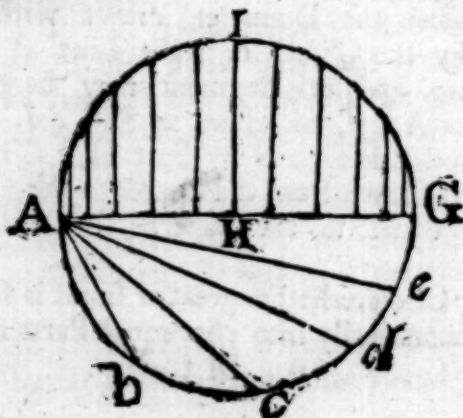
24. Every Circle, whether great or small is supposed to be divided Mathematically into 360 equal Parts called Degrees, and

and every Degree into 60 equal Parts called Minutes, every Minute into 60 other equal Parts, called Seconds, &c. the Reason of dividing the Circle into that Number is, because it will admit of more equal Parts than any other Number, as by the Work is plain.

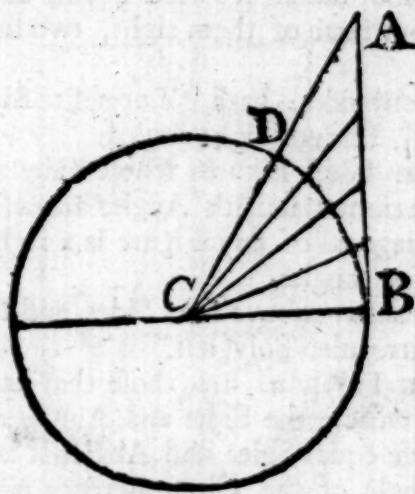
2)	360	(180 Degrees.
3)	————	(120
4)	————	(90
5)	————	(72
6)	————	(60
8)	————	(45
9)	————	(40
10)	————	(36

25. The Measure of a right Angle is an Arch of a Circle described on the Angular Point, by the Sweep or Chord of 60 Degrees, and lieth intercepted betwixt the two Sides that forms the Angle; and as many Degrees as there are in the Arch, so much is the Measure of the Angle, when that opening is applied to the Line of Chords, which is the most expeditious, but any right lin'd Angle may be measured by it's Sine, Tangent, or Secant.

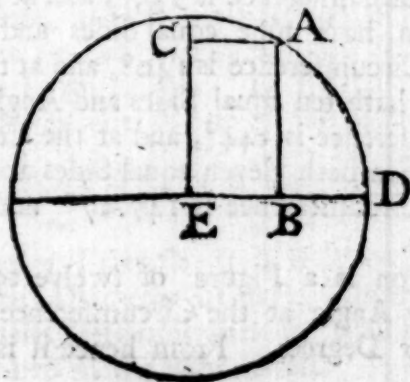
26. The right, or natural Sine of an Arch or Angle, is a Line perpendicular to the Diameter and continued till it touch the Periphery of the Circle; as HI is the Sine of the Quadrant, $AI = GI$, and likewise the other Perpendiculars are Sines of their respective Arches, &c. The Chord of an Arch is like the String of a Bow, and less than the Diameter, (for the Diameter is the Chord of 180°) being drawn to touch the Circle in two Points any where, as Ab , Ae , Ad , Ae , are all Chords of their respective Arches.



27. The Tangent of an Arch, is a Line perpendicular to the Diameter, and touching the Circumference of the Circle, meeting with a Line issuing from the Center, called the Secant : So that A B is the Tangent, and A C is the Secant of the Arch D B, and so of the others in this Figure, where the Secant cuts the Circle and meets with the Tangent, is determined the Tangent and Secant of that Arch.



28. The Complement of an Arch or Angle is what the Angle wants of a Quadrant, or of a Semicircle ; as suppose you have the Sine of 60= $\text{to } A B$, then it's Complement to a Quadrant is $A C$, and $B D$ is the versed Sine : Now 'tis plain from the Figure that the versed Sine and Co-sine, or Sine Complement, are always equal to Radius, for $C A = E B$, and $E B + B D = E D$ the Radius.



N. B.,

N. B. The Chord, Sine, Tangent and Secant of every Arch less than 90, is also the Chord, Sine, Tangent and Secant of so much more than 90 as it's Complement is short of 90 Degrees.

29. A Geometric Square, has four equal Sides, and the Angles all right, or 90 Degrees.

30. A Parallelogram, oblong or long Square, has it's opposite Sides parallel and equal and the Angles right.

31. A Rhombus has all it's Sides equal, and the opposite Angles equal, but none of them right, two being acute and two obtuse.

32. A Rhomboides, hath it's opposite Sides and opposite Angles equal, but not right angled.

33. All four-sided Figures whose Sides are not equal, are called Trapeziums, and it's Angles are also unequal.

34. The Diagonal of any Figure is a right Line drawn from the opposite Angles.

35. If Figures have above four Sides, and those unequal, they are called irregular Polygons.

36. Regular Polygons are those that have more than four Sides, in which the Sides and Angles are equal; so that if it has five equal Sides and Angles it is called a Pentagon, every Angle of the Circumference is 108 Degrees, and at the Center 72°.

A Hexagon has six equal Sides and Angles, every Angle at the Circumference is 120 Degrees, and at the Center 60 Degrees.

A Heptagon has seven equal Sides and Angles, the Angle at the Circumference is 128.5714° and at the Center 51.42857°.

An Octagon hath eight equal Sides and Angles, every Angle at the Circumference is 135°, and at the Center 45°.

A Nonagon hath nine equal Sides and Angles, every Angle at the Circumference is 140°, and at the Center 40°.

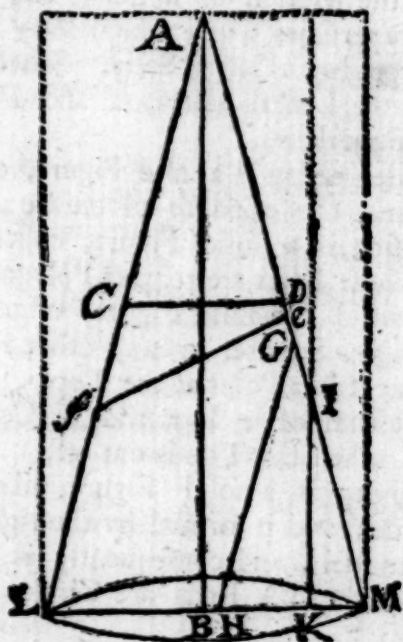
A Decagon hath ten equal Sides and Angles, every Angle at the Circumference is 144°, and at the Center 36°.

An Undecagon hath eleven equal Sides and Angles, every Angle at the Circumference is 147.27° and at the Center 32.72 Degrees.

A Dodecagon is a Figure of twelve equal Sides and Angles; every Angle at the Circumference is 150, and at the Center 30 Degrees. From hence it is plain, that all regular

regular Polygons derive their Name from their Number of Sides, be they ever so many, so that 'tis needless to mention any more of them.

37. A Cone is a solid Figure, rising from a Circle by the Rotation of a Triangle round one of it's Sides and terminates in a Point at the Top, called it's Vertex ; a Cone is equal to one third of it's circumscribing Cylinder, as is plain from the Figure.



Every Cone hath five Sections. (1.) If the Cone be cut by a Plain thro' it's Axis A B, that Section will be a Triangle. (2.) If the Cone be cut any where by a Line parallel to the Base, as C D, this Section will be a Circle. (3.) If the Cone be cut in any oblique Position, as f e, this Section will be an Ellipsis. (4.) If the Cone be cut by a Line as G H, parallel to one of the Sides, this Section will be a Parabola. (5.) If the Cone be cut by a Plain as I K, parallel to the Axis A B, this Section will be an Hyperbola.

Note, That every Triangle is half it's circumscribing Parallelogram, the Diameter of every Circle, is the Side of that Square that circumscribes it, which is double to that Square inscribed in the same Circle.

The
f

*By an oval the ends differing
in the Axis let an Ellipsis*

It differs from the Oval? see p. 152

The Ellipsis is a Geometrical Mean between two Circles described on the Transverse and Conjugate Diameters. The Parabola is $\frac{2}{3}$ of it's circumscribing Parallelogram. The Hyperbola, is $\frac{3}{2}$ of it's circumscribing Parallelogram.

38. There are three Kinds of Magnitudes, *viz.* Length, Breadth, and Thickness; that is to say, a Line, a Superficies, and a Solid.

A Line being generated by the Motion of a Point, a Superficies by the Motion of a Line, and a Solid by the Motion of a Superficies; and which Way soever a solid is moved, it still produces but a Solid. Therefore a Solid is that which hath Length, Breadth and Depth, and it's Bounderies are Superficies.

39. A Parallepipedon is a solid Figure, contained under six Parallelograms, the opposite which are parallel.

40. A Prism, is a solid Figure, contained under several Plains, whose Bases are regular Polygons.

41. A Pyramid is a solid Figure, whose Base may be either a Triangle, Square, or any other Polygon, whose several Planes meet in a Point at the Top.

42. The Frustum of a Pyramid or Cone, is only the remaining Part when the Top is cut off.

43. A Cylinder is a solid Figure, like the Rowling Stone of a Garden, and is formed by the right Line moving round parallel to itself, and consequently is in all Places of the same Diameter, it's Ends are Circles and parallel to each other.

44. A Spheroid is a solid Figure, made by the Motion of a Semi-Ellipsis, the transverse Diameter remaining fix'd.

45. A Sphere or Globe, is a Solid, perfectly round every where, produced by the Rotation of a Semi-circle, the Diameter remaining fix'd.

46. The Segment of a Sphere, or Globe, is a Piece cut off less than half the Globe; and the greater Part of it is called a Frustum.

47. A Parabolic Spindle, is a Solid, formed by the Rotation of an acute Parabola, about it's greatest Ordinate, remaining fixed.

48. A Parabolic Conoid, is made by the Rotation of a Semi-parabola, the Abscissa remaining fixed.

49. An Hyperbolic Conoid, is generated by the Motion of a Semi-Hyperbola, the transverse Diameter remaining fix'd.

50. The Tetraedron, is one of the Pythagorian, or Platonic Bodies, and is a Solid Figure, contained under four equal and equilateral Triangles, which may be deem'd a little Pyramid.

51. The Hexahedron, or Cube, is another of the Platonic Bodies, and is a solid Figure, contained under six equal Sides, or Geometric Squares.

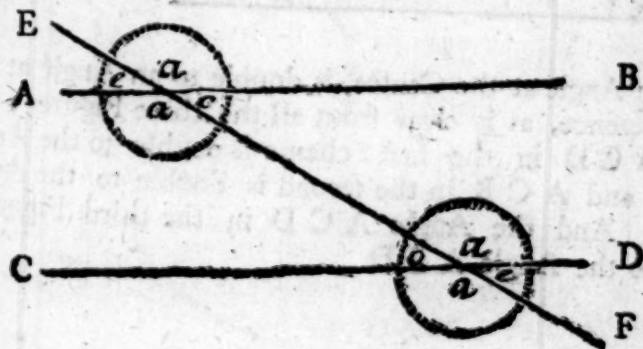
52. The Octahedron is another of the Platonic Bodies, and is a Solid, contained under eight equal and equilateral Triangles.

53. The Dodecahedron is another of the Platonic Bodies, and is a Solid, contained under twelve equal equilateral, and equiangular Pentagons.

54. An Icosahedron, is also one of the Pythagorian Bodies, and is a solid Figure, contained under twenty equal equilateral Triangles.

III. Of Geometrical Theorems.

1. IF two right Lines be parallel at any distance, as A B and C D, and be cut by a third E F, at any Angle whatsoever, I say the Angles of the one are equal to the Angles of the other. $a = a$ and $e = e$.

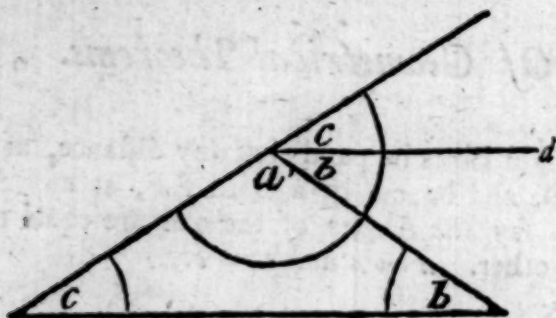


2. A

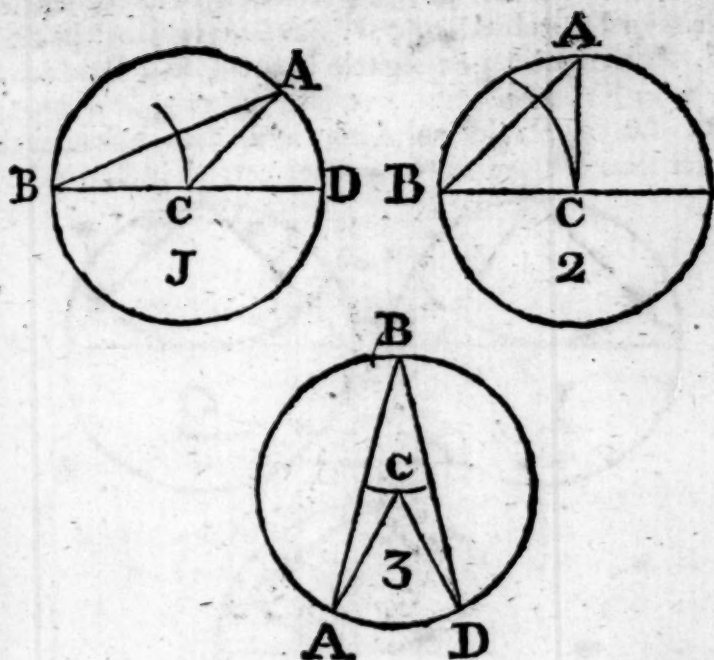
2. A right lin'd Triangle, made between two parallel Lines, the internal Angle is equal to the external; and that all the three Angles taken together are equal to a Semicircle. $b = b$, and $c = c$, and that a is = to Complement b and c .



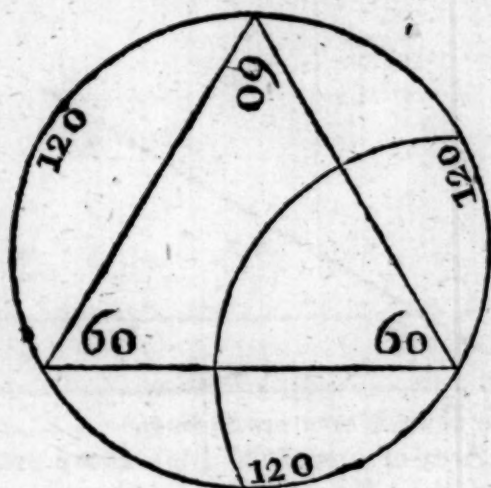
3. Any side of a right lin'd Triangle, being produced; the external Angle is equal to the Sum of the two acute internal Angles. And further, if we draw a Line parallel to the Base as $a d$, it will divide the external Angle into two other Angles, which will be in Quantity equal to the two internal Angles seperately.



4: An Angle at the Centre, is double to an Angle at the Circumference, as is clear from all the three Figures; the Angle $A C D$ in the first Scheme is double to the Angle $A B D$, and $A C B$ in the second is double to the Angle $A B C$. And the Angle $A C D$ in the third Figure is double to the Angle $A B D$.



5. Every Equilateral Triangle inscribed in a Circle, the internal Angles in the Triangle are equal to a Semicircle, and the external to the whole Circle, or 360 Degrees, as per Figure. For $60 \times 3 = 180$. And $120 \times 3 = 360$.



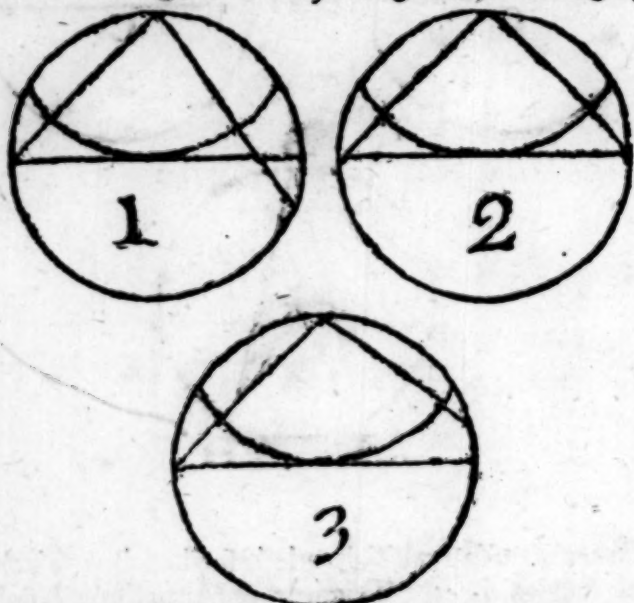
N

6. An

6. An Angle in an Arch greater than a Semicircle, is less than a Quadrant, or 90° . *Euclid* 31, 3. As in Fig. 1.

7. An Angle in a Semicircle is equal to a Quadrant, or 90° , as in Fig. 2.

8. An Angle in an Arch less than a Semicircle is greater than a Quadrant or 90° Degrees, as in Fig. 3.



9. Equiangular Triangles have the Sides about the equal Angles proportional. For as,

$AB : AD :: BC : DE$, or as $AB : BC :: AD : DE$.

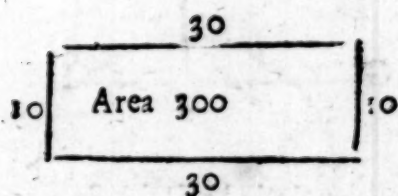
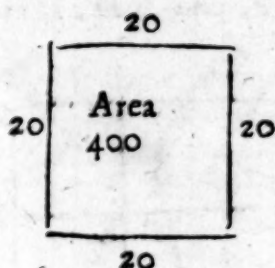


This is useful in Mercators Sailing.

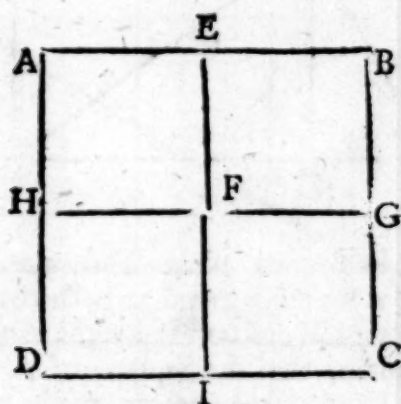
10. The Area of superficial Figures are proportional at the Square of their Sides that bound them.

For

For in these two Figures the Area of the Square, exceeds that of the Parallelogram, proportional to the Square of their Sides; although they be 80 each all round, yet the Area of the Square exceeds the Area of the Oblong by 100.



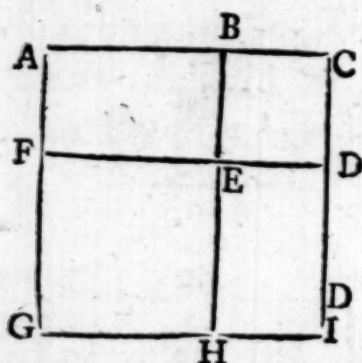
11. When a right Line is divided into two equal Parts; the Square made of those Parts are equal but to $\frac{1}{4}$ of the Square made of the whole Line A B, whose Square is = A B C D. And the Square of A E, is = A E F H, one fourth Part of the great Square. But the Squares made of A E and E B, are equal to half the great Square.



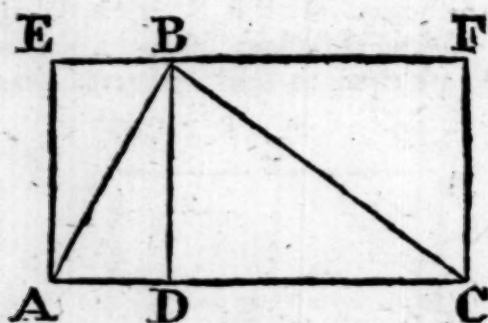
N 2

12. When

12. When a Line is divided by chance into two unequal Parts, the Square of the whole Line is equal to both the Squares made of the Parts, and to two Parallelograms comprehended under the same Parts also: For the Square $B C D E$, and the Square $F E H G$, with the Parallelogram $A B E F$, and $E D I H$ are equal to the Square $A C I G$.

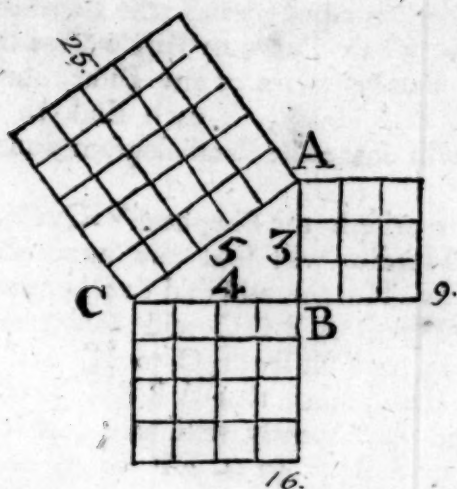


13. Every Triangle is half the Square or Oblong that circumscribes it. In the Triangle $A B C$, let fall the Perpendicular $B D$. Then 'tis plain from the Figure that $A B C$ is half of $A E F C$.



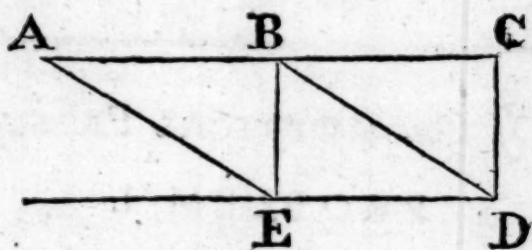
14. In all right angled plain Triangles, the Square made of the Hypotenuse, is equal to both the Squares made of the two Legs which contain the right Angle. For by the Figure it is plain, that the Square of $A B$ $3=9$, and the Square of $B C$ $4=16 + 9=25 =$ the Square of $A C$.

15. The



15. The diagonal Line of Every geometric Square, is double in Power to the Side of the same Square.

16. Parallelograms, which stand on the same Base or on equal Bases, and in the same Parallels, are equal one to the other. For $BCDE = ABDE$.



17. Every Polygon, is equal to a Parallelogram, whose Length is equal to half the Perimeter or Circumference thereof, and Breadth to a Perpendicular drawn from the Centre to the middle of any Side of the same. *Euclid 4th, 1.*

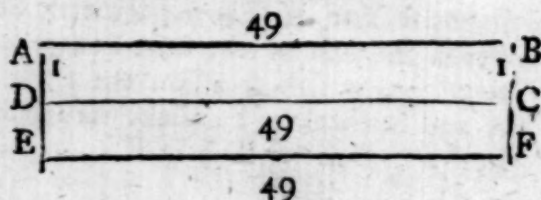
18. Every Circle is nearly equal to a Parallelogram, whose Length is equal to half the Circumference, and Breadth to the Semidiameter.

19. Every Circle's Sector is nearly equal to a Parallelogram, whose Length is equal to the Semidiameter of the Circle

Circle, and Breadth to half the Curve or Arch-line thereof. Or whose Length is equal to half the Semi-diameter, and Breadth of the whole Curve or Angle-Line thereof.

20. If any Parallelogram of any Dimensions whatsoever have for it's End Unity, if each End be increased by Unity, that will make the Parallelogram double to what it was before.

From hence ariseth the Shepherd's Question : *viz.* If an hundred Hurdles will fold one hundred Sheep, one hundred and two Hurdles will fold two hundred Sheep. For if the Parallelogram A B C D will hold 100 Sheep, take another Hurdle and set from C to F, and another from D to E, 'tis then plain, that A B F E is twice as big as A B C D, and consequently will hold double the Number of Sheep. For $AB = 49 + EF = 49 = 98. + BF 2,$
 $+ AE 2 = 4 + 98 = 102$ Hurdles.



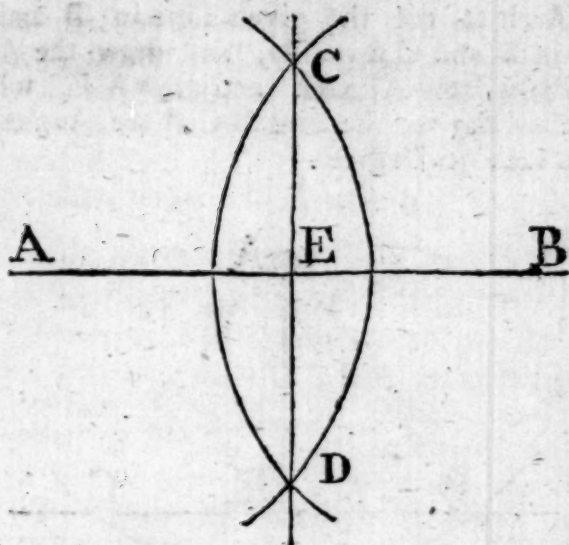
IV. Of GEOMETRICAL PROBLEMS.

PROBLEM I.

To divide a given Line into two equal Parts.

LET the Line given be A B, set one Foot of the Compasses in A, and open the other Foot to any Distance above half the Line, and sweep the Arch C D, with that Extent or Opening of the Compasses, set one Foot at B and draw the Arch C D, lay a Ruler to the Intersection C D, and draw the right Line C D E and it will divide the given Line A B in E, into two equal Parts.

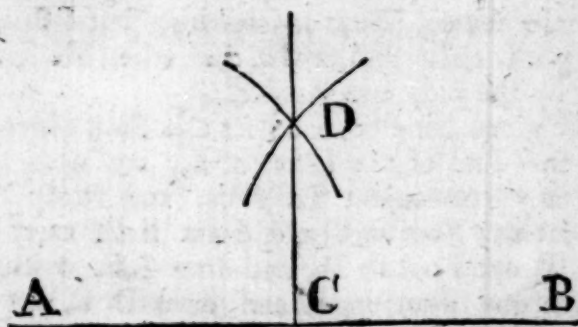
PROBLEM



PROBLEM II.

To erect a Perpendicular at the middle of a given right Line.

By the last Problem find the middle of the Line A B at E, set one Foot of the Compasses at A, and draw the Arch D, with the same Extent set one Foot in B, and draw another Arch at D, where these two Arches intersect at D lay a Ruler, and draw the Line D C, and 'tis done.

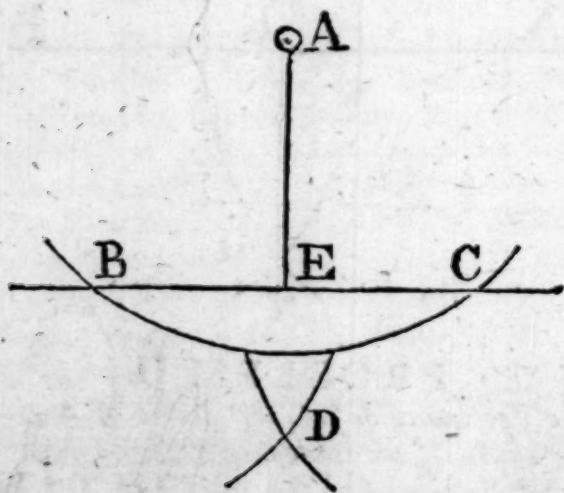


PROBLEM III.

To let fall a Perpendicular upon the Middle of a given Line.

Let the Point above be A, set one Foot of the Compasses in the given Point and open the other Foot and draw

draw the Arch to cut the given Line in B and C, set one Foot in B and C severally, and draw the Arches at D, lay a Ruler from A to D, and draw A E, which is a true Perpendicular to the Line B C; the Angles A E B, A E C are each 90 Degrees.



PROBLEM IV.

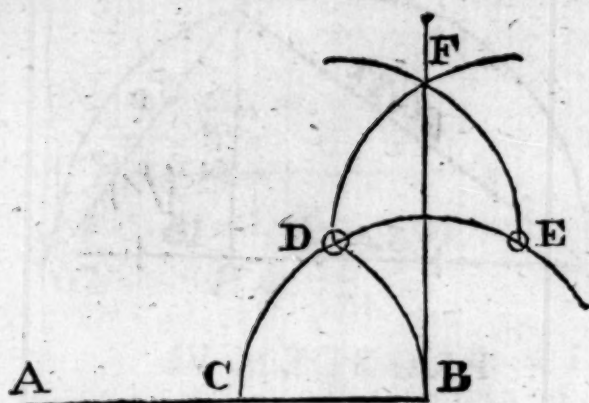
To erect a Perpendicular at the End of a given right Line.

There are several Ways to do this, but I shall content my self with one, and leave the other Methods to be practised by the judicious Reader.

Let the given Line be A B, set one Foot of your Compasses at the End of the Line at B, and open the other Foot to any convenient Distance, and sweep the Arch C D E, set one Foot in C and draw B D, carry that extent and set one Foot in D, and draw E F, with the same extent set one Foot in E and draw D F, lay a Ruler from F to B and draw F B, which shall be at right Angles to A B as was required.

See Theorem 2 of the 8. page 134. where this is performed according to *Euclid*.

PROBLEM



PROBLEM V.

We have a Problem directing us how to draw Parallel Lines, but since the Invention of the Parallel Rules, I rather chuse to advise the Young Practitioner to buy the Ruler, and make Use of that, before the Problem.

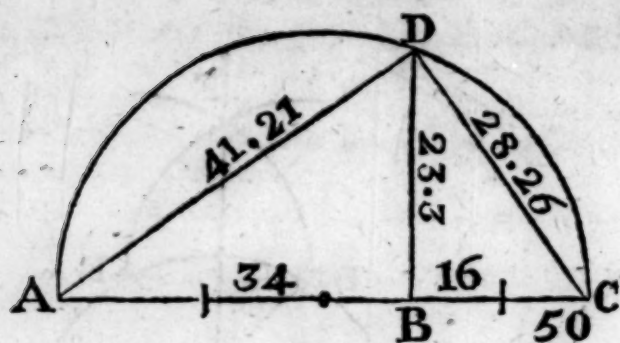
Also parallel or centric Circles are easily drawn without giving any Directions.

To find a Geometrical mean Proportion between any two given right Lines.

This we have shewn how to do in Numbers when we taught the Use of the Square Root; it now remains to shew the young Geometrician how to do it by Lines which will give him a bright Idea of many useful Things in Geometry.

Let the two Lines given be $AB = 34$, and $BC = 16$, draw an occult Line, and from a Scale of equal Parts take 34, and lay it from A to B. from the same Scale take 16 and set it from B to C, by which the first Problem bisect AC, and sweep the Semi-circle ADC, from B erect the Perpendicular BD, which measured on the same Scale you will find to be 23.3, and that is a true geometrical mean Proportional between 16 and 34. Draw AD and CD, so shall the Angle ADC (by *Theorem* 8.) be a right Angle; and by *Theorem* 14 AD will be found 41.21, and CD 28.26. For as $BC\ 16 : BD\ 23.3 :: BD = 23.3 : AB\ 34$.

PROBLEM

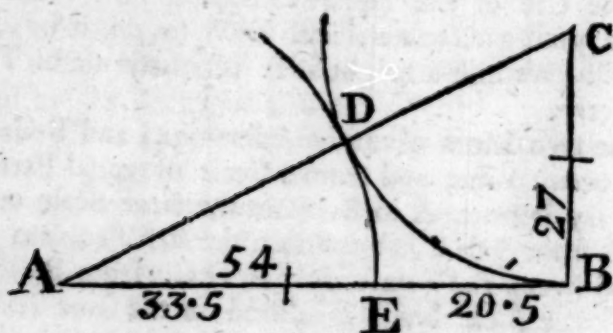


PROBLEM VI.

To divide a right Line in extreme and mean Proportion.

Let the given right Line be $AB = 54$, from a Scale of equal Parts, or from the Line of Lines on the Sector, (which how to do, we shall shew when we come to treat of the Use of that Instrument) take half of $AB = 27$ and set it at right Angles from B to C , set one Foot of the Compasses in C , and draw the Arch BD , joyn A and C , then $AD = AE$ measured on the same Scale will be found to be 33.5 , then $AB = 54 - AE\ 33.5 = EB\ 20.5$

For as $\begin{cases} EB. & AE. & AE. & AB. \\ 20.5 : 33.5 :: 33.5 : 54 \end{cases}$

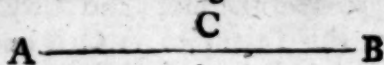


PROBLEM VII.

To divide a given right Line into any Proportion required.

Let the given Line be $AB = 60$, and I would have it divided in proportion as 40 to 50? Add

Add the two Numbers of your Proportion together, viz. $40 + 50 = 90$, then say, as $90 : 60 :: 33.33$ the greater Part of the Line. Then $AB\ 60 - 33.33 = 26.67$. the lesser Part of the given Line.

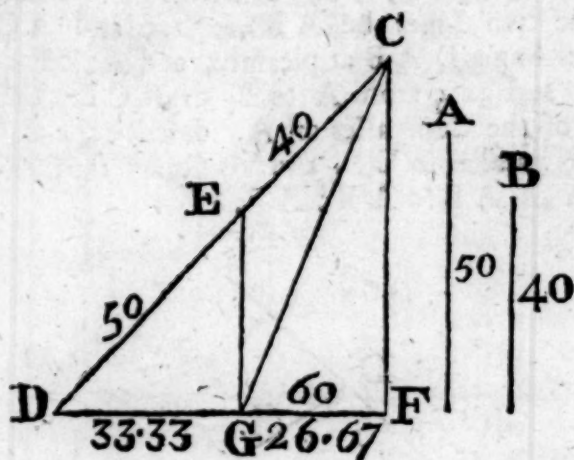


Or for C B the lesser Lines you may say

AC CB

As $50 : 40 :: 33.33 : 26.67$.

But this may be done Geometrically thus, Let DF be $= 60$ and be divided in the Proportion of A to B. Make the Angle CDF any Quantity at pleasure, and draw DC, erect the Perpendicular FC, take the Line A and set from D to E, and B set from E to C, draw EG parallel to CF, so shall DG and GF measured severally on the Line of equal Parts be 33.33 and 26.67, and in Proportion as 50 to 40; draw the Line GC, so shall the Triangle CDF, be to the Triangle CGF, as 50 to 40.

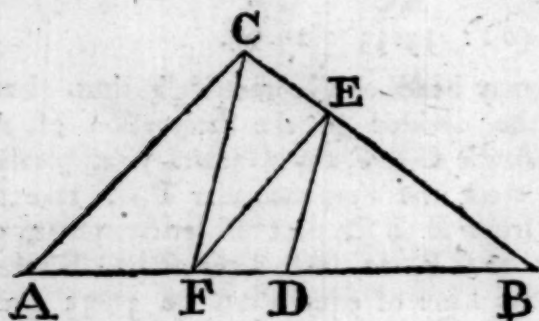


Note, From hence we learn to divide a Field into two equal Parts, so that each Person might have the Benefit of the Water (for his Cattle) which was placed on one Side thereof.

Let

Let $A C B$ be the Field, and a Bason of Water at F be required to be so divided into two equal Parts that each might enjoy an equal Share of the said Water.

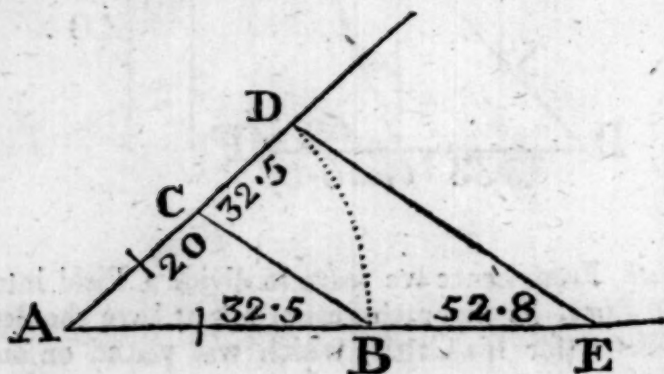
First divide $A B$ into two equal Parts at D , then draw $F C$, and $D E$ parallel thereto, and lastly draw $F E$, and 'tis done. Now the Triangles $A C B$ and $F E B$ are Similar, therefore $A C E F = F E B$.



PROBLEM VIII.

Two Lines given, to find a Third in Proportion.

Let the two Lines be $AB = 32.5$ and $AC = 20$. Make the Angle $D A B$ at pleasure, and set off 20 from A to C , and 32.5 from A to B , draw $C B$, then with one Foot of the Compasses on A , describe the Arch $B D$, draw $D E$ parallel to $C B$, and 'tis done; for then it will be, As $A C : A B :: A B : A E$.

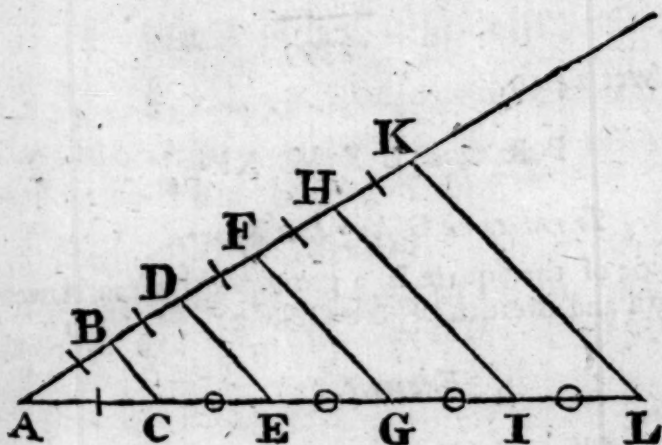


PROBLEM

PROBLEM IX.

To Three given right Lines to find a Fourth, Fifth, Sixth, &c. in Proportion.

Make the Angle KAL any Quantity at pleasure; let AB, AC, and AD be given to find others in Proportion; draw BC and DE parallel to BC. Then, as $AB : AC :: AD : AE$. Then from the same Scale of equal Parts that you laid off AB, set off AF, AH, and AK, and draw FG, HI, and KL parallel to BC, and you will have these Proportions, viz. As $AB : AC :: AF : FG$: and to HI and to KL, &c.



PROBLEM X.

To make a Square equal to a Triangle.

The Side of the Square is a geometrical Mean between the Base and half the Perpendicular.

Example.

Let the Base of a Triangle be 30, and the Perpendicular 40, what's the Side of a Square equal.

O

See

See the Work.

Base	- - -	30
$\frac{1}{2}$ Perpendicular		20
		600 (24.49
	44)	200
		176
	484)	2400
		1936
	4889)	46400
		44001
		2399

Answer 24.49.

P R O B L E M X I.

To reduce an Oblong to a Square.

The Side of the Square is a geometrical Mean between the Length and Breadth of the Oblong.

Example.

Let the Length of the Oblong be 40, and Breadth 18, what's the Side of the Square equal?

Operation.

	18
	40
	720 (26.8
	46)320
	276
	528)4400
	4224
	176

P R O B L E M

P R O B L E M XII.

To reduce a Rhombus to a Square.

The Side of a Square is a geometrical Mean between the Side of the Rhombus and it's Perpendicular.

Example.

First, the Angles of the Rhombus are always the same, viz. 60, 60, and 120, 120. which all added together, make 360; and if we put 1 for the Side of the Rhombus, the Perpendicular will be found to be .866, and the Side of the geometrical Square equal is 93.

P R O B L E M XIII.]

To reduce a Rhomboides to a Square.

The Side of the Square is a geometrical Mean between the longest Side and the Perpendicular.

Example.

Let the Side of the Rhomboides be 50, and the Perpendicular 22. What's the Side of the Square equal?

Operation.

$$\begin{array}{r}
 22 \\
 50 \\
 \hline
 1100 \quad (33.16 \text{ Side Square equal} \\
 9 \\
 \hline
 63 \overline{) 200} \\
 \underline{189} \\
 661 \overline{) 1100} \\
 \underline{661} \\
 6626 \overline{) 43900} \\
 \underline{39756} \\
 4144
 \end{array}$$

○ 3

PROBLEM

PROBLEM XIV.

To reduce a Trepezium to a Square.

Every Square's Side is a geometrical Mean between the Diagonal and half the Sum of the Perpendiculars.

EXAMPLE.

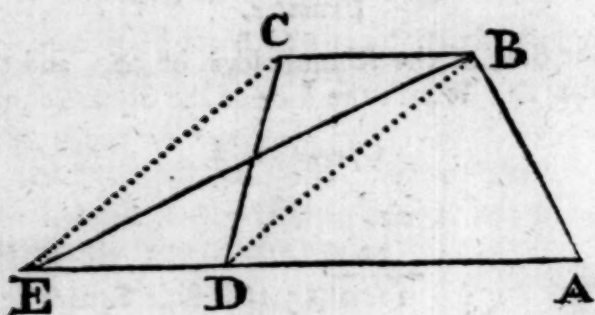
Let the Diagonal be 60, and the half Sum of the two Perpendiculars 23, what's the Side of the Square equal?

Answer 37.14.

PROBLEM XV.

To reduce a Trepezium to a Triangle.

Let the Trepezium be $ABCD$. Draw the Diagonal BD , and continue AD to E , draw CE parallel to BD , draw BE and 'tis done. So that the Triangle $EBA =$ the Trepezium $DCBA$. See Theorem 16.

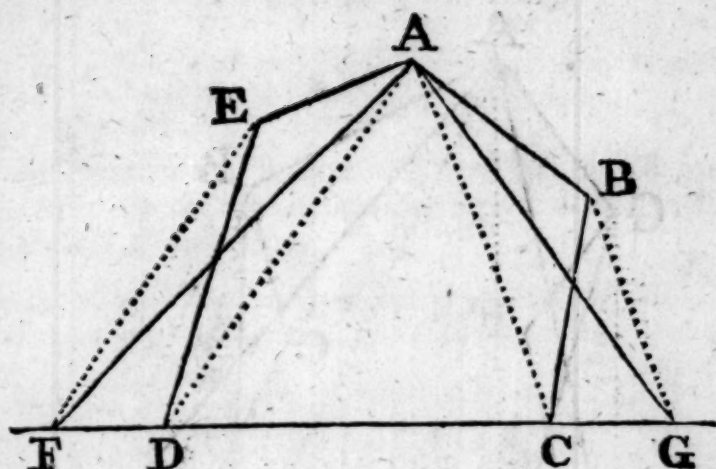


PROBLEM XVI.

To reduce an irregular Figure of Five Sides into a Triangle.

Let $ABCDE$ be an irregular Figure. First, continue the Side DC to F and G at pleasure: Draw AC and AD , then draw EF and BG parallel to AD and AC . Lastly, draw AF and AG , so shall the Triangle AFG be $=$ to the irregular Figure $ABCDE$.

PROBLEM



PROBLEM XVII.

To reduce an irregular Figure of Six or Seven Sides into a Triangle.

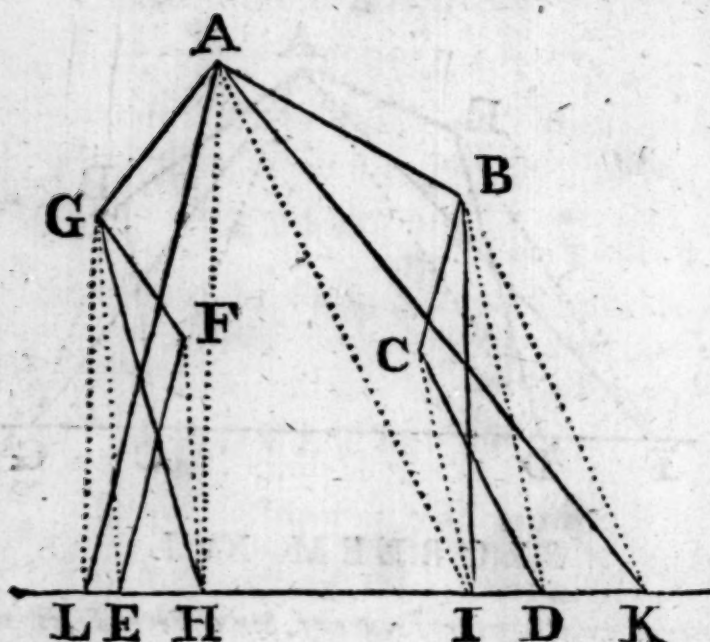
Let $ABCDEFG$ be a Plot to be reduced to a Triangle.

First, By the last Problem reduce the Seven Sides into Five, thus ;

Draw the Line GE , and parallel thereto FH , then draw the Line GH , whereunto the two Lines GF and FE are thereby reduced. Then draw the Line BD , and parallel thereunto CI , and BI , now are the two Lines BC and CD reduced to a streight Line BI , and the whole Plot to a Figure of five Sides

Secondly, To reduce this to a Triangle.

Produce the Side ED to K and L , and draw the Lines AI and AH , and parallel to them BK and GL , cutting the Line ED , (being extended) in K and L . Lastly, Draw AK and AL , and 'tis done, for I say, the Triangle, ALK is = to the irregular Figure $ABCDEFG$.



PROBLEM XVIII.

To reduce a Pentagon to a Square.

The Side of every Square is a geometrical Mean between half the Perimeter and the Perpendicular. As suppose the Side of the Pentagon be 1, the Perpendicular is .6882, then what's the Side of the Square equal?

See the Work,

Perpendicular	- -	.6882
Half N ^o Sides	- -	2.5

34410
13764

1.720500 (1.312 ferè.

23)	72
	69
261)	305
	261
262)	4400

PROBLEM

PROBLEM XIX.

Given, a Square of any Dimension whatsoever, to make another bigger or lesser, at pleasure.

In Theorem 15, I have told you that the Diagonal of one Square is the Side of another Square double in Power to the said given Square.

R U L E.

Multiply the Square of the Side by 2, 3, 4, &c. the Square Root of the Product will give the Side of a Square whose Area shall be 2, 3, 4 times greater than the given Square, so that if the Side of one Square be 12, you will find the Side of another Square twice as big to be 16.97, and thrice as large to be 20.78, and four times as large, the Side will be 24.

If the Side of a Square be given to find the Transverse and Conjugate Diameters of an Ellipsis, that shall be equal in Area to that of the Square.

First, Find the Diameter of a Circle thus, *viz.* Multiply the Side of the Square by 1.128379, or divide it by .886227 the Product, or Quotient, is the Diameter of a Circle equal in Area: Then find the Circumference of it thus, Multiply the Diameter always by 3.14159 and the Product gives the Circumference, then half Circumference multiply'd by half Diameter is the Area or Superficial Content equal to that of the given Square: Then because every Ellipsis is a geometrical Mean between two Circles, described on the two Diameters of the Ellipsis, those Diameters may be taken at Pleasure, and consequently various Ellipses may be found to answer in superficial Content to that of the given Square: For the Ellipsis being a Figure that flows between the Circle and Parabola.

But here {Note, That 1.128379 is the Diameter of a Circle, whose Area is equal to the Square, whose Side is Unity or 1. And .886227 is the Side of a Square equal when the Diameter of a Circle is one. And 3.141592 is the Circumference of a Circle, when the Diameter is one: Near the Truth.

PROBLEM

and any part of a Circle in it. p 152 X

PROBLEM XX.

To lay down an Ellipsis by the Line of Sines on the Sector, having given the Transverse and Conjugate Diameters.

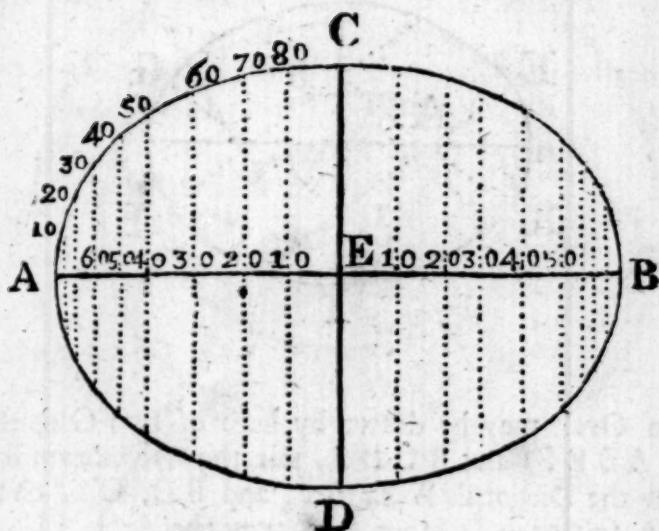
Take $AE = EB$ in your Compasses, and open the Sector at 90, 90 on the Line of Sines, and as the Sector now stands, take off the Sines 10, 20, 30, 40, 50, 60, 70, 80, and set them from E each Way towards A and B, draw occult Lines through those Points in the transverse Diameter parallel to the Conjugate, which done, set the Sector on the Sines 90, 90 to the Radius CE, and take in your Compass the Sine of 80, and set from 10 to 80, take the Sine 70, and set from 70 to 20 on each Side the Conjugate Diameter, the Sine 60, set from 30 to 60, the Sine 50 set from 40 to 50, the Sine 40 set from 50 to 40, the the Sine 30 set from 60 to 30, the Sine 20 set from 70 to 20, the Sine 10 set from 80 to 10, so will the Points 10, 20, 30, 40, 50, 60, 70, 80, CBDA be in the Ellipsis, which with an even Hand draw the Curve and 'tis done: Which is truly Mathematical, and has not any part of a Circle in it.

Note, The Ellipsis being delineated as above, upon Pastboard and neatly cut out, you may then draw the Ellipsis by help of that Pattern Pastboard, with Ink to a great Exactness, which is always my Method in drawing the Azimuths and Meridians in the Orthographick Projection of the Sphere.

Every Ellipsis has two Focus, or Navel Points, in the transverse Diameter of the Ellipsis, through which Lines drawn at right Angles thereto are called the *Latus Rectum*, the Length of which may be found by this Proportion.

As the Transverse Diameter is to the Conjugate, so is the Conjugate Diameter to the *Latus Rectum*.

PROBLEM



PROBLEM XXI.

Of Ovals.

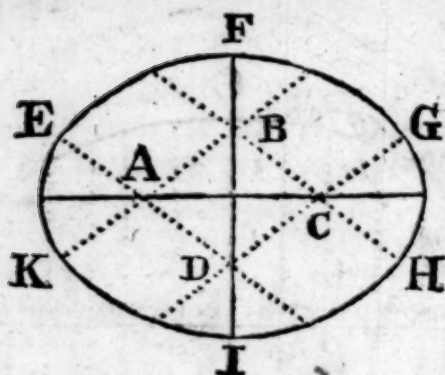
DEFINITION.

Ovals are something like Ellipses, of which there are divers Sorts, and are all made of Part of Circles, and consequently cannot be Ellipses, as noted above.

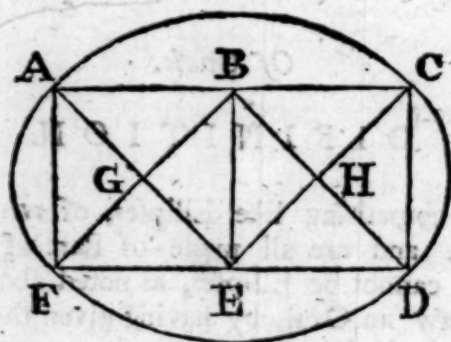
1. To draw an Oval, by having given the two Diameters, divide each Diameter into four equal Parts, and through those Parts draw the Lines A B C D, then set one Foot of the Compasses in D and extend the other Foot to F, and draw the Arch E F G; with this Extent of the Compasses set one Foot in B, and draw the Arch H I K; set one Foot of the Compasses in A and C severally, and draw the Arch G H and E K, and 'tis done.

2. Section of a Cone - and not of a cylinder - in the oval, one of the ends more pointed than the other

2. An



2. An Oval may be drawn by help of two Geometric Squares A B E F, and B C D E, join the two Sides in B E, and draw the Diagonals A E, B F, and B D, C E, on the Centre E, with the Distance E A draw the Arch A C, on the Center B, with the Distance B F draw the Arch F D, and you will have a neat Oval:

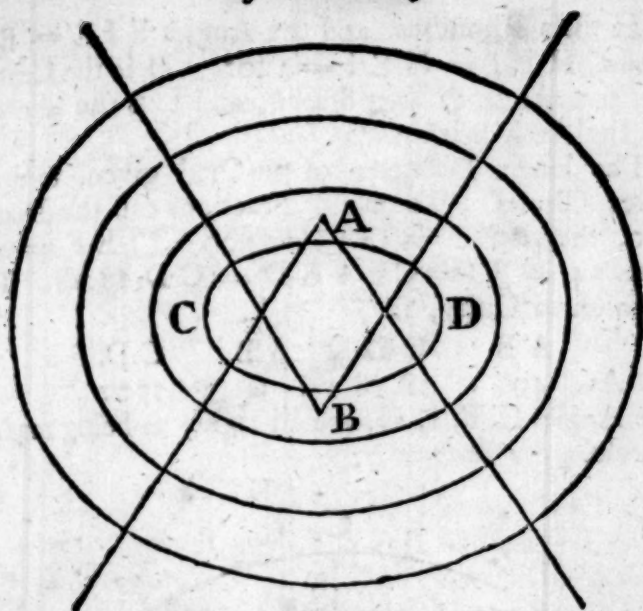


3. Several Ovals may be drawn in one Figure parallel to each other, by making the Angle C A D = 60 = to the Angle C B D, for the four Angular Points A B C D are the Center on which four Arches or Part of Circles are drawn, which joined together at the Legs or Lines A C, A D, B C, B D, being continued, you may draw as many Ovals as you please, as you may the better perceive by this Figure.

*Sy. then
from centres
H and G
respectively draw
arcs C D and
A F.*

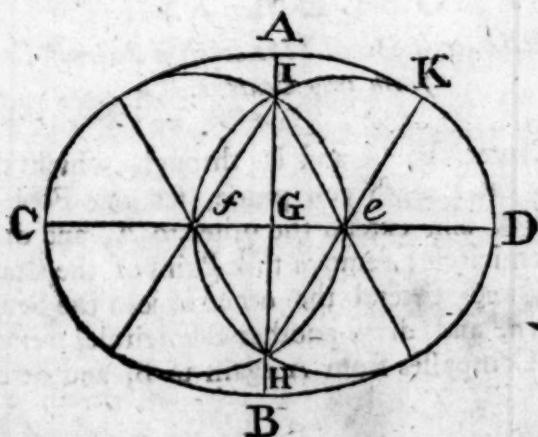
*Sy. are not the above ellipses
and not ovals?*

4. An



4. An Oval may be drawn (by having only one Diameter of the Oval given) on two Circles. If the Line CD be 1, then will AB be .756 thus proved. To avoid Fractions, suppose CD 12, then $FI = 4 \square = 16$. $FG = 2 \square = 4$. and $16 - 4 = 12 \square \sqrt{= 3.46 = GI. + GH} 3.46 = 6.92 = IH$. And $HK = CE = FD = 8 - HI = 6.92 = 1.08 = AI + IH = 6.92 + HB 1.08 = AB 9.08$. Then to reduce them to the Unity, say,

$$\begin{array}{cccc} CD & AB & CD & AB. \\ \text{As } 12 : 9.08 :: 1 : .756 \end{array}$$



FIEH

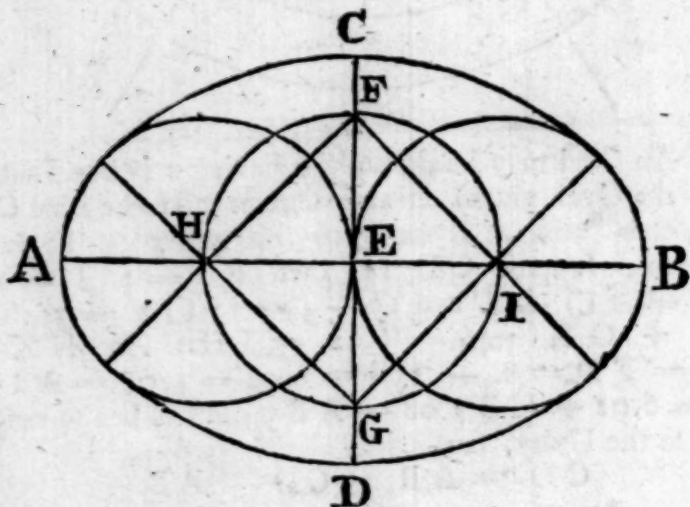
$FIEH$ is a Rhombus, and the Angles $FIE = FHE = 60$, $\angle HFI = HEI = 120^\circ$. H is the Center on which the Arch AK is drawn, and I is the Center on which the Arch B is drawn

5. The longer Diameter of an Oval given, to draw it on three Circles. To avoid Fractions call the Diameter AB 40, then is $EI = 10 \square = 100 + \square EF = 200 \square \sqrt{= 14.14 = FI = CE + ED = CD 28.28}$. To reduce which to Unity, say,

$AB \quad CD \quad AB \quad CD.$

As 40 : 28.28 : : 1 : .707

The Angles H, F, I, G , are all right, as being made in a Semicircle. See Theorem 7.

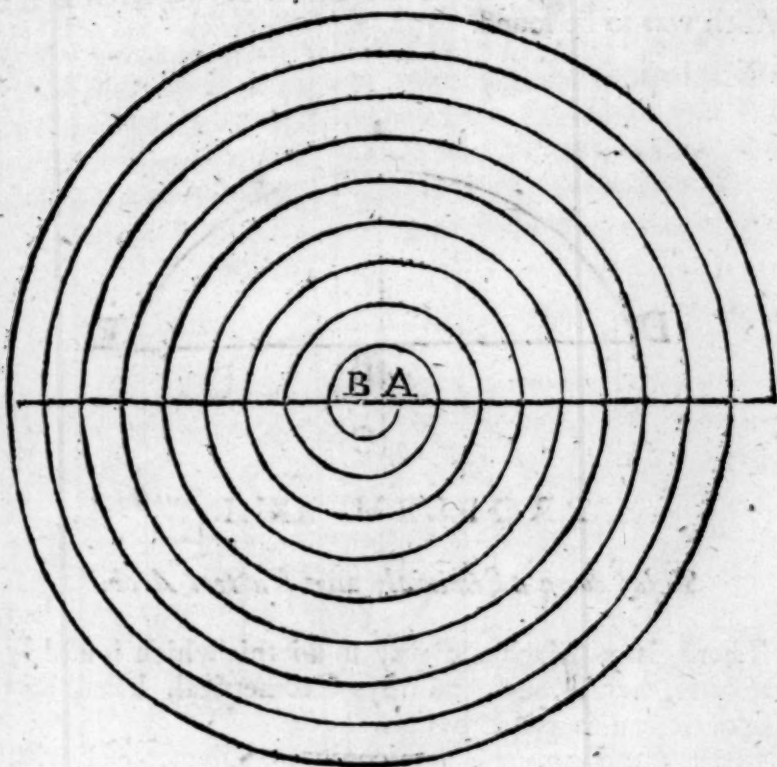


PROBLEM XXII.

To draw a Helix, or a Spiral Line with a Pair of Compasses from two Centers.

Let two Centers be A and B . through which draw a right Line, what Length you please, set one Foot of the Compasses in B , and extend the other to A , and draw the first little Semicircle; remove that Point of the Compasses from B to A , and extend the other to join the Semicircle just now drawn, and draw another Semicircle; remove the Point of the Compasses from A again to B , and extend the other

other Point to the last Semicircle, and there join it, and draw another Semicircle, do thus as long as you please, and you will have a delightful Spiral Line, rowling in several Circles, as per Fig.



PROBLEM XXII.

How to strike an Arch to any Height or Breadth.

Let it be required to make an Arch four Foot high, $= AB$, and the width on the Base 12 Foot $= DE$?

Now all the Matter lies to find the Center that will sweep this Arch, for our Assistance herein, we must call in Prob. 5, by which we find that $BE = BD$ is a geometrical Mean Proportional between AB and the rest of the Diameter, therefore, divide the Square of $BE = 36$, by $AB = 4$, and the Quotient 9 is the other Part of the Diameter, then

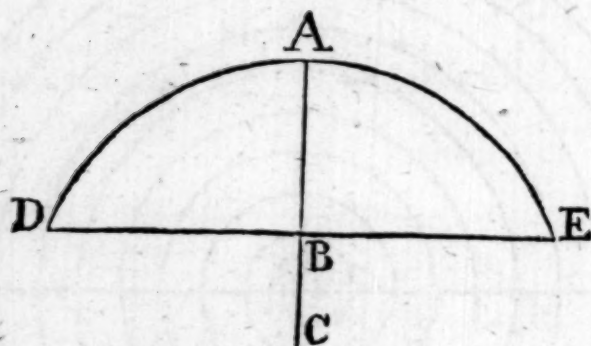
P

$AB \cdot 4$

AB 4 + 9 = 13, the whole Diameter of that Circle, whereof the Arch D A E is a Part of the Circumference. For,

AB DB DB.

As 4 : 6 :: 6 : 9 + 4 = 13 whose $\frac{1}{2} = 6\frac{1}{2}$ = the Semidiameter A C, so is C the Center of the Arch D A E which was to be found.



PROBLEM XXIII.

To lay down a Semi oval, alias Curteel Arch.

There is a Mechanic way to do this which is used by Artificers, but it being no ways Geometrical, I shall only hint of it and so pass it by.

Limit the Transverse Semiconjugate Diameter of what you please.

The Base or Transverse Diameter they divide into seven equal Parts, and the Semiconjugate into 15 (but not of the like Parts with the Transverse) then they take 11 of those 15 and set perpendicular to the Transverse at the Divisions 1 and 6, now have they three Points given to draw the Top of the Arch, to which three Points find a Center, and that Part is completed, the Ends of the Arch are drawn at the Distance of two of the Divisions of the Transverse Diameter.

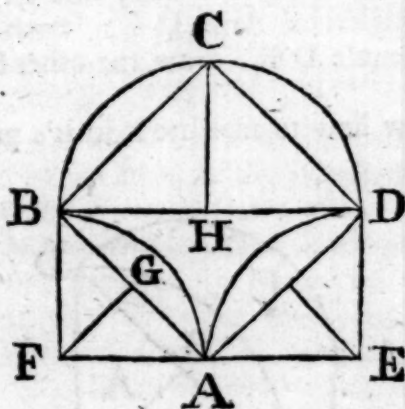
PROBLEM

P R O B L E M X X V .

To draw a Pelicoides.

D E F I N I T I O N .

A Pelicoides is a Figure, made up, or composed of a Parallelogram, a Square, a Semi-circle, Quadrants and Triangles, which how to measure will be shewn in their proper Places. And the young Geometer cannot miss laying of it down at the very Sight of the Figure.



P R O B L E M X X V I .

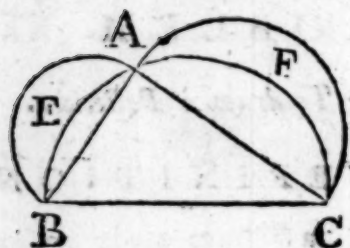
D E F I N I T I O N .

Lunes, or Lunulæ, are three Semi-circles described on the three Sides of a right lin'd right angled Triangle.

For the Triangle A B C is right angled for two Reasons, first because the Angles are made in a Semi-circle, (see Theorem 7.) and secondly, because the Sides are in Proportion as 3, 4 and 5. How this Figure is measured, will be shewn by and by.

P 2

P R O B L E M

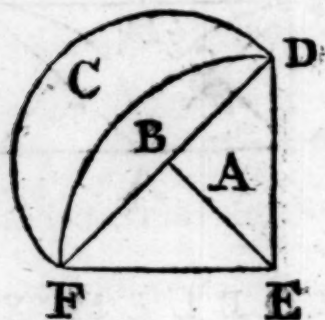


PROBLEM XXVII.

To describe a Lune in a Quadrant.

First draw the Triangle D E F, and on the Center E, describe the Quadrantal Arch D F; upon the Middle of the Hypothenufe D F, draw the other Semi-circle and 'tis done.

We shall shew how to measure it in it's proper Place.

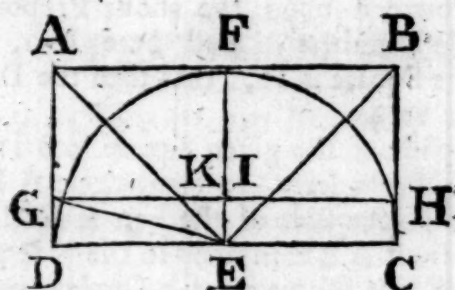


PROBLEM XXVIII.

A Sphere is $\frac{2}{3}$ of a Cylinder circumscribed:

Let A B C D be a Cylinder, D F C a Hemisphere, A E B an inverted Cone, to have the same Base and Altitude, and to be cut by infinite Planes all parallel to the Base, G H is one. I say the Square G I is every where equal to the Square F E (the Radius of the Sphere.) The $\square I E = \square I K$: and consequently, since Circles are

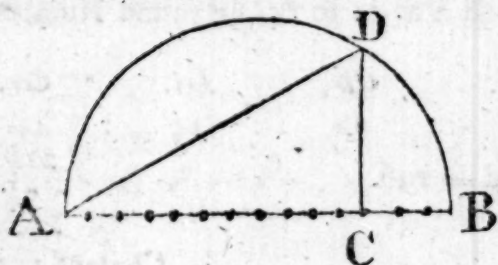
are to one another as the Square of their Radius, all the Circles of the Hemisphere will be equal to all those of the Cylinder, less all those of the Cone, $\mathcal{Q} E D$.



PROBLEM XXIX.

To Reduce a Circle to a Square.

This Problem is grounded upon *Archimedes's* Proportion of the Diameter of a Circle to the Circumference being as 7 to 22, this being not perfectly true, (see Page 151) but the nearest in whole Numbers, and may serve well enough our present Purpose, therefore let the Diameter AB , be divided into 14 equal Parts, at 11 of those Parts erect the Perpendicular CD , and draw AD , so is AD the side of the Square nearly equal in Content to the given Circle. And then if the Diameter be 14, the Side of the Square will be 12.41, as shall be shewn when we come to the Mensuration of Figures.

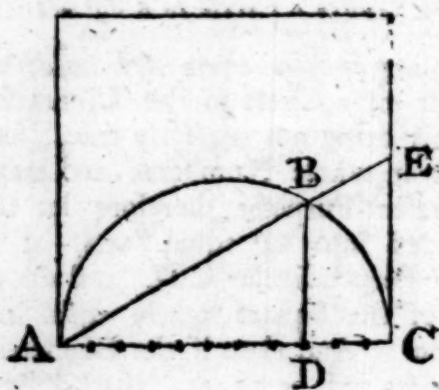


PROBLEM XXX.

To Reduce a Square to a Circle.

This is grounded upon the above Proportion for the Sake of whole Numbers. And here *Note*, That when the Side of the Square is 11, that then the Diameter of a Circle equal is 12.41.

Divide the Side of the given Square into 11 equal Parts, at 5.5 of those Parts draw the Semi-circle A B C, and at 8 of the Parts on the Side of the Square erect the Perpendicular D B, draw A B continued to the Side of the Square at E, so is A E the Diameter of a Circle equal in Content to the given Square, and if the Side be 11, the Diameter as 12.41.



By these two last Problems, we have proved the Proportion that a Circle hath to a Square, whose Diameter and Sides are equal is as 11 to 14, in round Numbers.

<i>Di.</i>		<i>Cir.</i>		<i>Di.</i>		<i>Cir.</i>	
As 7.	:	22	::	14	:	44.	Side 14.
Squared = 196.						$22\frac{1}{2}$ Cir.	
						$7\frac{1}{2}$ Diam.	

Content 154

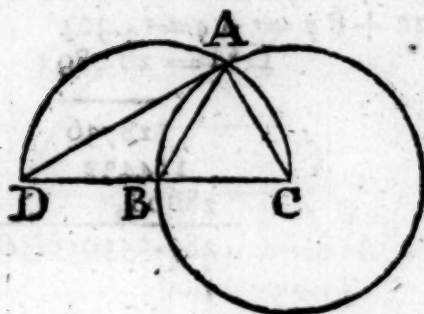
Now

Now say, As 11 to 14, so is 154 the Content of the inscribed Circle to 196, the Content of the circumscribed Square: Because you see the Diameter of the Circle inscribed in the Square, is equal to the Side of the Square, viz. each 14 as above, both being equal to A C.

PROBLEM XXXI.

To Divide a Circle into any Number of equal Parts.

1. Draw a Circle of any Radius whatsoever, and draw the Diameter A B, this divides the Circle into two equal Parts.
2. Erect the Perpendicular F C, and that shall be the Side of an Hexagon, or the sixth Part of the Circle = A D.
3. Set F C from A to D, and from D to E, draw A E, for the Side of an equilateral Triangle.
4. Draw A C for the Side of the Square inscribed, or the fourth Part of the Circle.
5. Bisect F B in G, and draw C G, make G H = G C, and draw C H for the Side of the Pentagon, or fifth Part of the Circle.
6. Joyn E G for the Side of a Heptagon, or one seventh Part of the Circle.
7. Bisect the Arch A C in I, and draw A I for the Side of an Octagon, or an eighth Part of the Circle.
8. Divide the Arch A D E into three equal Parts in K, and draw A K for the ninth Side or Part of the Circle.
9. The Line H F is the Side of a Decagon, or a ten sided Figure.
10. The Line F L is the Endecagon, or eleventh sided Figure. And by doubling or tripling these Lines, the Circle may be geometrically divided into more = Parts at Pleasure.



PROBLEM XXXIII.

To delineate a Parabola in Plano.

DEFINITION.

A Parabola is a conic Section, as mentioned in page 129
 A B standing at right Angles with the Base C D is called the
 Abscissa, and *a b, c d, e f, g h, i k, n o,* are Ordinates.

To find the Latus Rectum L M,

It will always hold,

$$\text{As } \left\{ \begin{array}{l} A B : C B :: C B : L M. \\ 70 \quad 37.5 \quad 37.5 \quad 20.089. \end{array} \right.$$

Note, The Focus through which the Latus Rectum
 must pass, is distant from the Vertex A a quarter of the
 Latus Rectum, which in this Example is 5.022.

To find the Ordinates.

R U L E.

The Rect Angle of the Latus Rectum, and it's inter-
 cepted Axe, is equal to the Square of the Ordinate.

In the Example above we have put 70 for the Abscissa
 A B, and 75 for the Base C D, from which the Latus
 Rectum L M is found to be 20.089, one 4th of which is=
 5.022 = A P, then $70 - 5.022 = 64.978 = P B,$
 $\div 7 = 9.282,$ the Distance of each Ordinate.

Now.

Now let us find the Ordinate N O, the Work stands thus.

$$A P = 5.022 + P q = A q = 14.304$$

$$L M = 20.089$$

$$\begin{array}{r} 128736 \\ 114432 \\ \hline 28608 \\ 287.353056(16.951 \\ 1 \end{array}$$

$$\begin{array}{r} 26)187 \\ 156 \\ \hline \end{array}$$

$$\begin{array}{r} 329)3135 \\ 2961 \\ \hline \end{array}$$

$$\begin{array}{r} 3385)17430 \\ 16925 \\ \hline \end{array}$$

$$\begin{array}{r} 33901)50556 \\ 33901 \\ \hline 16655 \end{array}$$

$$\begin{array}{r} N q = 16.195 \\ \times \quad 2 \\ \hline \end{array}$$

$$N O = 33.902$$

And after this Manner of working the other Ordinates are found, as is here set down; which being laid off at right Angles to A B, will give the Points *a, c, e, g, i, n, L, M, o, k, b, f, d, b*, which Points lye in the Parabola.

$$A B = 70$$

$$L M = 20.089$$

$$N O = 33.902$$

$$i k = 43.532$$

$$g b = 51.39$$

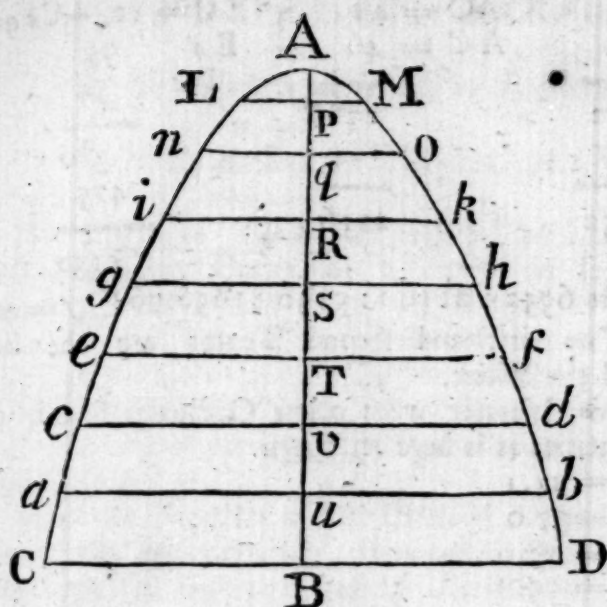
$$e f = 58.196$$

$$c d = 64.286$$

$$a b = 69.842$$

$$C D = 75.$$

PROB.



PROBLEM XXXVI.

To delineate an Hyperbola in Plano.

DEFINITION.

An Hyperbola is the fifth Section of a Cone cut by a Plane parallel to the Axis, and continued 'till it meet the other side of the Cone beyond the Vertex, which is here represented by E.

Let AB be 92, EC 104, and DC 63, then if we take six Ordinates. ga, bb, ic, kd, le, mf , every one will be 9, taking DC = 63. Then if AF be continu'd 'till it meet DE, DE will be 41, for EC 104 — DC 63 = DE 41. Then to find the Latus Rectum FD, it will hold,

As EF: fg :: ED: DF.

From which it is plain, that having the Latus Rectum and Latus Transversum ED, the Ordinates in any Hyperbola may be drawn.

For as $EC \times DC : \square AC :: E a \times D a : \square a g$.

See

See the Work for the Ordinate $g a = a s$.

$$E C = 104$$

$$A C = 46$$

$$E C = 104 - C a 9 =$$

$$D C = 63$$

$$A C = 46$$

$$E a$$

$$95$$

$$D a$$

$$54$$

$$312$$

$$276$$

$$624$$

$$184$$

$$380$$

$$6552$$

$$2116$$

$$475$$

$$5130$$

Now say, As 6552 : 2116 :: 5130 : 1656.76 $\square \sqrt{= 40.7}$

Note, The first and second Terms are the same through all the Work.

After this Manner were other Ordinates found, and their Quantities as is here set down.

$$m n = 24.1$$

$$l o = 37.0$$

$$k p = 48.0$$

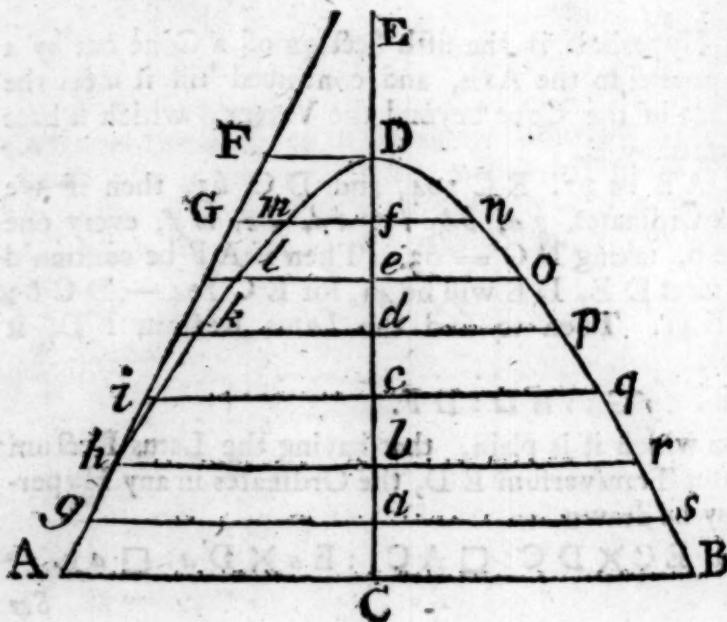
$$i q = 59.8$$

$$b r = 70.6$$

$$g s = 81.4$$

$$A B = 92.$$

Now these Ordinates being drawn at right Angles to CD will give the Points $g, h, i, k, l, m, n, o, p, q, r, s$, which are in the Hyperbola.



PROBLEM XXXVII.

To delineate a Cycloid, or Trochoid.

DEFINITION.

The Curve of this Figure is called by Mathematicians a Transcendant Curve, or a Line of an infinite Order, and may be best conceived to be generated by a Nail in a Coach Wheel; for in every Revolution of the Wheel, this Curve is described: And the whole Cycloidal Curve Line is equal to 4 Diameters of the generating Circle; and the Area of the Cycloidal Space is equal to three times that Circle.

Make the Diameter of the Circle A B of what Quantity you please, as suppose 60; then will the Circumference be 188.4954 (see page 151.) of the same Parts; and half the Circumference is 94.2477; therefore, set the Sector to 60 the Diameter of the Circle on the Line of Lines, (or on any Scale of equal Parts) and take off 94.2477, half the Circumference of the Circle, and set it from B to C and D; divide C B into any Number of equal Parts, as suppose 12, and also the Semi circle A k B into the like Number of equal Parts, and thro' those Parts in the Semi-circle draw Lines parallel to C D, take the Distance B E, and set from p to q, B F from o to r, B G from n to s, B H from m to t, B I from l to v, B K from k to u, B L from i to w, B M from h to x, B N from g to y, B O from f to z, &c. on each Side the Circle, so shall the Points q, r, s, t, v, u, w, x, y, z, be in the Curve of the Cycloid.



If the Cycloid be inverted, and a Body descend from Q R, or S, by it's own Gravity to A the lowest Point of the Curve, the Times of these Descents shall be equal.

If a Body descend from R to S in the Curve R T S by it's own Gravity ; the Times of it's Descent will be less than if it went in the streight Line Q V S : or less than if it went in any other Line ; therefore this is called the Line of swiftest Descent.

If the Curve C A D were cut into two equal Parts in the Line A B, and C B D were turn'd upwards, and a Pendulum hung at B, of a length of the Arch A C, counted from the Point of Suspension to the Center of Oscillation : I say that Center of Oscillation will describe the Cycloid DAC, and all the Vibrations, whether long or short, shall be performed in the same time ; for which Reason it is called the *Isocronal Curve*.

The first that applied Pendulums to a Movement, was the *Dutch Man*, Mr *Christopher Hugen*s, and since Pendulums are to each other as the Square of their Vibrations, and that a Pendulum vibrating Seconds, (or 60 times in a Minute) is, by Experience, found to be of the Length of 39.2 Inches ; from hence it is easy, according to these Principals and Experiments on Pendulums, to estimate nearly the Depth of a deep Well, by the Fall of a Stone from the Mouth into the Water : Or the Distance that any Ship at Sea, or that any Fort is off, by the Times between seeing the Flash of the Powder, and hearing the Report of the Gun, or the Distance that any Thunder Cloud is off ; For Sir *Isaac Newton* found that a Sound moves 968 Feet in a Second of Time : So that counting the Second, (that is, the Swings of the Pendulum, whose Length is 39.2 Inches) between seeing the Flash of Lightning and hearing the Report, and those multiply'd by 968 the Product are Feet, which divide by 5280, the Feet in an *English* Mile, will produce the Miles that the Cloud, Fort, &c. is distant from you.

Also, by the Vibrations of Pendulums, find the Length of any String, Chain, &c. that has a Weight hanging to it, without coming to measure it ; or without making Use of any Quadrant, or such like Instrument to take Heights. This may be put in Practice in taking the Height of Churches, Halls, &c. wherein hang Branches,

1760 yards = $\frac{1760 \times 5280}{968}$ = 950400

or Brass Candlesticks, being fastned to the Roof of the Church, or Theatre, for if you hang up a String and Plummet of any known Length, (as suppose 3 Foot) and make the Candlestick and Pendulum begin to swing both together, (which must be done by Help of a Correspondent;) The Vibrations that the Candlestick makes, while your Pendulum makes any competent Number, will easily help you to the Length of the String, Wire, or Chain that hold the Candlestick, and consequently the Height of the Roof of the Church will be known likewise.

EXAMPLE.

Suppose in the Quire, or Choir of St Paul's Cathedral, London, one of the Candlesticks be made to swing 12 times while your Pendulum of a Yard long makes 96 Vibrations, and the Candlestick hang 8 Foot from the Floor, how many Feet is it from the Floor to the Roof?

Operation.

$$\begin{array}{r}
 12 \\
 12 \\
 \hline
 144
 \end{array}
 \qquad
 \begin{array}{r}
 96 \\
 96 \\
 \hline
 576 \\
 864 \\
 \hline
 144) 9216 (64 \\
 864 \\
 \hline
 576 \\
 576 \\
 \hline
 0
 \end{array}$$

$27 + 8 \text{ feet}$
 $= 64 + 8$
 $= 200 \text{ feet}$
 $64 \times$
 96
 $+ 100$
 200

Answer 64 Yards = 192 Feet.

I shall conclude this Chapter with three Magick Squares.

Q2

2 7 6

Compare alleged origin of discovery of the Pendulum by Galileo.

2	7	6		5	10	3
9	5	1		4	6	8
4	3	8		9	2	7
						
8	28	27	11			
23	15	16	20			
17	21	22	14			
26	10	9	29			

The First Square makes 15 every way, the Second makes 18 every way, and the third makes 74 every way. The Method of filling all Sorts of Magical Squares with the Magick Cubes, may be seen at large in the *Memoirs of the Royal Academy of Sciences*, for 1710, page 124, by Monf. Saurier, in his *Construction Generale, des Quarres Magiques*.

CH A P. III.

Containing the Mensuration of all Manner of Superficies.

And first of a Circle.

IN Page 151 we have shewn that if the Diameter of a Circle be Unity, or 1, that then the Circumference is 3.141592, so that if the Diameter of any Circle be multiply'd by 3.141592; the Product will be the Circumference: Or if the Circumference of any Circle be divided by 3.141592 the Quotient will be the Diameter. Now it is to be observed that the Dimensions may be taken either in Inches, Feet, Yards, Poles, Chains, &c. and that there is often a necessity to reduce one Sort of Measure to another, as when the Dimensions are taken in Inches, the Area in Inches divided by 144, the Square of 12, gives Feet, &c.

If

If the Diameter of a Circle be 1, the Area of that Circle will be .785398 which is called the Area of Unity: Then if Unity be divided by it's Area, the Quotient will be 1.273241, so that if the Square of the Diameter of any Circle be multiply'd by .785398, or divided by 1.273241 the Product or Quotient will be the Area of that Circle in Square Measure. See p. 22.

If 144 be multiply'd by 1.273241, the Product will be 183.3467, the Divisor in Circular Measure: By which, if the Square of the Diameter of a Circle be divided by 183.3467, the Quotient is the Area in Feet.

The Square Root of 183.3467 is = 13.54, the Gauge Point on the Sliding Rule, which is also the Diameter of a Circle, whose Content is 144 Inches, and the Circumference is = 42.5365. The Circle is the most Capacious Figure of all others, for if the Sides of a Square, Triangle, or any other Figure be equal to the Circumference of a Circle, it will not contain so much as the Circle doth. Suppose the Circumference of a Circle be 75.4 it's Area is 452.4, the Fourth Part of the Circumference is 18.85, which squared is only 355.3225, that being 97.0775 less than the Circle. And this is the Ground and Foundation of the Error of Measuring round Timber, by taking $\frac{1}{4}$ of the Girt for the Side of the Square equal; which false way I think ought no longer to have Place, but to be banished from the Thoughts and Practice of all thinking Men, that have any thing to do with Mensurations. See my Royal Gauger, p. 61.

2. To Measure a Square.

Multiply the Side in itself, and the Product is the Superficial Content in the same Measure the Dimensions were taken in.

3. To Measure an Oblong.

Multiply the Length by the Breadth, the Product is the Area, or Superficial Content.

4. *To Measure a Rhombus.*

Multiply the Side by the Perpendicular Heighth, the Product is the Content.

5. *To Measure a Rhumboides.*

Multiply the Length by the Perpendicular Heighth or Breadth, and the Product is the Content.

6. *To Measure a plain Triangle.*

Multiply the Base, or longest Side, by half the Perpendicular, or the whole Perpendicular by half the Base, or the whole Base by the whole Perpendicular, and half this Product is the Content.

7. *To Measure a Trepexium.*

Multiply the Diagonal by half the Sum of the two Perpendiculars *& Contra*, and the Product is the Superficial Content.

8. *To Measure any irregular Figure.*

Reduce the irregular Figure into Trepazia and Triangles, and measure them as taught in the 6th and 7th hereof.

9. *To Measure any regular Polygon ; as a Pentagon, Hexagon, Heptagon, Octagon, Nonagon, &c.*

Multiply half the Sum of it's Sides into the Radius of the Circle inscribed in the Figure, or half that Radius into the Sum of the Sides, the Product is the Area or Superficial Content.

Note, The Radius of a Circle inscribed in any regular Polygon is equal to the Perpendicular of it, let fall from the Center to the Side of the Polygon.

I shall here subjoin a useful Table of regular Polygons shewing the Angles at the Center, Length of the Perpendicular, (or Radius of the Circle inscribing it) and Area in Square Inches.

Names.

Names.	N ^o Sides.	Angle at Center.		Per- pendi- cular.	Area in Square- Inches.
		°	′		
Trigon	3	120	00	.2872	.4308
Tetragon.	4	90	00	.566	1.0000
Pentagon	5	72	00	.688	1.72
Hexagon	6	60	00	.866	2.598
Heptagon.	7	51	26	1.038	3.633
Octagon.	8	45	00	1.207	4.828
Enneagon.	9	40	00	1.374	6.183
Decagon	10	36	00	1.539	7.695
Endecagon	11	32	44	1.702	9.361
Dodecagon	12	30	00	1.866	11.196

In framing of this Table, the Side of the Polygon is supposed to be 1. And as the Areas of like Figures are as the Squares of the Sides that bound them; therefore the Square of any Side multiply'd by the Area of the Polygon in the Table, will give the Area of the Polygon required, so that these Areas in the Table are common Multipliers.

10. To Measure a Spheric Triangle.

From the Sum of the three Angles subtract 180° , multiply the Superficies of the whole Globe by the Remainder, and divide by 720, the Quotient is the Superficial Content.

11. To Measure a Semicircle.

Multiply $\frac{1}{4}$ of the whole Circumference of the Circle by the Semidiameter, and you have the Area.

12. To Measure a Quadrant.

Multiply half the Arch of the Quadrant, by the Radius of the Circle, and it gives the Content.

To

To find the Length of an Arch of any Part of a Circle.

1. Find the Circumference of the whole Circle to the given Radius, as has been taught in p. 151, and also find the Angle at the Center made by the two Legs of the Sector. Then say,

If 360° give the whole Circumference in Inches, &c. what will the Angle at the Center give. Work by a direct Proportion and what comes out is the Length of the Arch Line in Inches, &c. or multiply the Chord of half the Arch by 8, and from the Product subtract the Chord of the whole Segment or Arch, divide the Remainder by 3, the Quotient is the whole Length of the Arch Line very near the Truth.

13. *To Measure the Sector of a Circle.*

Multiply half the Radius into the Arch, or half the Arch into the Radius, gives the Area.

14. *To Measure the Segment of a Circle.*

Of the Segment make a Sector, then 'tis plain; from the Area of the Sector, take the Area of the Triangle, and there will remain the Area of the Segment. See Fig. page 164.

Or by *Ward's New Theorem* p. 406, of his *Young Mathematician's Guide*.

Viz. $\left\{ \begin{array}{l} R = \text{the Radius.} \\ d = \text{the X between versed Sine and} \\ \text{Let } \quad \quad \quad \text{Radius.} \\ C = \frac{1}{2} \text{ Chord of Seg. Base.} \end{array} \right.$

Theorem $\left\{ \frac{2\frac{1}{3} R R - 1\frac{1}{3} R d - d d}{1\frac{1}{2} R + d} \right\} \times C = s.$ the Area of the Segment.

15. To

15. To Measure an Ellipsis.

Multiply the Transverse by the Conjugate, and that Product by the Area of Unity .785398 gives the Area of the Ellipsis in square Inches, or according to what your Dimensions were taken in.

16. To Measure a Parabola.

Note, Every Parabola is $\frac{2}{3}$ of it's Circumscribing Parallelogram.

Multiply the greatest Ordinate, or Base into the Abscissa, or Height, and that Product by 2 and divide by 3, gives the Superficial Content.

17. To Measure a Pelicoides. See the Figure in page 159.

Here you may observe that this Figure is a Complication of several Figures, as the Oblong, Square, Quadrant, Semicircle, Triangle, &c. which Figures measured separately, and their Contents added together, gives the Content of the Pelicoides. And here for the Information of the Young Student, I shall give the Dimensions of several Parts of it. Thus, Let the.

Side of the Square D E h e	=	21
Side of the Oblong F E	=	42
Semidiameter of the Circle	=	21
Chord B C	=	29.698
Circumference B C D	=	65.97339
Quadrant A D	=	32.986695
Perpendicular F G	=	14.849

According to these Dimensions the Superficial Content of the Parallelogram B D E F is 882 = to the Content of the Square A B C D. And the Content of the Semicircle B C D = 692.72.

18. To Measure a Lune. See the Figure in page 160.

First find the Area of every one of the Semicircles, and the Area of the Triangle B A C, then the Area of the Semicircle

Semicircle BAC — Area Triangle BAC = to the two Segments in that Semicircle. And the Semicircle AFC — the Segment = to the Lune F . Also the Semicircle BEA — that Segment = to the Lune E . If we put the Sides of the Triangle 24, 32 and 40, the Area of that Δ will be 384 = to the Area of the two Lunes E and F . And the Area of the Semicircle BAC = to the other two Semicircles, because they are made upon the three Sides of the Triangle ABC = to the Ratio 3, 4, 5, by the 47th of the 1st of *Euclid*.

19. *To Measure a Lune in a Quadrant. See the Figure, page 160.*

First, Measure the Triangle, and also the Quadrant, and Lastly, The Semicircle from the Area of the Quadrant, take the Area of the Triangle A , and there remains the Area of the Segment B , then from the Area of the Semicircle take the Area of the Segment B , and there remains the Area of the Lune C . $FE = DE = 30$.

Area of	{	Quadrant	-	-	706.858
		Triangle	-	-	449.999
		Segment	-	-	256.859
		Semicircle	-	-	706.644
		Segment subtract B	-	-	256.859
		Lune C	-	-	449.785

20. *To Measure a Cycloid, or Trochoid.*

Find the Area of the Circle inscribed in the Cycloid, which multiply by 3, and the Product is the Area of the Cycloid.

CHAP. IV.

To Measure all manner of Solids.

AS a Superficies has Length and Breadth, so has a Solid Length, Breadth, and Depth.

1. *To*

1. *To measure a Cube.*

Multiply the Side into itself, and that Product by the Depth, gives the Content.

2. *To measure a Parallelogram.*

Multiply the Length, Breadth, and Depth, one into another, gives the solid Content.

3. *To measure a Triangular Prism.*

Find the Area of the Triangular Base, which multiply by the Length, gives the solid Content.

4. *To measure a Pyramid.*

If the Base be a Triangle, Square, Pentagon, or any other Polygon, find it's Area accordingly, and multiply it by $\frac{1}{3}$ of the Heighth gives the solid Content.

5. *To measure the Frustum of a Pyramid, cut parallel to it's Base.*

Find the Area of each End, and multiply them together, the Square Root of the Product is a mean Area, which add to the other two Areas, and multiply that Sum, by $\frac{1}{3}$ of the Length gives the Content.

6. *To Measure a Cone.* See the Figure in page 129.

Find the Area of the Base, and Multiply it by $\frac{1}{3}$ of the Height gives the Solid Content.

7. *To Measure the Frustum of a Cone cut parallel to the Base.*

To three times the Rect-angle of the two Diameters, add the Square of their Difference. Multiply that Sum by $\frac{1}{3}$ of the Length, gives the Solid Content. Or as in the Frustum of the Pyramid.

8. *To find the Diameter in any Part of the Fruſum.*

Say, As the whole Length, is to the whole Difference of the two Diameters: So is any Length from the greater End, To a fourth Number; which subtract from the Diameter at the greater End gives the Diameter sought. Or you may work by the Distance from the lesser End, and add what comes out to the Diameter at the lesser End, gives the same thing.

Or Thirdly, divide the Difference of the two Diameters by the Length, and the Quotient is a common Multiplier; by which multiply any Length from either End, the Product added to the lesser, or subtracted from the greater Diameter gives the Diameter sought.

9. *To measure a Cylinder.*

Find the Area of the End as a Circle, and multiply it by the Length, gives the Solid Content. If the Dimension were taken in Inches, divide the solid Content in Inches by 1728, (the cubic Inches in a Foot) and the Quotient are Feet.

But more expeditiously, divide the Square of the Diameter by 2200 15872 (that is 1728 multiplied by 1.273241 See page 733) or multiply it by .00045451 the Quotient or Product is the Area in Feet, which multiply by the Length in Inches gives the Content in Feet.

10. *To measure a Globe.*

Here are four several Ways to do this.

1. Multiply the Diameter by the Circumference, gives the superficial Content, and that by $\frac{1}{6}$ of the Globe's Diameter gives the Solidity.

2. Multiply the Cube of the Globe's Diameter by 11, and divide by 21, gives the Solidity near the Truth.

3. Multiply the Cube of the Globe's Diameter by .523598 (that is the solid Content of a Globe whose Diameter is Unity) or it is $\frac{2}{3}$ of the Area of Unity .785398, and the Product is the solid Content. Or,

4. Divide

4. Divide the Cube of the Globe's Diameter by it's proper Divisors gives the solid Content.
See my *Royal Gauger*, page 106.

10. *To measure the Segment of a Globe.*

A Piece being cut off less than half the Globe is called a Segment.

Multiply the triple Height of the Segment by the Square of half the Chord, to which add the Cube of the versed Sine (or Segment's Height) this Sum multiply by .523598, and this gives the solid Content.

11. *To measure a Frustrum of a Globe.*

A Frustrum is more than half the Globe; and it is measured as has been taught in the Segment; the Contents of which two being added together will give the Content of the Globe of which they are Part.

Note, If the Diameter of a Globe be 1, then the Circumference and superficial Content are equal, viz. 3.141592, and if the Diameter be 6, then the Solidity and superficial Content are equal, viz. 113.097312.

12. *To measure a Spheriod.*

Multiply the Square of the Conjugate by the Transverse or Length, and that Product by .523598 gives the Solidity.

14. *To measure a Parabolic Conoid.*

A Parabolic Conoid is equal to $\frac{1}{2}$ of it's circumscribing Cylinder.

Multiply the Square of the Diameter of the Base by the Height, and that Product by .392698 (that is $\frac{3}{4}$ of .523598) and that Product is the Content.

14. *To measure a Hyperbolic Conoid.*

A Hyperbolic Conoid is $\frac{5}{8}$ of it's circumscribing Cylinder. Therefore to find a Divisor for Feet, say, as 5 : 12 or as 1 : 2.4 :: 2200.158 : 5820.3792.

R

Multiply

Multiply the Square of the Diameter at the Base by the Height, and divide by 5820.3792 gives the solid Content in Feet.

15. *To measure a Parabolic Spindle.*

This is $\frac{8}{15}$ of it's circumscribing Cylinder.

As 8 : 15 :: 1 : 1.875. And,

As 1 : 1.875 :: 2200.15872 : 4125.2976.

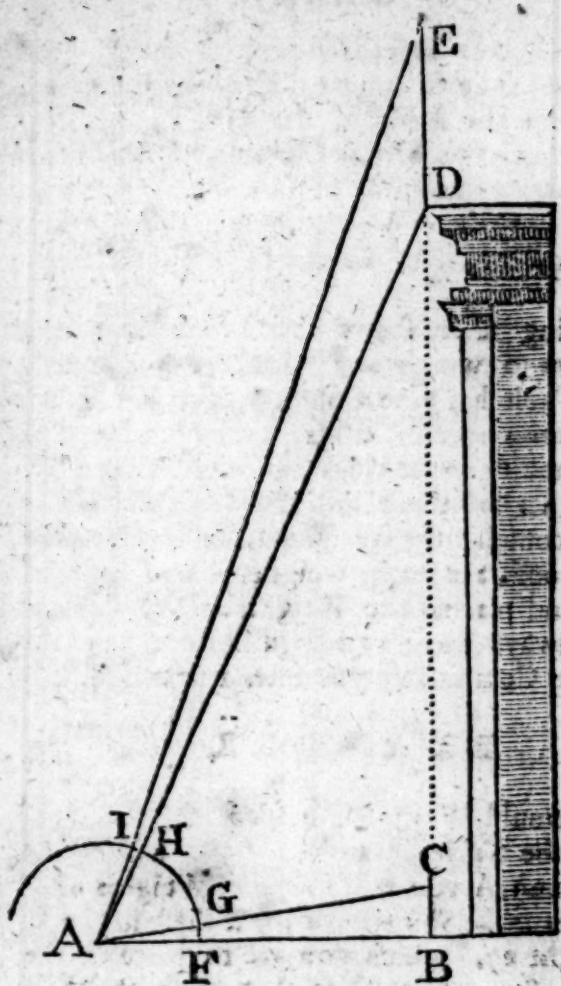
Multiply the Square of the conjugate Diameter by the Transverse, and divide by 4125.2976 gives the Content in Feet.

16. I shall conclude this Part with two useful and pleasant Propositions, the one in Statuary, and the other in Staticks; that is, 1. how to give the exact Height of a Statue placed on a Building, that the same shall appear equal to the common Height of a Man standing on the Ground.

E X A M P L E,

Suppose standing at A, B C be a Statue 6 Foot high, and is to be raised to D 200 Foot (more or less) high, how big must the Statue be made, to appear 6 Foot when it is placed at D.

Solution

*Solution.*

Open the Compasses to any convenient Radius, set one Foot in A, draw the Arch F I, and draw AC, and A D, now here is no more to be done but to make the Angle $I A H = \text{Angle } G A F$, because all Objects that we see, appear under a certain Angle, in proportion to the Distance of the Object from the Eye : Therefore take F G and set from H to I, and draw A I E so will D E appear to an Eye at A of the same height with B C : Then from the same Scale that B C is 6 Foot, measure D E, and what

R 2

you

you find that height on the Scale, so many Feet you must make the Statue to appear 6 Foot when elevated 200 Feet, &c. above the Level of the Eye.

2. For a Conclusion of this Chapter, I shall here shew how any quantity of any sort of Goods under 121 Pounds may be Weigh'd with 5 Weights only : The Weights are these 1, 3, 9, 27, 81, being in a geometrical Proportion.

The following table shews at first Sight how to use the Weights, so as to weigh any Number of Pounds under 121. The Table has nine Columns, the first, fourth and seventh Columns under N, contains the Number of Pounds of any Commodity to be weighed ; the Column A, contains the Weights to be put into the Weighing Scale ; the Column B, contains the Weights to be put into the Scale where the Goods are to be weighed, and by this means of changing and placing the Weights as the Table directs, you may always have an Equilibrium, or the true Weight of the Commodity you intend to weigh.

EXAMPLE,

Suppose I would Way 58 Pounds?

Look into the Table under N for 58, and right against it in the Column A you will find the Weights 81, 3, 1, which together make 85 ; right against these in the Column B is the Weight 27, which you are to place in the other Scale. Now $85 - 27 = 58$, so that if in the Scale B to the Weight 27, you put any Commodity 'till there be an Equilibrium, you will truly weigh 58 Pounds.

Secondly, If you would weigh 100 Pound, you must have 81, 27, 1 = 109, and 9 in the other Scale, for $109 - 9 = 100$, &c.

The Weighing Table.

N	A	B	N	A	B	N	A	F.
1	I.		41	8I.	27.9.3.I	81	8I.	
2	3.	I.	42	8I.	27.9.3	82	8I.I	
3	3.		43	8I.I.	27.9.3	83	8I.3	I
4	3.I		44	8I.	27.9.I	84	8I.3	
5	9.	3.I	45	8I.	27.9	85	8I.3.I	
6	9.	3.	46	8I.I	27.9	86	8I.9	3 I.
7	9.I.	3.	47	8I.3	27.9.I	87	8I.9	3
8	9.	I.	48	8I.3	27.9	88	8I.9.I	3
9	9.		49	8I.3.I	27.9	89	8I.9	I
10	9.I.		50	8I.	27.3.I	90	8I.0	
11	9.3	I.	51	8I.	27.3	91	8I.9.I	
12	9.3		52	8I.I	27.3	92	8I.9.3	I
13	9.3.I		53	8I.	27.I	93	8I.9.3	
14	27.	9.3.I	54	8I.	27.	94	8I.9.3.I	
15	27.	9.3	55	8I.I	27.	95	8I.27	9.3.I
16	27.I	9.3	56	8I.3	27.I	96	8I.27	9.3
17	27.	9.I	57	8I.3	27.	97	8I.27.I	9.3
18	27.	9.	58	8I.3.I	27.	98	8I.27	9. I
19	27.I	9.	59	8I.9	27.3 I	99	8I.27	9.
20	27.3	9.I	60	8I.0	27.3	100	8I.27 I	9.
21	27.3	9.	61	8I.9.I	27.3	101	8I.27.3	9 I
22	27.3 I	9.	62	8I.9	27.I	102	8I.2.73	9
23	27.	3.I	63	8I.9	27.	103	8I.27.3.I	9
24	27.	3.	64	8I.9.I	27.	104	8I.27	3 I
25	27.I	3.	65	8I.0.3	27.I	105	8I.27.	3
26	27.	I.	66	8I.9.3	27	106	8I.27.I	3
27	27.		67	8I.9.3 I	27	107	8I.27.	I
28	27.I		68	8I.	9.3.I	108	8I.27.	
29	17.3	I.	69	8I.	9.3	109	8I.27.I	
30	27.3		70	8I.I	9.3	110	8I.27.3	I
31	27.3.I		71	8I.	9.I	111	8I.27.3	
32	27.9	3.I	72	8I.	9.	112	8I.27.3.I	
33	27.9	3.	73	8I.I.	9.	113	8I.27.9	3 I
34	27.9 I	3.	74	8I.3	9.I	114	8I.27.9	3
35	27.0	I.	75	8I.3	9.	115	8I.27.9 I	3
36	27.9		76	8I.3.I	9.	116	8I.9.27	I
37	27.9 I		77	8I.	3.I	117	8I.9.27	
38	27.9.3	I.	78	8I.	3.	118	8I.27.9.I	
39	27.9.3		79	8I.I	3.	119	8I.27.9.3	I
40	27.9.3.I		80	8I.	I.	120	8I.27.9.3	
						121	8I.27.9.3.I	

CHAP V.

Of the SECTOR.

THE Sector being an Instrument of such universal Use, that to pass it by with Silence were a Crime too great to be pardoned : Many learned Men have wrote upon the Use of the Sector, as *Gunter, Forster, Stone, Cunn,* &c. to which I refer the inquisitive Reader for general Satisfaction, and sha'll content myself only to shew the Use of such Lines as are now put upon the Sector, in laying down or measuring Angles, in Multiplying and Dividing, and making Proportions.

Upon one Face of the Sector, there is generally a Line of Chords to 60° issuing from the Center to the End of each Leg, with a Line of 10 equal Parts decimally divided, issuing from the Center, and extends itself to the End of each Leg; by which Means this Line is divided into 100 equal Parts, and Numbered with 1, 2, 3, 4, &c. to 10, and marked at the End with Lin for Line, so 'tis known by the Name of the Line of Lines.

Between the Line of Chords and the Line of Lines, is placed a small Line of Secants, beginning at two brass Centers at (or near) 25 of the Line of Lines and ending near the Ends of the Sector, where is wrote *Se.* signifying Secant. There is also a Line of Latitudes answering a Line of Chords of the same Radius upon the Foot Sector, with a line of Regular Polygons, and sometimes a Meridian Line for projecting Mercators Chart.

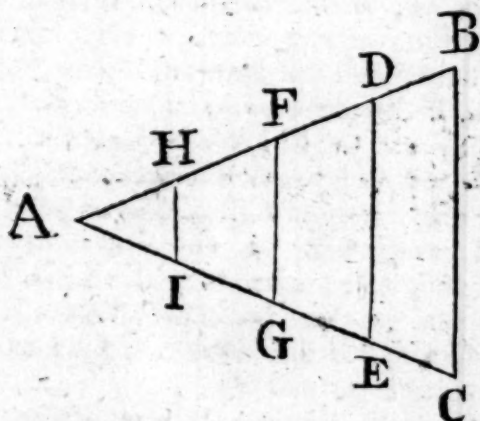
On the other Face of the Sector, is a Line of Sines issuing from the Center upon each Leg, numbered with 10, 20, 30, &c. to 90 at the End of the Sector; also a Line of Tangents to 45° , Numbered with 10, 20, 30, 40, 45° at the End.

Between the Lines of Sines and Tangents, on each Leg is a small Line of Tangents beginning with two brass Centers,

Centers, and N^o. with 45, 50, 60, 70, 75 near the End, marked with *Tan.* for Tangents.

On this Face is also a Line of Inclination, fitted to a Scale of six Hours, these with the Chords and Line of Latitude on the other Face, are useful in Dialling; how these Lines are made, with their Use, I have fully explain'd in my *Mechanic Dialling*, which See.

There is likewise a Line of Numbers, commonly called *Gunter's Line*, with the artificial Sines and Tangents, (for the first mentioned Lines, are natural Sines and Tangents) with a Line of Inch Measure, or a Foot divided into 12 equal Parts decimally divided; how the Line of Numbers is made, see my *Royal Gauger*, where you will meet with ample Satisfaction.



The Figure of the Sector is well represented by the Isosceles Triangle A, B C. Here the Point A is the Joynt or Center of the Sector to which all the sectoral Lines are drawn, and they are of an equal Length as A B, A C, and those that bear the same Name, are equal and alike divided and numbered. And therefore in all the Operations wrought by the sectoral Lines, the Points B C, D E, F G, H I, representing the same Divisions, are equally distant from the Center A, the same is true of any other two Points in the sectoral Lines A B, A C, &c. when they be

be equal Parts, Chord, Sines, Tangents or Secants. And therefore as $AC : CB :: AE : ED$, and as AE is to ED : so is AG to GF . and so is AI to IH . See Prob. IX. page 145 Euclid 2, and 4 of the VIth Book.

Note, The Lengths AB , AC , AD , AE , AF , AG , AI , are called Laterals, or Crurals, because they lye or are measured in the Legs of the Sector,

And the Lines BC , DE , FG , HI , are called Parallels; because all the Proportions wrought by the Sectorals, the two Extrems between the Legs, will be two such Parallel Lines.

If AC on the Line of Lines measure 60 equal Parts, and so AB likewise; then is AC and AB said to be 60 Crural Parts, and BC 45 parallel Parts: And so of the Sines, Tangents, and Secants.

Here is one Thing more to be observed before we come to the Use of the Lines, and that is, sometimes a Number falls so near the Center of the Sector, that it cannot well be taken off, or measured; to remedy this, multiply it by any convenient Number at Pleasure, and take that Product instead of the Number itself,

As suppose BC were 5 = Parts, now that, with other in proportion, are to be laid down by the Sector, if I take BC in my Compasses and apply it parallel to 5, 5 on the Line of Lines, it will be too long for the Sector, therefore I multiply it by 12 (or any other Number) and the Product is 60, then take the Line BC and apply it parallel to 60, 60, and now all other Lines depending will be in a just Proportion, the same as if I had laid down the Number 5.

Again, Suppose I have a Line whose Length exceeds 100, or the whole length of the Line; to remedy this Inconveniency, Divide your given Line by 2, 3, 4. (or any other convenient Number at pleasure) and apply the Quotient of the given Line, instead of the Line itself, and by this Means you will bring the Line upon the Sector, which otherways it would not. Suppose I have a Line of 118' 120" to lay down by the Sector, I divide it by 2, and it gives 59' 6", then take half of the given Line, and apply it parallel to 59' 6", and now the
Sector

Sector stands true all one as if I had had a Line of $118^{\circ}12'$ equal Parts, your own Reason with a little Practice will soon make Master.

2. The Use of the Sector.

1. To open the Sector that the Line of Lines may stand to a Right, or to an Angle of 90 Degrees. Take the Whole Lateral Length of the Line of Lines 10, in your Compasses, and set one Foot in 6 and the other in 8, in the Line of Lines, and then they will be opened to a right Angle, because $\square 8 + \square 6 = 100, = \square 10$. by 47 of the 1st of *Euclid*, for now you have a right angled Triangle of 6, 8, and 10 Sides = 3, 4, 5. See page 137.

2. The Line of Lines may be opened to a right Angle, if the Lateral Sine of 90° be apply'd over parallel, between 45 and 45 on the Sines, or if the Lateral Sine of 45 be apply'd parallel over the Sine 30, 30.

3. The Line of Chords may be opened to any particular Angle, by taking out the Lateral Chord of the Angle required, and applying it over the Parallel of 60° , 60, and then those Lines will stand open at the Angle required.

EXAMPLE.

Suppose I would open the Line of Chords to an Angle of 30° ?

Take the Lateral Chord of 30, and apply it parallel over 60, 60, and then the Line of Chords do stand open at an Angle of 30 Degrees.

4. If the Sector be opened to any Angle at venture, to find the Quantity of the Angle the Lines of Chords then make.

This is but the Converse of the last, for take the parallel Chord of 60, and Measure it on the Lateral Chord, and that gives you the Angle sought. Observe the same of the Line of Sines, by considering that the Sine is half the Chord of the double Arch See the Figure, page 126, where A G is the Chord of the Arch A I G, and A H = H G, it's half, being the Sine of the Arch I G.

Suppose it were required to open the Sector in the Line of Sines to an Angle of 40° ?

Take

Take out the Lateral Chord of 40, and apply it parallel over 60, 60, so shall the Line of Sines be opened to an Angle of 40 Degrees.

Or if the Semiradius, as the Sector now stands to an Angle of 40, (that is, divide the Lateral Chord of the given Angle into two equal Parts) be applied over, parallel between 30, 30, on the Sines, it will open the Lines of Sines to the given Angle. Thus, In the last Example where it was required to open the Lines of Sines to an Angle of 40, divide the Lateral Chord of 40 into two equal Parts, and lay that Extent parallel over 30, 30, on the Sines, and then will the Lines of Sines be opened to an Angle of 40.

Note, It is one Thing to open the Edge of the Sector to an Angle, and another thing to open the Lines on the Sector to the same Angle: For when the Sector is close shut, the Edges of it make no Angle, but the Lines of Lines, Sines, Tangents, and Secants, make an Angle of about 6 Degrees.

5. If you would examine the Lines of Chords, Sines, Tangents and Secants, whether they be truly made, project them from a Circle of the same Radius. See *Definitions*.

Open the Sectoral Lines (Sines) not the Legs of the Sector, and take the Sine 10, 10 in your Compasses on each Leg in a straight Line, carry this Extent to the Line of Chord, set one Foot in the Center, and the other Foot will exactly reach to the Chord 20° ; because every Sine is half the Chord of the double Arch, as has been noted above; the same observe of any other Degree.

6. To lay down an Angle of any Quantity of Degrees.

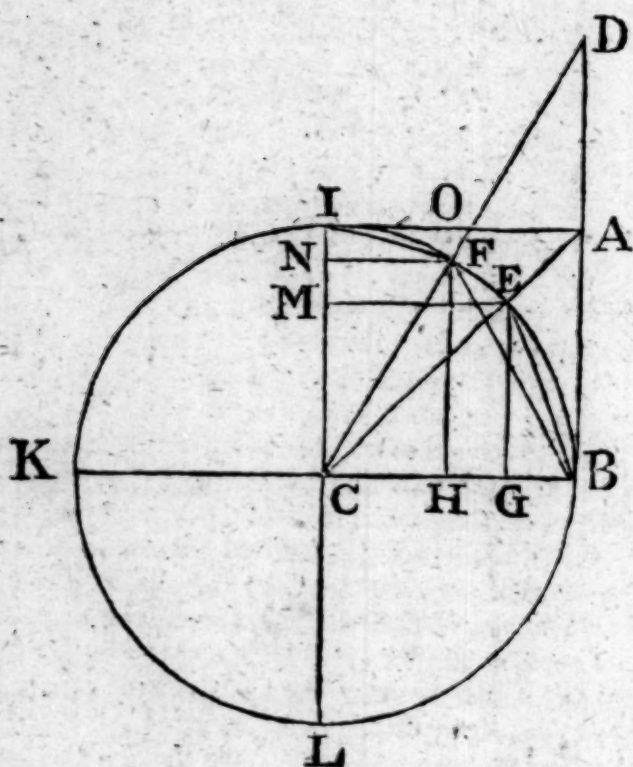
This may be perform'd, either by the Line of Chords, Sines, Tangents, or Secants, due Regard being had to each particular Line.

1. By the Chords, in the annexed Figure, take the Radius CB in your Compasses, and therewith sweep the Circle KIBL, make this Radius a Parallel of 60 on the Sectoral Chords, then take the parallel Chord of 45 from the Sector and set it from B to E, and draw CE A, and B A, so shall the Angle A C B be $= 45^\circ$, of which B E is the Chord B A is the Tangent, G E the Sine, and C A the Secant of the Arch B E, which measures the Angle

Angle $A C B$. Again take the Radius of the Circle $C B$, and make it a parallel Radius upon the Sectoral Chord 60 , 60 , also draw $B F$, and $C F D$, so shall the Angle $D C B$, be 60 Degrees, which is measur'd by the Arch $B E F$, of which $B F$ is the Chord, $B D$ the Tangent, $F H$ the Sine, $C D$ the Secant, and $H B$ the Versed Sine of Arch $B E F$, or Angle $D C B$. 2. And now as the Sector stands opened to the Radius $C B$, take the Tangent $B A$ in your Compasses, and apply it parallel on the Sectoral Tangent, and you will find it just reach to 45° . 3. Take the Sine $E G$ in your Compasses and apply it parallel on the Sectoral Sines, and you will find it reach just to 45 . The Chord $B E$ will be a Parallel of 45 on the Sectoral Chords. 4. But for the Secant $C A$, because it is part of the Radius $C E$, the Sector must always be opened to two Brass Centers which lye near the Joint of the Sector, and that made equal to the Radius of the Circle, and then you will find $C A$ taken and made a Parallel on the Sectoral Secants to be 45° . And thus you will find all the Parts of the Triangle $D C B$ measured on their respective Lines as above directed to be $60^\circ =$ to the Angle $D C B$. So that you see the Measuring an Angle is only the Converse of laying down an Angle, as has been shewn in both.

And here you are to Note, That the Radius, or Sine of $90 = C I$ is always equal to the Chord of $60 = B F$, and $=$ Tangent $45^\circ B A$. That is, the Sine of 90 , the Chord of 60 , and Tangent of 45 are always equal in the same Circle as above has been proved. And that the Radius is a Mean Proportional between the Tangent of an Arch, and the Tangent Complement of the same Arch: As suppose in Fig. 1, page 142, you make $A B = B D$ the Tangent of the Arch $B E F$ in this Figure above, and $B C = I O$ the Tangent Complement of the same Arch, then will the Geometrical Mean $B D$ in Fig. 1, page 142, be $= C I$ the Radius of this Circle, and so of any other.

Q. E. D.



A Synopsis of the Scheme above.

Of the Arch BEF $\left\{ \begin{array}{l} BF \\ FH \\ BD \\ CD \\ HB \end{array} \right\}$ is the $\left\{ \begin{array}{l} \text{Chord.} \\ \text{Sine.} \\ \text{Tangent.} \\ \text{Secant.} \\ \text{Verfed Sine.} \end{array} \right.$

Of the Arch IF $\left\{ \begin{array}{l} IF \\ NF \\ IO \\ CO \\ IN \end{array} \right\}$ is the $\left\{ \begin{array}{l} \text{Chord.} \\ \text{Sine.} \\ \text{Tangent.} \\ \text{Secant.} \\ \text{Verfed Sine,} \end{array} \right.$
or Difference between Radius and Co-Sine.

7. *To multiply on the Sector.*

The RULE on the Line of Lines.

As 1 is to either Factor laterally, and set to 10, 10 parallel, So is the other Factor taken off parallel with your Compasses, To the Product laterally.

E X A M P L E.

What is the Product of 4 by 7?

Operation.

Take 7 in your Compasses from the Center of the Sector laterally, and lay it parallel over 10, 10, as the Sector now stands, take off 4, 4, parallel, and carry this Extent of the Compasses laterally, and the other Point will reach to 28, the Product sought.

Or take off 4 laterally, and lay it parallel on 10, 10, then take off 7 parallel, and apply it laterally, will give 28 as before. This being so plain, more Examples are needless.

8. *To divide on the Sector.*

R U L E on the Line of Lines.

As the Dividend laterally, Is to the Divisor parallel, So is the Parallel 10, 10, To the Quotient.

E X A M P L E.

What's the Quotient of 28 divided by 4?

Operation.

Take 28 the Dividend off laterally in your Compasses from the Center of the Sector, and apply it parallel to the Divisor 4, 4, then take off 10, 10 parallel, and it will reach from the Center of the Sector laterally to 7, the Quotient sought.

Note. If the lateral Distance be too long for the opening of the Sector, then take one Half, one Fourth, &c. or multiply them by 2, 3, 4, &c. of it at Pleasure, then Work as above. Suppose I would divide 120 by 4? Now 120 falls so near the Center of the Sector, the Divisions are scarce perceptable, therefore to have the Number to fall upon the Line, I multiply both Dividend and Divisor by 2, and then they are 240 and 8, with these Work as above is taught, and the Quotient will be 30, the same as if you had divided 120 by 4.

9. *The Single Rule of Three on the Sector.*

R U L E on the Line of Lines.

First observe, that the Question must be Stated as has been taught in *Chap. iv. page 34.* Then,

As the lateral Middle Term, Is to the Parallel first Term,
So is the Parallel third Term, To the lateral fourth Term.

E X A M P L E.

Suppose 7 Yards of Cloth cost 22s. what will 35 Yards cost at that Rate?

Operation.

Take 22 laterally and lay it parallel over 7, 7, then take the Parallel 35 and apply it laterally will give 110s. and so much doth the 35 Yards come to at the given Price.

These Things I mention more for Curiosity than Use, which is only to shew what may be done by this useful Instrument; but those that are curious in Proportions, I refer them to my *Royal Gauger* for the Use of the Sliding Rule.

Here is yet one Beauty of this Line of Lines, *viz.* that it is a Scale of equal Parts of all Lengths; and is therefore preferable to any Scale where the Divisions are only one particular Length.

E X A M P L E.

EXAMPLE.

Suppose I would form a Scale of 1 Foot 6 Inches, that may represent 124 28 Yards.

Here I cannot open the Sector wide enough to receive $1\frac{1}{2}$ Foot; therefore I multiply both by any Number, suppose 4, and there will be produced 6 Feet and 497.12 Yards, then I divide all by 6, and I have 1 Foot and 82.85 Yards; therefore opening the Sector 'till one Foot reaches from 82.85 to 82.85 parallel on the other Leg, it will be a Scale of 100 Yards of the Bigness required. The Lines of Polygons are very near the inner Edge of the Sector, at the End stand the Figure 6, and the other small punch'd Holes in it are number'd with 7, 8, 9, 10, 11, 12 towards the Center.

To inscribe a regular Polygon of any Number of Sides in the given Circle.

* Open the Sector to the Radius of the given Circle, and as the Sector now stands take the Side of the given Polygon, viz. if 'tis a Pentagon take 5 off parallel, if a Hexagon take 6, if a Heptagon take 7, &c. and lay that Extent of your Compasses upon the given Circle, and it will inscribe your Polygon required. In page 175, I have given a Table of regular Polygons, with the Quantity of the Angle at the Center of each; by which you may lay down any of them, for there be some Sectors that have not the Polygons on them; therefore, as before, set the Sector at Chord of 60 to the Radius of the given, in which you would inscribe the Polygon, and with your Compasses take off the parallel Chord of the Angle at the Center of the given Polygon, (as in the Table, page 175) and 'tis done.

In page 142 we have promised to shew how that Figure is laid down by the Sector, which is thus; take the given Line $AB = 54$ in your Compasses and lay it off parallel to 54, 54 on the Line of Lines on the Sector, then take the Line AE in your Compasses, and apply it parallel on the Line of Lines on the Sector, and you will find it to be 33.5, and EB you will find by the same way of working to be 20.5. After this Manner are the Lines in any Figure either measured or laid down. As for the other Sectoral Lines, their Use shall be shewn in the next Chapter when we come to treat of *Trigonometry*.

CHAP. VI.

Of Plain Trigonometry.

DEFINITION.

TRIGONOMETRY is the Art of Measuring Triangles, or of calculating the Sides or finding the Angles of any Triangle sought; and this is either Plain, or Spherical: of which I shall speak distinctly; and first of the Plain.

The Art of Trigonometry doth much depend on the Knowledge of the Lines in the Figure page 192, which the Learner must carefully observe to understand well, and then he may proceed.

First then, in that Figure page 192, it is plain, that as the Co-Sine is to the Sine, so is Radius to the Tangent. That is, As $NF : HF :: CB : BD$.

2. As Radius is to the Sine, so is the Secant to the Tangent. That is, $CF : FH :: CD : BD$.

3. As the Sine is to Radius, so is Radius to the Co-Secant. That is, As $FH : FC :: FC : CO$.

4. As the Tangent is to Radius, so is Radius to the Co-Tangent. That is, As $BD : CB :: CB : IO$.

Therefore the Rectangle between the Tangent and Co-Tangent of any Arch is equal to the Square of the Radius.

5. Every Triangle hath six Parts, of which three are Sides, and three Angles; and of these if we have three given we can find the rest, (except in the Case where the three Angles of a plain Triangle are only given.) For from thence the Sides cannot be found because two Triangles may be equi-angular, and yet have the Sides by no means of the same Length. We can find the rest, I say, if suppo-
sing

fining the Radius divided into any Number of equal Parts, we can but discover how many such Parts, any Sine, Tangent, or Secant of any Arch or Angle doth contain. Now this is readily done to our Hands, in the Tables of Sines, Tangents, and Secants, which we have with prodigious Industry, in Books, (of which *Sherwin's* is the best) ready calculated (to our Hands) for this Purpose.

6. When therefore any Triangle is given to be resolved, the first Thing we have to do, is to consider that there is in the Table of Logarithms, Sines, Tangents, and Secants, a Triangle exactly Similar, and equal to that we are required to solve, and whose Sides are to one another in the very same Proportion of those of the Triangles proposed.

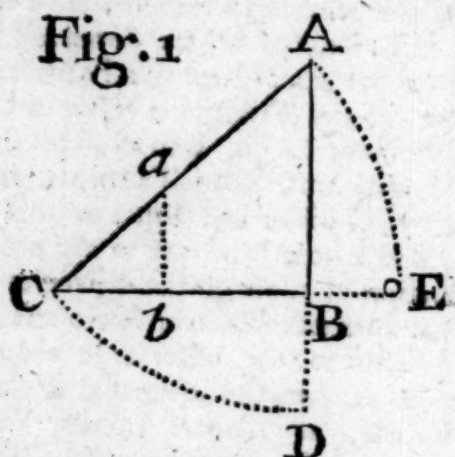
7. We must understand whatever Ratio one Side of the Triangle given, hath to the other Side about the same Angle, considered as Lengths estimated or number'd by any known Measure, as suppose Inches, Yards, Miles, Leagues, &c. the very same hath the two Sides about the same Angle in the Triangles in the Tables of Logarithms, &c. which two Things well understood, do lead us into the Whole Mystery of Trigonometrical Calculation.

8. There is one general *Axiom* which solves all the Cases in plain Trigonometry, which is this.

In a right-angled plain Triangle, if any of the Sides be put, or supposed the Radius, Whole Sine or Semi-diameter of a Circle, then the other two shall be as Sines, Tangents, or Secants, as by them represented.

And the same Proportion that the Side put for Radius, has to Radius, the same hath the other Sides to the Sine Tangent, or Secant, by them represented.

1. If Hypotheneuse be made the Radius of a Circle, every Side is proportionable to the Sine of it's opposite

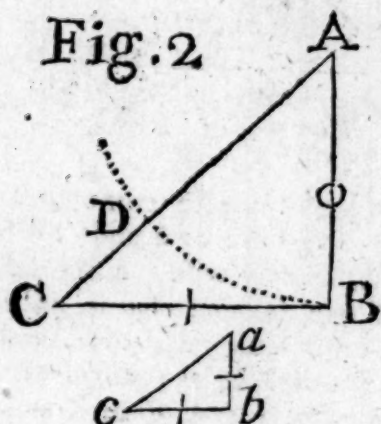


Angle. For setting one Foot of the Compasses in A, extend the other to C, and draw the Arch C D, (which you may suppose to be a perfect Circle of which A C is the Semidiameter, or Radius). Now 'tis plain from the Figure page 192, that C B, the Base, is the Sine of the Arch C D, and the Arch C D measures the Angle C A B, consequently the Base C B is the Sine of the Angle at A. Again, make C the Center of a Circle, and with the Radius C A draw the Arch A E, here the Arch A E measures the Angle A C B, and because A B is the Sine of the Arch A E, therefore A B the *Cathetus*, or Perpendicular is also the Sine of the Angle at the Base, viz. Angle C.

For hence it is evident that every Side is the Sine of it's opposite Angle, and so will always hold, as Radius or Sine of the Angle B is to A C, the Side made Radius, so is the Sine of the Angle A to C B the Base, and so is the Sine of the Angle C to A B the Perpendicular, & *Contra*.

Secondly, Set one Foot of your Compasses in A, and extend the other to B, now is A B made Radius, or Semidiameter of a Circle, with which draw the Arch B D, which is sufficient for our present Purpose, as well as if we had drawn the Whole Circle; now the Arch B D measures the Angle at A, and 'tis plain C B is the Tangent of

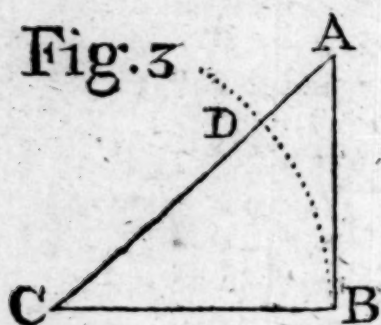
of the Arch DB , therefore CB is also the Tangent of the Angle at A , and CA is the Secant of the same Angle, now it will hold as Radius, or Sine of 90° is to AB (which is now made Radius) so is the Tangent of the Angle A to CB , and so is the Secant of the Angle A to CA . Or as AB is to Radius, so is CB to the Tangent of the Angle A , and so is CA to the Secant of the Angle A .



Thirdly, Set one Foot of the Compasses in C , and extend the other to B , and with that Extent draw a Circle, or the Arch DB will do, now the Arch BD measures the Angle at C , and BA is the Tangent of the same Arch, consequently BA is the Tangent of the Angle C . Now it will hold, As Radius, Sine of 90 , is to CB (now Radius) so is the Tangent of the Angle C to the Perpendicular BA ; and so is the Secant of the Angle C to the Hypotenuse CA . Or as the Base CB is to Radius, so is the Perpendicular BA to the Tangent of the Angle at the Base, viz. the Angle ACB .

Note, When we express an Angle by three Letters, the middle Letter always denotes the Angle.

And



And as in the Rule of three in common Numbers, so here you must observe that the First and Third Terms be Things of one Name, and likewise the Second and Fourth Terms must be of one Name; that is, if the First be a Side, the Third Term must be a Side, and the Second and Fourth Terms will be Angles; but if the First Term be an Angle, the Second and Fourth will be Sides: And when you seek an Angle begin with a Side, but when you seek a Side begin with an Angle. For the Sum of the Logarithms of the Second and Third Terms, take the Logarithm of the First Term, and the Remainder is the Logarithm of the Fourth Term, or Answer. But when Radius comes not in the Proportion, (as it will not in oblique Triangles) take Complement Arithmetical of the Logarithm of the First Term, and add them all into one Sum. (always rejecting Radius) gives the Logarithm of the Fourth Term, or Answer. The Complement Arithmetical is found by subtracting each Figure of the Logarithm severally from 9. but the Unites Place from 10, and the Remainder is the Complement Arithmetical of the Logarithm sought.

EXAMPLE.

What's Complement Arithmetical of the Sine of $20^{\circ} 20'$

See the Work.

From	-	-	-	-	.99999990
Take the Logar. Sine of $20^{\circ} 20'$	-				.95409314
Co. Arith. of	-		$20^{\circ} 20'$		<u>0.4590686</u>

What's the Co. Arith. of the Logarithm of 60?

From	-	-	-	-	.9.9999990
Take the Logarithm of 60	-				.1.7781512
Co. Arith. of the Logarithm 6 is	=				<u>8.2218488</u>

Note, The Complement Arithmetical of a Tangent, or Tangent Complement, is the Tangent Complement of that Tangent, or the Tangent of that Tangent Complement.

EXAMPLE.

What's the Com. Arit. of the Logarithmetical Tangent of $20^{\circ} 20'$?

See the Work.

Logarithmetical Tangent $20^{\circ} 20'$ is	=	9.5688735
Tangent Compl. or Co. Arith.	=	10.4311265
And Co Ar. of the Tangent of $69^{\circ} 40'$	=	9.5688735

e like observe of any other.

If you work with Tables wherein there is no Logarithmetical Secants, that defect may readily be supplied by subtracting the Co-Sine out of the double of the Radius, the Remainder is the Secant of that Arch, or Angle, whose Co-Sine you made Use of in the Work.

EXAMPLE.

What's the Logarithmetical Secant of $20^{\circ} 20'$?

Operation.

Operation.

Double of the Radius = 20.0000000

Co-sine of $20^{\circ} 20'$ = 9.9720579

Secant of $20^{\circ} 20'$ = 10.9279421

By what goeth before, it appears that the Secant and Co-Sine of any Arch, are the Complement one of the other to the double Radius of artificial Numbers, so that if one be given, the other is also known.

The versed Sine may be found by adding .3010300 unto the double of the Sine of half the Arch, and rejecting Radius, gives the versed Sine sought.

EXAMPLE.

What's the versed Sine of $20^{\circ} 20'$?

First, set down - - - .3010300

Half of $20^{\circ} 20' = 10^{\circ} 10'$ it's Sine 9.2467746

The same again - - - 9.2467746

The versed Sine of $20^{\circ} 20' = 8.7945792$

To find the Logarithm of any Number under 101000.

The largest and best Tables of Logarithms now in being, are those known by the Name of *Sherwin's* Tables; which I advise all Students in the Mathematics to make use of, as being the largest and most correct.

First then, It is to be observed, that the Characteristic or Index is always less by one than the Number of Places of the absolute Number, and that all Numbers with their Logarithms under 1000, are found in each Column answering each Number, and so you will find the Logarithm.

$$\text{of } \left. \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\} \text{ to be } \left\{ \begin{array}{l} 0.0000000 \\ 0.3010300 \\ 0.4771212 \\ 0.6020600 \end{array} \right.$$

and

$$\text{And of } \left. \begin{array}{l} 10 \\ 20 \\ 30 \\ 40 \end{array} \right\} \text{ to be } \left\{ \begin{array}{l} 1.0000000 \\ 1.3010300 \\ 1.4771212 \\ 1.6020600 \end{array} \right.$$

Here you see that the Logarithm of 1 and of 10 is the same, only changing the Index as was noted above.

To find the Logarithm of any Number, from 1000, to 101000.

If your given Number has only four Places, the Logarithm is in the next Column under 0, by placing it's proper Index 3. If the Number has five Places, as suppose 11814, find 1181 in the first Column, and 4 at the Top; and in the common Angle is .0723970, to which prefix it's Index 4, and the Logarithm of 11814 will be 4.0723970; after this Manner may the Logarithm of a decimal or mixt Number be found, as is made manifest by these Examples.

Numb.	Logar.
1181.4	3.0723970
118.14	2.0723970
11.814	1.0723970
1.1814	0.0723970
.11814	1.3723970
.011814	2.0723970

If you would know the Logarithm of $27\frac{1}{10}$, set it down as whole thus, 27.1, and look for the Logarithm thereof, all one as if it were whole, only put one for the Index and 'tis done; so I find the Logarithm of 27.1 to be 1.4329693.

If you would have the Logarithm of a vulgar mixt Number, as suppose $27\frac{1}{2}$?

Reduce it into an improper Fraction, and subtract the Logarithm of the Denominator, from the Logarithm of the Numerator 55, and the Remainder is the Logarithm sought.

Operation

Operation.

Numerator - - 55	Logarithm 1.7403627
Denominator - 2	Logarithm 0.3010300
Logarithm of $27\frac{1}{2}$ is =	1.4393327

After the same Manner I find the Logarithm

$$\text{of } \left\{ \begin{array}{l} \frac{1}{4} \\ \frac{1}{2} \\ \frac{3}{4} \end{array} \right\} \text{ to be } \left\{ \begin{array}{l} 0.6020600 \\ 0.3010300 \\ 0.1749337 \end{array} \right.$$

See my *Compleat System of Astronomy*, page 287, for further Satisfaction.

In finding a Sine or a Tangent, if your Degree be under 45 you must seek it on the Head of the Table, and the Minute in the first Column on the left, down to 60; but when your Degree is more than 45, you must have it at the Bottom of the Table, and the Minutes on the right Hand ascending to 60.

Let it be required to find the Logarithm, Sine and Tangent of $29^{\circ} 45'$?

Sine = 9.6956712, Tangent = 9.7570520, with their Differences 2210 and 2932; by Help of these Differences we find the Seconds answering any Quantity of Seconds under 60, thus,

Suppose I wanted the Sine of $29^{\circ} 45' 38''$, you must always say,

If $60' : 2210 \text{ Diff.} :: 38'$

$$\begin{array}{r} 38 \\ \hline 17680 \\ 6630 \\ \hline 60) 83980 (1399\frac{2}{3} \\ \hline 23 \\ \hline 59 \\ \hline 58 \\ \hline 40 \end{array}$$

To the Sine of $29^{\circ} 45'$ - - - 9.6956712

Add the proportional Part 1399 $\frac{2}{3}$

Logarithm Sine of $29^{\circ} 45' 38'' = 9.695811\frac{2}{3}$

The like observe for Tangents to any Quantity of Seconds.

But if you have a Logarithm, Sine, Tangent or Secant, and want the Arch or Angle answering thereto, it is but the Converse of the former Operation; for suppose the Sine were 9.6958111, and the Angle answering required?

See the Work.

Given Sine = - - - - - 9.6958111

Nearest less Sine $29^{\circ} 45' = 9.6956712$

Difference = 1399

Now say, If 2210 Diff. : $60'$: 1399 Diff.

60

2210) 83940 (38 $\frac{1}{2}$

693

1764

1768 Ferè

By which I find the Arch answering is $29^{\circ} 45' 38''$. And here it will not be amiss to inform the Reader that instead of saying Sine Complement or Tangent Complement, we say for shortness, Co-Sine, or Co-Tangent, &c. and the Word Complement signifies a filling up, or what any Arch or Angle wants of 90, or of 180, &c.

Here are seven Cases, which we shall briefly illustrate as follows; in which Things given are marked with a Dash of the Pen thus X, and Things required with a Cypher thus $\ominus$. And further

Note, That when the Angles are required, a required Side cannot be made Radius. And in the Solution of the following Cases we shall perform the same.

1. By Construction.

2. By Logarithms, &c.

T

3. By

3. By *Gunter's Scale*, or *Slider*.
4. By the *Sector*.
5. By the *Natural Sines*, *Tangents*, and *Secants*,
6. By *Natural Numbers*.

Case I.

Given, The Angles, and Base, to find the Cathetus, or Perpendicular, and the Hypothenufe.

Let the Angle at the Perpendicular be $48^{\circ} 30'$, then the $\angle$ at the Base will be it's Complement to 90, viz. $41^{\circ} 30'$, and the Base 56 Feet, Yards, Miles, or any other Measure at Pleasure. I demand the Perpendicular and the Hypothenufe?

The Hypothenufe made Radius. Construction, Fig. I. page 198.

Open the Sector to any convenient Distance, on the Line of Lines, and take off 56 and lay it from B to C, take the Chord of 60 (of any Radius whatsoever) set one Foot of the Compasses in C, and sweep the Arch A E, as the Sector now stands take off from the Chords $41^{\circ} 30'$ the Quantity of the given Angle at the Base, and set it from E to A, draw C A and B A, and the Triangle is completed; then measure C A and B A severally upon the Line of Lines on the Sector (being first set the Base 56) and you will find A B to measure 50 *ferè*, and A C 74.8 Feet, &c.

Analogy for the Hypothenufe, C A.

As S $\angle$ A	$48^{\circ} 30'$	9.8744561
To C B	56	1.7411880
So Radius	90 00	10.0000000
To A C	74.77	1.8737319

For

For the Perpendicular B A:

As Radius	90° 00'	10.0000000
To A C	74.77	1.8737319
So S < C	41 30	9.8212646
To B A	49.545	1.6949965

Secondly, In Case I, the same Things given to find the required, A B made Radius. See Fig. II. page 199.

Analogy for the Perpendicular.

As Tang. < A	48° 30'	10.0531916
To B C	- 56	1.7481880
So Radius	90 00	10.0000000
To A B	49.545	1.6949964

For the Hypotenuse A C.

As T < A	48° 30'	Co. Ar.	9.9468084
To C B	56		1.7481880
So Sec. < A	48 30		10.1787354
To C A	74.77		1.8737318

Or you may say thus, A B being found.

As Radius	90° 00'	10.0000000
To A B	49.545	1.6949964
So Sec < A	48 30	10.1787354
To A C	74.77	1.8737318

3. *To Work the same upon Gunter's Scale.*

Extend the Compasses in the Line of Sines, from 48° 30', to 46 in the Line of Numbers, and the same Extent will reach from Radius or Sine of 90°, to 74.8 in the Line of Numbers. For the Perpendicular A B, extend the Compasses from the Sine of 90 to 74.8 in the Line of Numbers, and the same Extent will reach from the Sine 41° 30' to 49.5 in the Line of Numbers.

There are Sliding Gunter's made, which worketh far better than with Compasses.

4. By the Sector.

This most useful Instrument is fully described in Chap. V. with it's Use in many Cases; I shall here compleat it's Use in the Solution of Triangles.

Case I.

To find the Hypothenuſe A C Analogy.

$$\text{As } S A : C B :: R : A C.$$

Take the Base 56 in your Compasses, and make it a lateral Distance in the Line of Lines, carry this Extent to the Sines, and make it a Parallel of $48^{\circ} 30'$ the Quantity of the Angle at the Perpendicular, then take off the Parallel Line of 90° , and that Extent will reach laterally in the Line of Lines, from the Center of the Sector, to 74.8, the Hypothenuſe A C, as required.

Secondly, For the Perpendicular A B Analogy.

$$\text{As } R : A C :: S. C : B A:$$

Take the lateral Hypothenuſe A C 74.8 on the Line of Lines, and make it a parallel Radius on the Sines $90, 90$, then take the parallel Sine of $41^{\circ} 31'$, the Angle at the Base, and it will reach latterally from the Center in the Line of Lines to 49.54, the Perpendicular sought.

Secondly, The Perpendicular A B made Radius, for the Perpendicular A B Analogy.

$$\text{As } T. A : C B :: R : A B.$$

Take the Base C B = 56 laterally on the Line of Lines, and make it a Parallel on the small Tangents of $48^{\circ} 30'$, then the Parallel of the small Tangent 45° will reach from the Center of the Sector laterally to 49.54, the Perpendicular sought.

5. By the natural Sines, Tangents, and Secants.

In *Sherwin's* Tables you have the Natural Sines and Tangents ; answering to every Degree and Minute of the Quadrant : The whole Sine, or Semi-diameter of the Circle = Radius being = 10000000, which are no other than the natural Numbers of the artificial Sines and Tangents : And this Way of working was used before the Logarithmic Canon was invented, but it being troublesome, and now quite out of Use, I offer it here only for Curiosity, and not for Practice.

Case I. Scheme 1. page 198.

$$\text{Given } \left\{ \begin{array}{l} \text{CB } 56 \\ \angle \text{CAB } 48^\circ 30' \\ \angle \text{ACB } 41^\circ 30' \end{array} \right\} \text{To find } \left\{ \begin{array}{l} \text{AC} \\ \text{AB} \end{array} \right.$$

First, *The Hypotenuse AC, Analogy.*

$$\text{As SA : CB : R :: AC.}$$

Seek in the Tables for the natural Sine of the Angle CAB $48^\circ 30'$, and you will find it to be 7489557, which must be the first Term in the Rule of Three; the Base = 56 the second Term, and Radius the third Term ; which being placed in due Order will stand thus.

$$\begin{array}{rcl} \text{Sine A.} & \text{CB} & \text{Radius.} \\ \text{As } 7489557 & : 56 & :: 10000000. \\ & & 10000000 \end{array}$$

$$7489557)56000000(74.77 = \text{AC}$$

$$\underline{52426899}$$

$$35731010$$

$$\underline{29958228}$$

$$57727820$$

$$\underline{52426899}$$

$$53009210$$

$$\underline{52426899}$$

$$\underline{582311}$$

The

The Analogy for the Perpendicular A B.

As S A : C B :: S C : A B. Or,
As R : A C :: S C : A B.

Operation.

S. A C B S. C.
As 7489557 : 56. :: 6626201

56

39757206

33131005

7489557)371067256(49.54=B A.

29958228

71484967

67406013

40789630

37447785

33418450

29958228

3460222

Case I.

The Perpendicular made Radius. Analogy for the Hypothenuse A C.

As T A : C B :: Sec. A : A C.

Operation.

Of Plain Trigonometry.

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Operation.

$$\begin{array}{lcl} \text{T. A.} & \text{C. B.} & \text{Sec. A.} \\ \text{As } 11302944 & : 56 & :: 15091605 \\ & & 56 \end{array}$$

$$\begin{array}{r} 90549630 \\ 75458025 \\ \hline \end{array}$$

$$\begin{array}{l} 11302944)845129880(74.77=A.C. \\ 79120608 \end{array}$$

$$\begin{array}{r} 53923800 \\ 45211776 \\ \hline \end{array}$$

$$\begin{array}{r} 87120240 \\ 79120608 \\ \hline \end{array}$$

$$\begin{array}{r} 79996320 \\ 79120608 \\ \hline \end{array}$$

$$875712$$

Analogy for the Perpendicular A B.

$$\text{As T. A. : C B :: R : A B.}$$

Operation.

$$\begin{array}{lcl} \text{T. A.} & \text{C B.} & \text{R.} \\ \text{As } 11302944 & : 56 & :: 10000000 \end{array}$$

$$\begin{array}{l} 10000000 \\ 11302944)560000000(49.545 \text{ fereet} \end{array}$$

$$45211776$$

$$107882240$$

$$101726496$$

$$61557440$$

$$56514720$$

$$50427200$$

$$45211776$$

$$52154240$$

$$56514720 \div$$

In

Construction.

Let the equal Leg'd and right angled plain Triangle ABC be given. Then with the Radius CA = Hypothenufe describe the Semicircle DFG, and make GH = CG. Draw DE parallel to BA, as a Tangent to the Angle ACB, and draw EAH; then is BA the Sine, DA the Chord, and DE the Tangent of the Angle ACB. DH is the Triple of the Radius AC or longest Side of the Triangle ABC.

Demonstration.

Because DE is parallel to BA, the Triangles HBA, HDE as standing on the same Base are Similar, and therefore proportional. Then,

As HB : BA :: HD : DE = DA because DE is the Tangent, and DA is the Chord of the same Arch.

The Chord of the Angle ACB, must be reckoned an Arch of a Circle whose Circumference is 360° , it's half is = DG, being the Chord of 180° , but we must find the Diameter in such Parts as the Circumference is 360° . Thus,

As $3.141592 : 1 :: 360 : 114.59158 = DG$ whose half is = $57.25979 = DC$, whose Triple is $171.88737 = DH$, now because of the great Fraction, the 171 is made 172 Whole, whose half is 86 called a general Number made Use of in finding of Angles, as will be shewn by and by. And $DC = CG = GH =$ to the Hypothenufe CA. Now we suppose the Leg BC, to be the longest of the Two, and therefore it will be, as twice the Hypothenufe CH + B : HD 172 thrice the Radius; so is the shortest Leg BA to DE = $\angle BAC$. In short, as $\frac{1}{2} BC + AC : AB :: \frac{1}{2} DH = 86 :$ to the Angle opposite to the shortest Side.

Now by this general Number 86 and the 47th of the First of *Euclid*, are all the Cases in right angled plain Triangles solved, as will more plain appear by what follows.

Case 1. Given, The Angles and Base, to find the Hypothenufe and Perpendicular.

EXAMPLE.

EXAMPLE.

Figure 1, page 198, $CAB 48^{\circ} 30'$ $ACB 41^{\circ} 30'$, $CB, 56$, I demand AC and BA ? First, therefore when any right angled plain Triangle is proposed for Solution by this Method, having the Angle given, you must first assume another right angled plain Triangle as abc , and the Leg opposite to the lesser Angle as ab , to be 1.00, &c. where by finding the Hypothenufe and other Leg, (the greater Leg) by the Rule here given; which done, these two Triangles ABC and abc are Proportional. See *Theorem 9*.

That is, (observing what Side is given in the proposed Triangle) As any one Side in the assum'd Triangle abc , is to it's corresponding Side in the proposed Triangle ABC , so is any other Side in the assumed Triangle to it's corresponding Side in the proposed Triangle. And this Method will be exact enough for common Use, if you work it to two or three Decimal Places, ever remembering to reduce the Minutes of the Angles to Decimals, which is done by Inspection in the following Table.

A Table to Convert Minutes and Seconds into Decimals, & Contra. One Degree the Integer.

Sec.	Decim.	Min.	Decim.
15	.0042	29	.4833
30	.0083	30	.5
45	.0125	31	.5167
Min.		32	.5333
1	.0167	33	.55
2	.0333	34	.5667
3	.05	35	.5833
4	.0667	36	.6
5	.0833	37	.6167
6	.1	38	.6333
7	.1167	39	.65
8	.1333	40	.6667
9	.15	41	.6833
10	.1667	42	.7
11	.1833	43	.7167
12	.2	44	.7333
13	.2167	45	.75
14	.2333	46	.7667
15	.25	47	.7833
16	.2667	48	.8
17	.2833	49	.8167
18	.30	50	.8333
19	.3167	51	.85
20	.3333	52	.8667
21	.35	53	.8833
22	.3667	54	.9
23	.3833	55	.9167
24	.4	56	.9333
25	.4167	57	.95
26	.4333	58	.9667
27	.45	59	.9833
28	.4667	60	1.

Now

$$\begin{array}{r} C a = 151 \\ \times \text{ by } 2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Subtract} \quad 3.02 \\ \text{From} \quad 4.14 \\ \hline \end{array}$$

$$C b = 1.12$$

Now for C A. As $C b$ CB. $C a$.
As $1.12 : 56 :: 1.51$

$$\begin{array}{r} 56 \\ \hline 906 \\ 755 \\ \hline 1.12) .84.56(75.49 = C A. \end{array}$$

$$\begin{array}{r} 784 \\ \hline 616 \\ 560 \\ \hline 560 \\ 448 \\ \hline 1120 \\ 1008 \\ \hline 112 \end{array}$$

Note, The Side opposite to the lesser Angle is always called 1.

Secondly, For the Leg A B.

$$\begin{array}{r} C b \quad C B \quad a b. \\ \text{As } 1.12 : 56 :: 1. \end{array}$$

$$1.12) 56.00(50 = A B.$$

$$\begin{array}{r} 560 \\ \hline 0 \end{array}$$

U

After

After this Manner are the 2d and 3d Cases solved; but the 4th, 5th and 6th Cases must be wrought by Natural Arithmetic, thus; For the Sides, the Square of the Hypothenufe, is equal to the Square of the Sum of the Squares of the other two Legs. *See Theorem 14.*

And to find an Angle, you take half the longer of the two lesser Sides, *i. e.* Base and Perpendicular, and add it to the Hypothenufe, then say, As that Sum, is to the shortest Side, so is the general Number 86, to the Quantity of the Angle opposite to the shortest Side: As we will shew by and by, when we come to the Fourth Case.

Case 2. *Given, the Angles and Hypothenufe, to find the Base and Perpendicular.*

N. B. The Datas are the same in all the following Cases as was assum'd in the First Case.

Analogy. A C made Radius.

As R : A C :: S A : C B. Base.

90 : 74.77 :: 48° 30' : 56 = C B.

As R : A C :: S C : A B Perpendicular.

90 : 74.77 :: 41° 30' : 49.545 = B A.

Case 3. *Given, the Angles and Perpendicular, to find the Hypothenufe and Base.*

Analogy. A B made Radius.

As R : A B :: T A : C B.

90 : 49.54 :: 48 30 : 56 = Base.

As R : A B :: Sec. A : A C.

90 : 49.54 :: 48 30 : 74.77 Hypothenufe.

Case 4. *Given, the Base and Perpendicular, to find the Angles and Hypothenufe.*

Analogy. A B made Radius.

As A B : R :: C B : T. < A.

49.54 : 90 :: 56 : 48° 30'.

Of Plain Trigonometry.

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As R : A B :: Sec. A : A C.

90 : 49.54 :: 48 30 : 74.77.

By Natural Arithmetic.

1. First for the Hypothenuſe A C.

C B = 56	A B = 49.54
<u>56</u>	<u>49.54</u>
336	19816
<u>280</u>	<u>24770</u>
	44586
□ C B = 136	19816
□ A B = 2454.2116	<u> </u>
Sum = 5590.2116	2454.2116

5590.2116 (74.77 ſerè. = A C.

49

144) 690

576

1487) 111421

10409

14947) 101216

104629+

For the Angles. $\frac{1}{2}$ Longest of the two leſſer Sides = 28

$\frac{1}{2}$ C. B = 28	
A C = 74.77	
<u> </u>	
Sum = - - -	102.77

Of Plain Trigonometry.

Sum. AB
 Now say, As 102.77 : 49.54 :: 86.

29724
 39632

102.77) 4260.4400 (41°.45 = <C.

41108

14964
 10277

46870
 41108

57620
 51385

6235

<B = 90° 0'
 <C = 41 27
 <A = 48 33

.45 = 27'. See Table, page 314.

Case 5. Given, the Base and Hypothenufe. to find the Angles and Perpendicular. Analogy, Base made Radius.

As CB : R :: CA : Sec. C.

56 : 90 :: 74.77 : 41° 30' = C.

As R : CB :: TC : AB.

90 : 56 :: 41° 30' : 49.54 = AB.

Case

Case 6. Given, the Hypothenuſe, and Perpendicular, to find the Angles and Baſe. Analogy, Hypothenuſe made Radius.

As $AC : R :: AB : S. C.$

$74.77 : 90 :: 49.54 : 41^{\circ} 30' = \angle C.$

As $R : AC :: S. A : CB =$

$90 : 74.77 :: 48 30 : 56 = \text{Baſe.}$

Case 7. Given, all the Sides, to find the Angles. Analogy, Hypothenuſe made Radius.

As $AC : R :: CB : SA$ whoſe Complement to a Quadrant $= 90^{\circ}$ Angle $B = \angle C 41^{\circ} 30'.$ Or,

As $AC : R :: AB : C.$

2. For the ſame Angles, AB made Radius.

As $AB : R :: CB : T. A.$

3. CB made Radius.

As $CB : R :: AB : T. C.$

Note. When you work upon *Gunter*, a Proportion that has Secants, this may be ſupplied by the Co-Sines, for by reaſon the Secant begins not 'till the Sines ends (See the Scheme in page 192) ſo becauſe the Secants are not placed upon *Gunter*, the Line of Sines ſupplies that Defect, viz. by their Co-Sines, thus,

Sines - - - - - 90, 80, 70, 60, 50, 40, 30, 20, 10,

Co-Sines, or Secants, 0, 10, 20, 30, 40, 50, 60, 70, 80,

So in *Case 4*, where AB is made Radius to find AC , extend the Compaſſes from the Radius, or Sine 90° , to the Sine of $41^{\circ} 30'$, Co-Sine of Angle A , and the ſame Extent will reach from 49.5 the Side AB , to 74.8 in the Line of Numbers, and ſuch is the Hypothenuſe AC , as was required.

Before I cloſe this Part, I ſhall add ſome uſeful Problems, as a Touch ſtone to try the Learner's Ability.

PROBLEM I.

Given, the Angles in a right angled plain Triangle, with the Sum of the Sides, to find the Sides severally.

EXAMPLE.

In the Triangle A B C, Fig. 1. page 198 the Sum of the Sides have already been found to be 180.31; and $\angle A 48^\circ 30' < C 41^\circ 30'$, I demand the Sides severally.

For a Solution, you may suppose any of the Sides what Length you please, as suppose the Base to be 40, now with this false Base 40, and the true Angles lay down the Triangle, and work with it as if it were the true Base thus,

A C made Radius.

$$\begin{array}{l} \text{As S. A : CB :: R : AC.} \\ 48^\circ 30' : 40 :: 90 : 53.408. \end{array}$$

Secondly, For A B.

$$\begin{array}{l} \text{As S. A Co-Ar. : CB :: S C : A B.} \\ 48^\circ 30' : 40 :: 41^\circ 30' : 35.389. \end{array}$$

Now we have found the other Sides in proportion to the guessed Base.

$$\begin{array}{rcl} \text{A C} & = & 53.408 \\ \text{C B} & = & 40 \\ \text{A B} & = & 35.389 \\ \hline \text{Sum.} & & 128.797 \end{array}$$

Now with this feigned Sum of the Sides find the true Base thus,

If the feign'd Z Sides	---	128 793	C A.	7.8901179
Give for their Base	---	40	---	1.6020600
What true Z Sides give	---	180.31	---	2.2560198
Answer, true Base C B	---	56.001	---	1.7481977

Now

Construction.

Draw the Base C B 56 of any convenient Length, take it in your Compasses and make it a Parallel upon the Line of Lines on the Sector, erect an occult Perpendicular, as B D, drawn at Pleasure: Now because the Sum of the Hypothenufe and Perpendicular amounts to 124.31, this exceeding the Bounds of the Sector, take half that Number, viz. 62.155 in your Compasses from the Line of Lines, (as the Sector was just now set to the Base 56) and lay it twice upon the occult Line B D, and it will terminate at D; draw B D, and C D, Bisect C D in E by *Prob. I.* and draw E A at right Angles to C D, then doth the Point A determine the true Triangles; to complete which, draw C A, and 'tis done.

Now by Construction the Angle C D B is just half the Angle C A B.

For the Angle C A B, D B made Radius.

As D B	124.31	.20945061
To Radius	90 00	10.0000000
So C B	56	1.7481880
To T < C D B	24 15	9.6536819
Doubled=CAB	48 30	Comp=< A C B 41° 30'

Now by working as in *Case 1.* I find A C to be 74.77, and A B 49.54 which two Sides added together, make 124.31 as was at first proposed, which proves the Work to be right. The < A C D = A D C, because A C = A D.

PROBLEM III.

Given, the Perpendicular and Sum of the Hypothenufe and Base, to find the Angles.

EXAMPLE.

Let the Perpendicular A B = 49.54, and the Sum of the Hypothenufe and Base be 130.77; I demand the Angles? and the Sides A C and C B severally?

Construction

P R O B L E M IV.

How by the Proportion of a Man's Shadow unto his Height, or other Shadow to it's Gnomon, set perpendicular to the Horizon, to find the Hour of the Day.

1. It is by this Problem that the Table is calculated to find the Hour of the Day by a Staff divided into 10 equal Parts, which Table is printed in many Books, but will only serve for one Latitude, *viz.* that for which it was calculated. In order then to make such a Table for any Latitude, you must have the Sun's Altitude at every Hour and Quarter of the Day, for every Day in the Year, for your given Latitude, and then it will hold, As Radius, To the Length of the Staff in Parts, as suppose 10, or 100, or any other Number, So is the Co-Tangent of the Sun's Altitude, To the Length of the Shadow of the Staff in the same Parts. Which is inserted into a Table right against the Day of the Month, and in a Column noted at the Top with the given Hour; this is done for every Hour and Quarter of the Day, and for every 5th Day of the Month for your given Place. Thus the Table being made, set up your Staff perpendicular on plain level Ground, and mark where the Shadow of the Top of the Staff falls, measure the Distance from the Place where the Staff stood and the mark just made, which note in Staff's Length and Parts, then look in your Table for the same Number; and at the Top of the Table is the Hour of the Day when the Observation was made.

2. By the Altitude of a walking Cane only, &c. set perpendicular to the Horizon, to find the Height of any Tree, Castle, Tower, or Steeple, the Sun shining.

The Solution of this, is from similar Triangles; as suppose your Cane, or Stick be divided into any Number of equal Parts, as 10, 36, &c. and you walking along seeing an Object cast a Shadow (the Sun shining) set up your Stick, and with it measure the Length of it's Shadow which we will suppose to be 40 Parts, that is one Stick's Length and 4 Parts more; with it also measure the Length of the Shadow of the Object, which let be $13\frac{1}{2}$ Sticks Length, that is 504 Inches; now it will hold, (See Fig. II. page 199)

As $c b$ the Length of the Shadow of the Stick, in the little Triangle, is to $b a$ the Height of the Stick ; So is CB in the great Triangle, the Length of the Shadow of the Tower, to B A the Height thereof.

Operation.

$c b.$ $b a.$ C B. B A.^r
As 40 : 36 :: 504 : 453.6 = 37.75 Yards,
and such is the Height of the Object. And thus by the Help of a Canē or Stick, and plain Triangle, we can tell the Height and Distance of any Place.

II. Of Oblique Angled plain Triangles.

OBLIQUE Triangles, are such as have in them no right Angle, but are either acute or obtuse, or both. See the *Geometrical Definitions*.

An Oblique angled plain Triangle has always all the Angles acute, or else two acute, and one obtuse, whereof the Sum of any two of them is always a Complement of the Third to 180° , or a Semi-circle, with which you must work instead of the obtuse Angle itself. And herein are three *Axioms*, on which the whole Fabric of this Doctrine is laid.

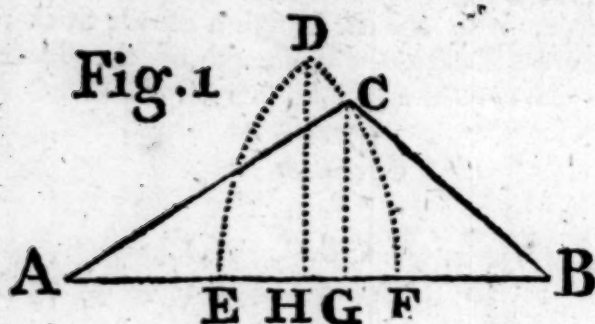
Axiom I.

The Sine of every Angle is in direct Proportion to it's opposite Sides, and every Side is in direct Proportion to the Sine of it's opposite Angle.

Construction.

Draw the oblique Triangle A C B,

M



Make $BD = AC$, with the Radius BD sweep the Arch DE , with the same Extent set one Foot of the Compasses in A , and draw the Arch CF , let fall the Perpendiculars CG and DH upon the Side AB .

Now CG is the Sine of the Angle A , and DH , the Sine of the Angle B ; the Triangles ACG , and BDH are Similar and Proportional. See *Euclid* 6, 4, where 'tis demonstrated, that the Sides of equiangular Triangles are proportional. Therefore as $CG : SB :: GB : S. CGB$: and $CB : SCGB$. The like in the Triangle ACG . And from hence it will always hold in any oblique angled Triangle, (whether plain or spherical) As $S. ACB : AB :: SCAB : CB$, and so is the $S. ABC : AC$; & *Contra*.

Axiom II.

As the Sum of two Sides including an Angle, is to their Difference; So is the Tangent of half the Sum of the other two Angles, (that is the Angles opposite to the given Sides) to the Tangent of half the Difference of the said Angles. Which added to the half Sum, gives the greater Angle sought; but subtracted from the half Sum gives the lesser Angle.

It is by this Axiom that we find the geometric Place of a Planet.

Construction.

Make $CD = CB$. and draw BD ; on which, let fall the Perpendicular CE , and it will Bisect DB in E ; draw EF

EF parallel to AG , and it will Bifect AD in F , and is the Difference of the two Sides AC and CB .

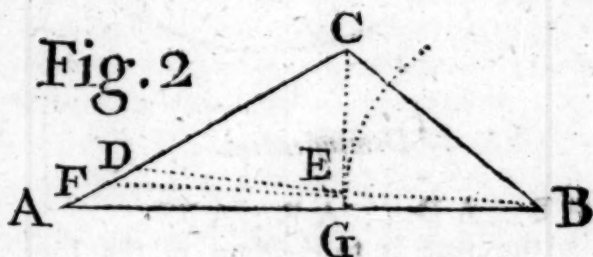
Demonstration.

The Angle CBE is half the Sum of Angles CBA and BAC , and EBG half their Difference.

Now if BE be made the Radius of a Circle, EC will be the Tangent of $\frac{1}{2}$ the Sum, and EG of half their Difference. The Triangle CGA is cut proportional by the Line EF . For,

$$\text{As } CF : FA :: CE : EG.$$

Note, Half the Difference of any two Numbers added to, and subtracted from their half Sum, gives the greater and lesser Number, that is, the Numbers themselves. As suppose the two Numbers be 10 and 14, their Sum = 24, $\frac{1}{2} = 12$, their Difference = 4, $\frac{1}{2} = 2$, then $12 + 2 = 14$ the greater Number, and $12 - 2 = 10$ the lesser Number.



Axiom III.

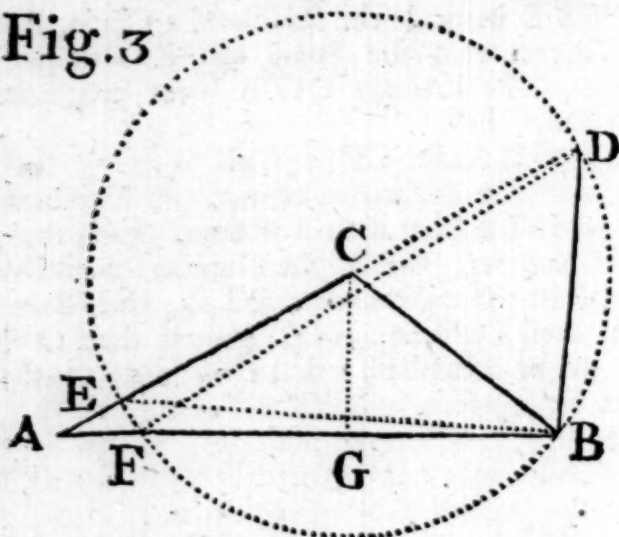
As the true Base, Is to the Sum of the other two Sides, So is the Difference of the said Sides, To the alternate Base. That is; to the Difference of the Segments of the Base made by a Perpendicular let fall from the Angles opposite to the Base.

Construction.

Lay down the Triangle ABC by Help of the Sector, (as has been already taught) by making $AB = 92$ on the Line of Lines, $AC = 60$, and $CB = 50$. Then on the
X Center

Center C, with the Distance CB, describe the Circle, which will intersect both of the other Side of the Triangle at E and F; and then DF will represent the Sum of the Legs AC and CB, AE will represent their Difference, and AF will represent the Difference of the Segments of the Base made by the fall of the Perpendicular CG.

Fig. 3

*Demonstration.*

I say $AB : AD :: AE : AF$.

That is, the Base is to the Sum of the Legs, as the Difference between the two Sides, is to the Difference of the Segment of the Base.

Note, If the two Legs including an Angle be equal, then the two unknown Angles are known; because they are equal; whose Complement of the known Angle to 180, is their Sum, the half of which is each Angle severally.

If the three Sides be equal, the Angles are all acute, and each 60° , and the Sum of all three = 180.

In any oblique angled plain Triangle whatsoever, where the Three Sides are given to find, an Angle. See my *Uranoscopia*, page 55, 56.

RULE

R U L E.

From the Sum of the three Sides subtract the Side opposite to the Angle required, and Note the Remainder; then to the Co. Ar. of the two Sides, including the required Angle, add the Logarithm of half the Sum of the Sides, and the Logarithm of half the Remainder; half the Sum of these Four Logarithms will be the Co-Sine of half the Angle required.

Here are four Cases which we shall illustrate in the following Order.

Case 1. *Given, All the Angles and one Side, to find the other two Sides.*

E X A M P L E.

Let the Angle A be 30° , B $36^{\circ} 52'$ and ACB $113^{\circ} 8'$, the Side CB = 50, I demand the other two Sides?

1. First for the Side A C. See Fig. 1, page 228.

As S < A	$30^{\circ} 00'$	Co. Ar.	0.3010299
To CB	- 50	-	1.6989700
So S < B	$36^{\circ} 52'$	-	9.7781186
To A C	59.995	-	1.7781185

2. For the Base A B.

As S < A	$30^{\circ} 00'$	Co. Ar.	0.3010299
To CB	- 50	-	1.6989700
So S < C	$66^{\circ} 52'$	-	9.9635957
To A B	91.959	-	1.9635956

Case 2. *Given, Two Sides and an Angle opposite to one of them, to find the Angles and the other Side.*

E X A M P L E.

Let CB be = 50, AB = 91.959 and the Angle CAB 30° , what are the other Angles, and the Side AC? See Fig. 1, page 228, by Axiom 1. For the Angle B.

X 2.

Operation

Operation.

As CB	-	50	Co. Ar.	8.3010299
To S < A	30° 00'	-		9.6989700
So AB	91.959	-		1.9635956
To S < C	66 52	-		9.9635955
From	180 00			
Angle ACB	=	113 08		
Angle CAB	=	30 00		
Sum	=	143 08		
		180 00		
Angle ABC	=	36 52		

2. For the Side A C.

Operation.

As SA	-	30° 00'	Co. Ar.	0.3010299
To CB	-	50	-	1.6989700
So SB	-	36 52	-	9.7781186
To AC	-	59.995	-	1.7781185

Case 3. Given, Two Sides, and the Angle included, to find the Angles and the third Side.

EXAMPLE.

Suppose the Angle $ABC = 36^\circ 52'$, the Base of $AC = 91.995$, and $CB = 50$, I demand the other Angles, and the Side AC .

Operation.

By Axiom 2, Fig. 2, page 229.

The Three Angles	=	180° 00'
Given Angle B	=	36 52
Sum of < A and C	=	143 08
Half	=	71 34

Secondly,

Secondly,

$$\begin{array}{rcl}
 \text{Given Sides.} & \left\{ \begin{array}{l} AB \\ CB \end{array} \right. & \begin{array}{rcl} - & - & 91.995 \\ - & - & 50. \end{array} \\
 \text{Sum} & = & - & - & 141.995 \\
 \text{Difference} & = & - & - & 41.995
 \end{array}$$

Analogy.

$$\begin{array}{rcl}
 \text{As Sum of the Sides Co. Ar.} & 141.995 & - & 7.8477728 \\
 \text{To their Difference} & - & 41.995 & - & 1.6231976 \\
 \text{So Tangent } \frac{1}{2} \text{ Sum Angles} & 71^\circ 34' & - & 10.4771621 \\
 \text{To Tangent } \frac{1}{2} X < < \text{ fought} & 41 & 35 & - & 9.9481325
 \end{array}$$

$$\begin{array}{rcl}
 \text{Sum} & = < A C B & - & 113 & 9 \\
 \text{Difference} & = < C A B & & 29 & 59
 \end{array}$$

Lastly, for the Side A C by Axiom 1.

Analogy.

$$\begin{array}{rcl}
 \text{As } S < A & - & 29^\circ 59' \text{ Co. Ar.} & 0.3012489 \\
 \text{To } C B & - & 50 & - & 1.6989700 \\
 \text{So } S < B & - & 36 & 52 & - & 9.7781186 \\
 \text{To } A C & - & 60.026 & - & 1.7783375
 \end{array}$$

Case 4. Given, The Three Sides, to find the Angles,
Fig. 3, page 230.

EXAMPLE.

Let A B be 91.959, A C 60, and C B 50, I demand the Angles?

By Axiom III.

Operation.

$$\begin{array}{rcl}
 \text{Two shortest Sides.} & \left\{ \begin{array}{l} A C \\ B C \end{array} \right. & \begin{array}{rcl} 60 \\ 50 \\ \hline 110 \\ 10 \end{array} \\
 \text{Sum} & = & - & 110 \\
 \text{Difference} & = & - & 10 \\
 & & X & 3
 \end{array}$$

Analogy.

Analogy.

As the Base A B	-	91.959	Co. Ar.	8.0364044
To the Sum of A C and C D	110.	-		2.0413927
So is the Difference	10	-		1.0000000
To the alternate Base A F	11.967	-		1.0777971
From the true Base A C	=	91.959		

Remains F B = - 79.992

Now F G = G B, half of F B = 39.996 is the true Base to the right Angle plain Triangle G C B. Then F G 39.996 + A F 11.967 = A G 51.963, which is the true Base to the right angled plain Triangle A C G.

Now the oblique angled Triangle A C B, being reduced into two right angled plain Triangles G C B, and A C G; there are given C B = 50, and G B = 39.996 to find the Angles C B G, which by the 5th Case of right angled plain Triangles, will be found to be $36^{\circ} 52'$.

Secondly, In the Rect-angle plain Triangle A C G, there is given A C = 60, and A G = 51.963, to find the Angle of G A C; which by the said 5th Case of right angled plain Triangles, will be found to be 30° : And the Angle A C B $113^{\circ} 8'$.

Note, Because C D = C B, the Angle C D B = $\angle$ C B D = 56, 34 it being half of the Angle A C B, and the $\angle$ B C D = $66^{\circ} 52'$, it being the Complement of the Angle A C B to a Semicircle, or 180° . The Angle A B D = $93^{\circ} 26'$. But the Angle E B D is right, as being made in a Semicircle.

Here follows two Problems Extraordinary.

Given, one Side, and the Angle adjacent, with the Sum of the other two Sides, to find the Angles, and the Sides severally.

EXAMPLE.

In Fig 3, page 230. Let A B = 91.959, and the Angle at A 30° , the Sum of A C and C B = 110, I demand the Angles and Sides severally?

Construct

Construct the Triangle as has been already taught, make $AB = 91.959$, and the $\angle BAC = 30^\circ$, draw an occult Line AD , take the Sum of the two shortest Sides, viz. 110, and set from A to D , and draw DB , now in the oblique $\triangle$ plain Triangle ADB , we have given $AB = 91.959$, $AD = 110$, and the included Angle at A , to find the other Angles, and the Sides AC and AB , which work by the Second Axiom, as has been taught, the $\angle ACB = 113^\circ 8'$, the Angle $ABC = 36^\circ 52'$, and the Side $AC = 60$, and $CB = 50$.

PROBLEM II.

Given, one Side and the Angle opposite thereto, with the Difference of the other Two Sides, to find the Two Angles and the Two Sides.

EXAMPLE.

Let $AB = 91.959$, and the Angle $ACB = 113^\circ 8'$, the Difference of the Two unknown Sides $= 10$, I demand the Angles CAB , ABC , and the Side AC and CB severally?

Make $AB = 91.659$, and the Angle $ACB = 113^\circ 8'$, then will $CB =$ Radius of a Circle, which drawn, will cut AC in E , draw EB , now because the Triangle ECB (by Construction) is Equi-crural, the Angle $CEB =$ to the $\angle CBE$, and it's Quantity will be known from Theorem 4, that is $\angle E$ and B , will be each half the Complement of $\angle C$, to a Semicircle, and consequently are each $33^\circ 26'$. Now in the Triangle AEB , are known $AB = 91.959$, $AE = 10$, and the $\angle AEB = 146^\circ 34'$ it being the Complement of the Angle CEB to 180° : Then by Axiom 1, I find the $\angle ABE = 2^\circ 26' + \angle CBE = 33^\circ 26' = \angle ABC = 36^\circ 52' + \angle ACB = 113^\circ 8' = 150^\circ$. And $180 - 150 = \angle CAB = 30$. Also by the same Axiom I find $AC = 60$. $CB = 50$.

As right angled plain Triangles are useful in taking of Altitudes, so oblique Triangles are applied to the taking of Distances of all Kinds, as shall be shewn when we come to treat on Surveying of Land.

CHAP. VII.

Of Right angled Spheric Triangles.

DEFINITION.

SPHERIC TRIGONOMETRY teaches to measure the Sides and Angles of Spheric Triangles.

1. A great Circle of the Sphere is that which cuts the Globe in two equal Parts, of which there are six principally, *viz.* the Equinoctial, Ecliptic, Meridian, Horizon, the Equinoctial, and the Solstitial Colours. But you may imagine an infinite Number of great Circles at pleasure.

2. A Spheric Triangle is made by the Interfection of three great Circles of the Sphere.

3. As (on a plain Surface) a Right Line is the nearest Distance between any two Points; so on a Spheric Surface, an Arch of a great Circle is the nearest Distance between any two Points, or Places on the Terraqueous Globe.

4. A Spheric Angle is the same with the Inclination of the Planes of those two Circles which constitute the Angle; hence the opposite Angles at the Interfection of two Circles are equal. $ACB = ICH$.

5. The greatest Angle is always opposite to the greatest Side.

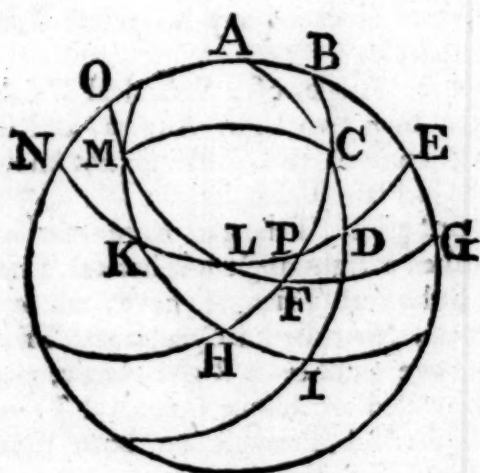
6. A Spheric Angle is measured by the Arch of a great Circle described on the Angular Point at the Distance of a Quadrant. So that with one opening of the Compasses you may describe a Spheric Triangle, all the Angles right, and every Side a Quadrant, as in the Triangle HMC the Sides are all Quadrants, and the Angles right.

7. What we have said in Plain Trigonometry, holds good here, *viz.* that the Radius is a Mean Proportional between

between the Tangent of an Arch, and the Tangent Complement of the same Arch, &c. which see.

8. A great Circle of the Sphere passing through the Poles of another great Circle, cut one another at right Angles.

9. The Sides of a Spheric Triangle may be turned into Angles, and the Angles into Sides, the Complement of the greatest Side, or greatest Angle to a Semicircle being taken in each Conversion. For in this Scheme. Let ABC be a Spheric



Triangle obtuse angled at B, then is DE the Measure of the Angle DAE, because AD and AE are Quadrants, by the 6 hereof; and FG is the Measure of the acute Angle FBG, which is the Complement of the obtuse Angle ABC. Let HI be the Measure of the Angle at C: KL is equal to the Arch DE, because KD and LE are Quadrants, and their common Complement is LD; LM is = to FG, because LG and FM are Quadrants, and their common Complement is LF, KM is equal to the Arch HI, because KI and MH are Quadrants, and their common Complement is KH. Therefore the Sides of the Triangle MKL are equal to the Angles of the Triangle ABC, taking for the greatest Angle ABC it's Complement, viz. the Angle DBE. And by the like Reason it is demonstrated that the Sides of the Triangle ABC are equal to the Angles of the Triangle MKL.

For

For the Side $AC = DI$, and DI is the Measure of the Angle HKR , which is the Complement of the obtuse Angle MKL . The Side $AB = NO$; now NO is the Measure of the Angle KLM . And Lastly, $BC = FH$ which is the Measure of the Angle at M , for AD and CI are Quadrants, so are AN and BO , BF and CH ; and CD , AO , and CF , are the common Complements of those Arches, therefore the Sides of a Spheric Triangle may be chang'd into Angles, and the Angles into Sides, *Q. E. D.*

10. The three Sides of any Spherical Triangle are less than two Semicircles.

11. The three Angles of a Spherical Triangle, are together greater than two right Angles, and therefore two Angles being known, the third is not known by consequence as it is in Plain Triangles.

12. If a Spherical Triangle have one or more right Angles 'tis called a right angled Spherical Triangle.

13. If a Spherical Triangle have one or more of it's Sides Quadrants, 'tis called a Quadrantal Triangle.

14. If it have neither a Right Angle, nor any Side a quadrant, 'tis called an oblique Spherical Triangle.

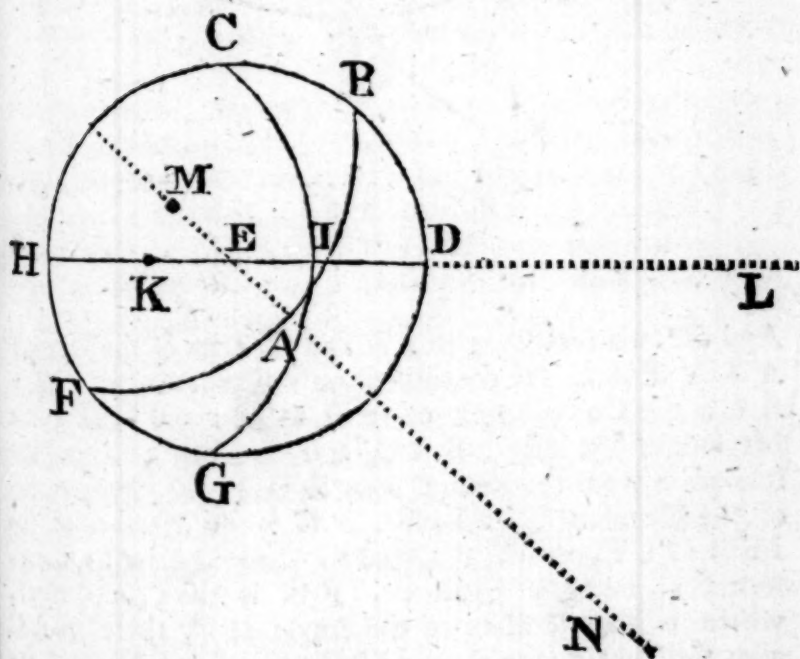
15. If a Spherical Triangle be both right angled and quadrantal, the Sides thereof are equal to the opposite Angles. As in the Triangle HMC .

Note, Mention is often made of Quadrantal Sides, that is, such or such a Side is a Quadrant: Now how they are known to be Quadrants, the Learner is to remember that in Plain Trigonometry we have shewn that the Sine of 90° , the Chord of 60° , and the Tangent of 45° are equal to each other, and also to the Radius of a Circle; therefore in the Scheme above, the Primitive Circle $NOABEG$ is drawn upon the Center L , with the Radius $LE = LG = LN = LO$, consequently all those Sides are Quadrants, as being equal to the Radius of the Primitive Circle; and so by the same Rule is the Side of an oblique Circle (in this Scheme) known to be more, equal, or less than a Quadrant.

16. Every Circle, whether right or oblique hath two Poles, if it is the Primitive they both lie together in the Center thereof; if it is a right Circle as HD , they lie in the Primitive Diametrically opposite at C and G . If

an

an oblique Circle, as C A G, one Pole lyeth within the Primitive, and the other without; which how to find Geometrically, see my *System of Astronomy*, Vol. I. page 62. But by the Sector it is performed thus: Set the Sector on the Tangents 45° , 45° , to the Radius of the Primitive Circle = E D; then let the Pole of the oblique Circle C A G be sought, take H I in your Compasses and measure it on the Tangents, you will find it to be 25° , this subtract from 45° = E D, there remains 20° ; which take from the Tangents on the Sector, and set from E to K, so is K the Pole of the oblique Circle C I G, and lyeth within the Primitive. For the other Pole add 25° that you just now found to 45° , and it make 70° , set the Radius of the Circle = E D to the lesser Tangents on 45° , and take off the Tangent of the Sum = 70° , and set it from E to L. So is L the other Pole of the said oblique Circle C I G, and falls without the Primitive Circle. Likewise the Poles of the oblique Circle B A F, will be found by the Rule, to fall at M, within the Primitive, and at N without the Primitive Circle. For the laying down



ments of AB is AH in the Triangle AGH , and the Complement BC is CI in the Triangle CID ; so that the two Legs AB and BC including the Right Angle at B , are never Complements, but the Sine itself, or the Tangent itself.

In the Solution of right angled Spheric Triangles, there are always two Things given, besides the Right Angle, to find out a fourth Term, in which the Right Angle is always set aside, in being any of the Circular Parts, and consequently doth not separate the two Legs AB and BC , but are always said to lye together. And of the Three Things or Parts concern'd in the Question, one of them is always called the Middle Part, and the other two are called Extreams either conjunct, when they lye together, or disjunct, when they are separated by any other Part.

Now in Order to know which is the Middle Part, observe, if the Things given and required lye together, the Middle of them Three is the Middle Part, as suppose AC and AB were given, and Angle A required; here, because the Things given and required lye together, Angle A being the middlemost of the Three is the middle Part, or if A had been given, and AC , or AB sought, the middle Part would have been the same because they lye together. But when the Three Things concern'd in the Question are disjoyned, that which stands by itself is the middle Part. As suppose AB and AC were given, to find BC . Here AB and BC lye together, (for the Right Angle at B makes no separation) and AC is separated by the Angle A and C which have nothing to do in the Question, therefore AC standing alone is the middle Part. The middle Part being thus known, the Solution of right angled Spheric Triangles are perform'd by one universal Proposition. *viz.*

The Sine of the middle Part with Radius is equal to the Tangents of the adjacent Extream; and with the Co-Sines of the opposite Extreams. That is,

As Radius, to the Tangent of one of the Extreams conjunct, so is the Tangent of the other Extream conjunct, to the Sine of the middle Part.

Also, as Radius, to Co-Sine of one of the opposite or disjunct Extreams, so is Co-Sine of the other Extream disjunct, to the Sine of the middle Part.

Y

When

242 . *Of Spheric Trigonometry.*

When the middle Part is sought, then Radius must be in the first Place of the Proportion; but if one of the Extream Parts be sought, then the other Extream must be in the First Place, and Radius in the Second or Third Place, it matters not which.

Illustration.

In this Synopsis of the foregoing Triangle A B C, I shall set down the Quantities of each Part thereof, with their Complements, together with their Sines, Tangents, &c. of each Part design'd as a Help to the young Student for the right Understanding of this most Useful Part of the Mathematicks.

Sides and Angles.		Sines.	Co-Sines.
Sides.	$\left\{ \begin{array}{l} A C = 66^{\circ} \ 31' \ 00'' \\ C B = 51 \ 32 \ 00 \\ A B = 50 \ 09 \ 52 \end{array} \right.$	9.9624527	9.6004090
		9.8937452	9.7938317
		9.8852967	9.8065773
Angl.	$\left\{ \begin{array}{l} C = 56 \ 50 \ 55 \\ A = 58 \ 36 \ 49 \end{array} \right.$	9.9228440	9.7378698
		9.9312925	9.7166767

	Tangents.	Co-Tangents.
A C	10.6320437	9.6379563
C B	10.0999135	9.9003459
A B	10.0787190	9.9212810
C	10.1849771	9.8150229
A	10.2146157	0.7853843

In the right angled Spherical Triangle A B C, I have shewn how A B and A C are Extreams conjunct, or adjacent

acent to the Angle at A, and that A C and Angle A and C are always Complements: Therefore by the Fundamental Axiom, or Catholick *Proposition*. The Co-Sine of the Angle C, added to Radius is equal to the Co-Tangent of A C, added to the Tangent of C B, which is briefly expressed thus, *viz.* $CS < C + R = CT. AC + T. CB$, in Numbers, thus,

$$\begin{array}{rcl} CS. < C & = 56^{\circ} 50' 55'' & = 9.7378698 \\ \text{Radius} & 90 & 0 00 = 10.0000000 \\ \hline \text{Sum} & = & - - - 19.7378698 \end{array}$$

$$\begin{array}{rcl} CT. AC & = 66 31 00 & = 9.6379563 \\ T. CB & = 51 32 00 & = 10.0999135 \\ \hline \text{Sum} & = & - - - 19.7378698 \end{array} \left. \vphantom{\begin{array}{rcl} CT. AC & = 66 31 00 & = 9.6379563 \\ T. CB & = 51 32 00 & = 10.0999135 \end{array}} \right\} \begin{array}{l} \text{The same} \\ \text{as above.} \end{array}$$

Secondly, The Angle A is the Middle Part and required, therefore from the above Equation, this Analogy will arise, *viz.*

$$As R : T. AB :: CT. AC : CS : A.$$

Or, As R : CT. AC :: T. AB : CS. A, the Middle Part and Thing required.

Again by the Second Part of the Fundamental Axiom: Let A C and C B be given, to find A B, here the Side standing alone is the Middle Part, 'tis an opposite Extream, and this Equation will arise, *viz.* $CS. AC + R = CS. AB + CS. CB$. Now because one of the Extreams disjunct, is the Middle Part, therefore the other Extream, *viz.* C B must come first in the Analogy, Radius in the Second Place, and the Middle Part in the Third Place; or Middle Part in the Second Place, and Radius in the Third, it matters not which, because 'tis all one if we add the Second Term to the Third, as if we add the third Term to the Second. Thus,

As CS CB : R :: CS : AC : CS CB.

Or, As CS CB : CS. AC :: R : CS A.B.

The Equation Illustrated.

$$\begin{array}{rcl} \text{CS. A C } 66^{\circ} 31' & = & 9.6004090 \\ \text{Radius } - & - & 10.0000000 \\ \hline \text{Sum } - & - & 19.6004090 \end{array}$$

$$\begin{array}{rcl} \text{CS. A B } 50^{\circ} 9' 52'' & = & 9.8065773 \\ \text{CS. C B } 51 32 00 & = & 9.7938317 \\ \hline \text{Sum } - & - & 19.6004090 \end{array}$$

The same with the other above.

Note, When a Complement in the Proportion falls upon a Complement in the circular Part, you must take the Sine itself, or the Tangent itself, because CS of CS. = S. and CT of CT = Tangent.

Illustration.

In the Triangle A B C, page 240. Let A B and Angle C be given, to find Angle A. This is an opposite or disjunct Extream, and C is the Middle Part, (because it stands alone) and because the other Extreams fall upon CS, the Angle A becomes a Sine for the Reason above. Therefore CS. C + R = SA + CS A.B.

Analogy.

$$\begin{array}{rcll} \text{As CS A B} & - & 50^{\circ} 09' 52'' & - & 9.8065773 \\ \text{To Radius} & - & 90 00 00 & - & 10.0000000 \\ \text{So CS. C} & - & 56 50 55 & - & 9.7378698 \\ \text{To S. A} & - & 58 36 49 & - & 9.9312925 \end{array}$$

That this Work may want nothing to make it plain to the meanest Capacity, I shall here put down the Equations

tions and Proportions of five adjacent, and five opposite Extrems.

1. The Five adjacent Extrems.

In the Triangle A B C, page 240.

Given, $\left\{ \begin{array}{c} A B \\ C \end{array} \right\}$ To find C B.

Equation, $S. C B + R = T. A B + C T. C.$

Proportion, As $R : C T. C :: T. A B : S. C B.$

2. Given, $\left\{ \begin{array}{c} C B \\ A \end{array} \right\}$ To find A B.

Equation, $S. A B + R = T. C B + C T. A.$

Proportion, As $R : T C B :: C T. C : S A B.$

3. Given, $\left\{ \begin{array}{c} A B \\ A \end{array} \right\}$ To find A C.

Equation, $C S. A + R = T A B + C T A C.$

Proportion, As $T A B : R :: C S A : C T. A C.$

4. Given, $\left\{ \begin{array}{c} A C \\ A \end{array} \right\}$ To find C.

Equation, $C S. A C + R = C T A + C T. C.$

Proportion, As $C T A : R :: C T. A C : C T. C.$

5. Given, $\left\{ \begin{array}{c} C B \\ C \end{array} \right\}$ To find A C.

Equation, $C S C + R = C T A C + T C B.$

Here the Reader must remember what has been taught before, *viz.* that the Radius being a Geometrical Mean proportional, between the Tangent of an Arch, and the Tangent Complement of the same Arch: Therefore in any Case where Tangent or C T comes in the First Term of the Proportion, you may change it into the Second Place and put Radius in the First Place. As for instance, in the Third adjacent Extream, the Proportion is there as T. A B to R: You may say, as Radius is to the C T A B, and so of any other where Tangents come in the Proportion.

Of the Five opposite Extrems.

1. Given, $\left\{ \begin{array}{c} C B \\ C \end{array} \right\}$ To find A.

Equation, $CSA + R = S.C. + CS. CB.$

Proportion, As $R : S.C. :: CS CB : CSA.$

2. Given, $\left\{ \begin{array}{c} A C \\ C B \end{array} \right\}$ To find AB.

Equation, $CS. CA + R = CS. CB + CS. AB.$

Proportion, As $CSCB : R :: CSAC : CS AB.$

3. Given, $\left\{ \begin{array}{c} A B \\ C \end{array} \right\}$ To find AC.

Equation, $S, AB + R = SAC + S.C.$

Proportion, As $S, C : AB :: R : AC.$

Here Co-Sines fall upon Co-Sines, therefore I take the Sines themselves.

4. Given, $\left\{ \begin{array}{c} A B \\ C B \end{array} \right\}$ To find AC.

Equation, $CS, AC + R = CS, AB + CS, CB.$

Proportion, As $R : CSAB :: CSCB : CS, AC.$

5. Given, $\left\{ \begin{array}{c} A C \\ C \end{array} \right\}$ To find AB.

Equation, $S, AB + R = SAC + S.C.$

Proportion, As $R : SAC :: SC : SAB.$

Let the Young Student as he goes over these ten Extrems, draw Triangles upon his Slate, or on waste Paper, and mark the Parts given and required, as expressed above, and this will greatly help him to the Knowledge thereof.

What we have spoke hitherto has been of Sines and Tangents only, but if the Reader pleases, he may use Secants also; and for the doing of this let him observe that in Fig. page 192, are these Proportions, viz.

1. As

1. As the Tangent is to the Radius, so is the Radius to the Co-Tangent.
2. As the Sine is to the Radius, so is Radius to the Co-Secant.
3. As, the Tangent of an Arch, is to the Tangent of an Arch; so is the Co-Tangent of the latter Arch, to the Co-Tangent of the former.
4. As the Sine of an Arch is to the Sine of an Arch, so is the Co-Secant of the latter Arch, to the Co-Secant of the former.

EXAMPLE.

In the first Case of the opposite Extream, let it be required, to work the same by Secants.

Given, $CB\ 51^{\circ}\ 32'$, $\angle C\ 56^{\circ}\ 50'\ 55''$, to find the Angle at A

First, You are to observe that for a Sine in the Proportion, you must always add a Co-Secant and Contra, and in this Case it will be,

As Co-Sec.	$\angle C\ 56^{\circ}\ 50'\ 55''$	-	10.0771560
To Radius	90 00 00	-	10.0000000
So CS CB	51 32 00	-	9.7938317
To CS $\angle A$	58 36 49	-	9.7166757

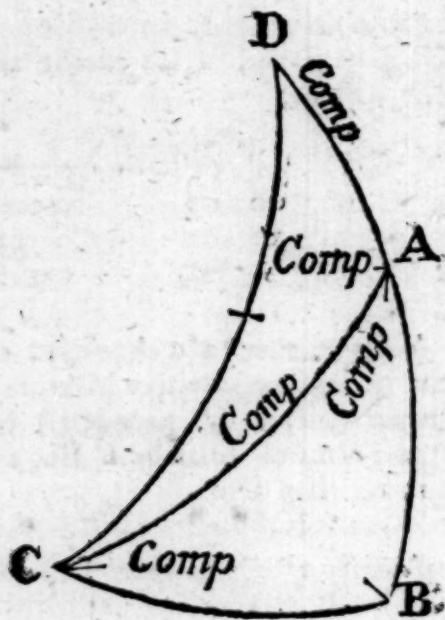
And in like Manner may any Analogy, consisting of Sines and Tangents, be perform'd with a Mixture of Secants, (*Martin's Trigonometry, Vol. II, page 103.*) by inverting and transposing the Terms of Analogy. But these Things are more for Curiosity than Use.

2. The Doctrine of Quadrantal Triangles.

What a Quadrantal Triangle is, has been already shewn, and here is to be observed that the three Parts that lye farthest from the Quadrantal Side are Noted with their Complements. As in the right angled Spheric Triangle ABC , the three Parts, viz. AC , Angle A and Angle C, which

which lye farthest from the right Angle at B are Noted with their Complements; so likewise in the Quadrantal Triangle A C D, the three Parts, *viz.* A C, A D and Angle C A D being most remote from their Quadrantal Side C D, are Noted with their Complements, for D A is the Complement of A B to a Quadrant, and the Angle C A D is Complement of the Angle C A B to a Semi-circle or 180 Degrees. And here the Universal Proposition holds true, *viz.* that the Sine of the Middle Part and Radius is equal to the Tangent of the adjacent Extream, and to the Co-Sine of the opposite: Also the Middle Part is found by it standing alone, as in the right Angle has been explained.

For the Learners further Improvement herein, I shall set down the Quadrantal Triangle with all it's Parts, as in the following Table.



Sides and Angles.				Sines.	Co-Sines.
CD	-	90°	00' 00"	10.0000000	
DB	-	90	00 00	10.0000000	
DA	-	38	28 00	9.7938317	9.8137452
AB	-	51	32 00	9.8137452	9.7938317
BC	-	50	9 52	9.8852967	9.8065773
AC	-	66	31 00	9.9624527	9.6004090
CAD	-	123	9 5	9.9228440	9.7378698
CDA	-	50	9 52	9.8852967	9.8065773
ACD	-	31	23 11	9.7166767	9.9312935
ACB	-	58	36 49	9.9312925	9.7166767
CAB	-	56	50 55	9.9228440	9.7378698
ABC	-	90	00 00	10.0000000	

1. In the Quadrantal Triangle ACD, let CDA be $50^{\circ} 9' 52''$, and ACD $31^{\circ} 23' 11''$, I demand the Side DA?

$$\text{Equation. } sD + R = ctDA + tc.$$

Analogy.

As t C	$31^{\circ} 23' 11''$	-	9.7853843
To Radius	90 00 00	-	10.0000000
So S. D	50 9 59	-	9.8852967
To ct DA	38 28 00	-	10.0999124

2. Another Example shall be of an opposite Extream, in the Quadrantal Triangle CDA.

Let the Angle ADC be $= 50^{\circ} 9' 52''$, and AD $= 38^{\circ} 28'$, I demand the Side CA?

$$\text{Equation, } cs.AC + R = csC + csD.$$

Analogy.

As Radius	$90^{\circ} 00' 00''$	-	10.0000000
To S. AD	38 28 00	-	9.7938317
So CS. D	50 9 52	-	9.8065773
To CS, CA	66 31 00	-	9.6004090

In

In this Triangle C D A, the Side C A is a Middle Part because it stands alone, being separated by the Angle D A C, therefore Radius comes in the first Place in the Analogy. And here the Reader may observe that all Questions in Quadrantal Triangles may be solved in the right angled Spheric Triangle A B C. As for instance, in the last Question where D and D A are given, to find A C, because C D and D B are Quadrants, the Side C B is the Measure of the Angle D, and D A is Complement of A B to a Quadrant, therefore in the Triangle A B C, we know A B and B C, and A C is required: From which this Equation doth arise, $\text{cs A C} + R = \text{CS A B} + \text{cs C B}$.

Analogy.

As Radius	-	90° 00' 00"	-	10.0000000
To C S A B		51 32 00	-	9.7938317
So C S B C		50 9 52	-	9.8065773
To C S A C		66 31 00	-	9.6004090

This gives us the same Answer you see as when it was solved in the Triangle C D A. All Spheric Triangles may be projected *Stereographically*, and the Sides and Angles truly laid down and measured; and in this Work there are but three Kinds of Circles, *viz.*

1. The Primitive which is always drawn with the Chord of 60°.

2. A Right Circle, and this always passes thro' the Center of the Primitive, cutting it at right Angles, and is the same with the Diameter of the Primitive Circle.

3. An Oblique Circle is that which neither lyeth in the Primitive Circle, nor in the Diameter, or Right Circle, but cuts them both at Oblique Angles, and the Points of it's Intersection with the Primitive are always opposite, so that a Line drawn from one Intersection to another, will pass through the Center of the Primitive Circle.

All Degrees are laid upon the Primitive Circle by the Line of Chord. And so any Quantity of the Primitive Circle is measured by the Chords also, as 38° 28' from Z to P in the following Scheme.

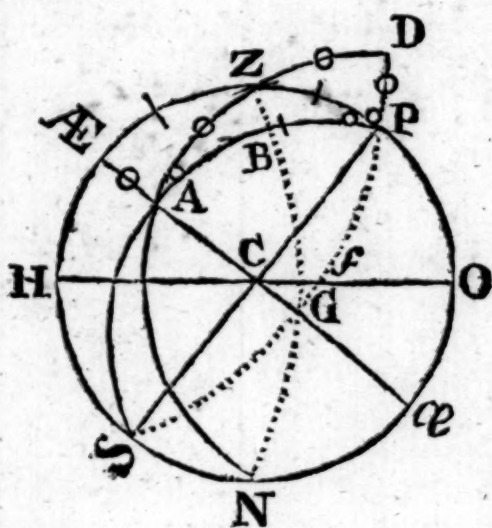
Any

Any Quantity of a right Circle, as HO , Æ , SP , &c. is measured by the half Tangents; and the Degree that an Oblique Circle makes with the Primitive, are laid down by the Secants.

And the several Parts of Oblique Circles, or their Angles are measured, by reducing of them to the Primitive Circles from their Poles.

III. Of Oblique Angles Spheric Triangles.

WRITERS on this Subject generally reckon twelve Cases, the ten first may be resolved by the Universal Proposition; if first the Oblique Triangle be reduced into two right angled Spherical Triangles, which may be done by letting fall a Perpendicular, which will sometimes fall within, and sometimes without the given Triangles, and to know when it will fall within, and when without, observe.



Let

Let the Perpendicular fall from the End of a given Side, being adjacent to a given Angle. Whether the Perpendicular fall within or without the Triangle, it must always be opposite to a known Angle. And the Perpendicular thus let fall, either divides the Oblique angled Triangle, $A Z P$ into two right angled Triangles, $A Z B$, $B Z P$, or produces two by adding of one to it $A D P$, and $Z D P$, by adding of one right angled Triangle $Z D P$, to the Oblique-angled Triangle $A Z P$ first given. If in the Oblique Triangle $A Z P$, there be given $A P$, $Z P$, and $A Z P$, to find $A Z$, here the Perpendicular $Z B$ cannot be let fall to be of any Service in this Case, therefore the Perpendicular must fall without the Triangle, as $A \bar{E}$, or $P D$; for in the Rect-angle Triangle $A \bar{E} Z$, $A Z$ may be found, or if the Side $A Z$ be continu'd to D , 'till $A Z D$ is a Quadrant, and let fall the Perpendicular $D P$, so that it may pass through the Poles of the two Oblique Circles at F and G , the Angles $Z D P$, and $A P D$ will be right; then in the Rect-angled Triangle $Z D P$, $Z D$ may be found, which is the Complement of $A Z$ to a Quadrant. And thus by forming of other Triangles adjacent to the given Oblique one, may the Things required be found: As for instance, in the Oblique angled Spherical Triangle, let there be given $A P = 90^\circ$, $Z P = 38^\circ 28'$, and the Angle $A Z P = 150^\circ$ to find $A Z$? Therefore in the right angled Spheric Triangle $Z D P$, it will be;

As t $Z P$	-	$38^\circ 28'$	-	10.0999135
To Radius	-	$90^\circ 00'$	-	10.0000000
So t $P Z D$	-	$30^\circ 00'$	-	9.9375306
To t , $Z D$	-	$34^\circ 32'$	-	9.8376171

Now $AD = 90 - ZD = 34^\circ 32' = AZ 55^\circ 28'$ as was required. Or it may be found in the Triangle $A \bar{E} Z$, for there is given $\bar{E} Z = 51^\circ 32'$, and the Angle $\bar{E} Z A = 30$, it being the Complement of the Angle $A Z P$, to a Semicircle, therefore I say,

As t , $\bar{E} Z$	=	$51^\circ 32'$	-	10.0999135
To Radius	=	$90^\circ 00'$	-	10.0000000
So t $\bar{E} Z A$	=	$30^\circ 00'$	-	9.9375306
To t $A Z$	=	$55^\circ 28'$	-	9.8376171

the same
as before. And for the Benefit of the young Student, I shall

Of Oblique Spherical Triangles. 253

shall here set down all the Parts of the Triangles AZP , and also of it's adjacent right ones $AÆZ$ and ZDP .

1. In the Triangle AZP .

$$\begin{aligned} AP &= 90^\circ 00 \\ ZP &= 38 \quad 28 \\ AZ &= 55 \quad 28 \\ AZP &= 150 \quad 00 \\ APZ &= 24 \quad 19 \quad ZPD = 65^\circ 41' \\ PAZ &= 18 \quad 07 = DP. \end{aligned}$$

2. In the Triangle $AÆZ$.

$$\begin{aligned} ÆZ &= 51^\circ 32' \\ AZ &= 55 \quad 28 \\ AÆ &= 24 \quad 19 \\ AÆZ &= 90 \quad 00 \\ AZÆ &= 30 \quad 00 \\ ÆAZ &= 71 \quad 53 \end{aligned}$$

In the Triangle ZDP .

$$\begin{aligned} ZP &= 38^\circ 28' \\ ZD &= 34 \quad 32 \\ PD &= 18 \quad 07 \\ ZDP &= 90 \quad 00 \\ ZPD &= 65 \quad 41 \\ PZD &= 30 \quad 00 \end{aligned}$$

There are four Axioms, by which all the Cases, commonly called 12, of Oblique Spherical Triangles are solved, which are these.

Axiom 1. As the Sine of a Side, is to the Sine of it's opposite Angles, so is the Sine of another Side, to the Sine of it's opposite Angle. Or, as the Sine of an Angle is to the Sine of it's opposite Side, so is the Sine of another Angle, to the Sine of it's opposite Side.

Axiom 2. As the Sine of half the Sum of two Sides including an Angle, is to the Sine of half the Difference, so is the Co-Tangent of half the contain'd Angle, to the Tangent of half the Difference of the other two Angles.

Z

2. As

2. As the Co-Sine of half the Sum of two Sides, including an Angle, is to the Co-Sine of half their Difference, so is the Co-Tangent of half the included Angle, to the Tangent of half the Sum of the other two Angles.

Axiom 3. First say, As the Sine of half the Sum of two Angles, is to the Sine of half the Difference, so is the Tangent of half their interjacent Sides, to the Tangent of half the Difference of the other two Sides. This reserve, and say again,

Secondly, As the Co-Sine of half the Sum of two Angles is to the Co-Sine of half their Difference, so is the Tangent of half the interjacent Side, to the Tangent of half the Sum of the other two Sides, which added to the first Arch just now found, gives the Answer. This shall be more fully explain'd in the next Chapter.

Axiom 4. As the Rect-angle of the Sines of the two containing Sides, is to the Square of Radius, so is the Rect-angle of the Sines of half the Sum of the three Sides, and of the Difference of the Sides opposite thereto, to the Square of the Co-Sine of half the contain'd Angle sought.

These four Axioms comprehend the whole-Business of Spherics; yet by Help of the Catholick Proposition in Rect-angular Triangles, and the *Homogeneal* in the Oblique, the four Axioms will be much shortned and made plain and easy to the meanest Capacity.

When the Perpendicular is let fall in, or without the Oblique Triangle, proceed to find what you are seeking, in one of the right angled Triangles, which being done, the next Thing is to proceed to a second Analogy, thereby to find the Answer to the Question, and this is easilicst done by the two *Homogeneal* Parts, or Things of the same Kind in each Triangle, by comparing them with the Vertical Angles, or with the Base, for they are proportional. As suppose in the Triangle AZP , the Perpendicular ZB reduces it into two right angled Triangles, ABZ , and PBZ , I say, that in the Sines and Tangents of the Circular Parts, it will be, as $AB : A :: BP : P$. for by the Catholick Proposition,

$$\text{In } \left\{ \begin{array}{l} A B Z \\ Z B P \end{array} \right\} \text{ As } \left\{ \begin{array}{l} t B Z : R :: S A B : ct A \\ t B Z : R :: S B P : ct P. \end{array} \right.$$

Therefore, As $S A B : ct A :: S B P : ct P$.

Thus you see that having found according to the Condition of the Question a Vertical Angle, or Base in the first Operation, you must next proceed to order your second Proportion aright among the *Homogeneous* Terms; as Base, or Hypothenuse, &c. compare the Perpendicular with each Homogeneous Term, whether it be given, or required in each Triangle, to distinguish between the Middle Part and the Extream, and to fit the artificial Sines and Tangents to them accordingly. Then reject the Perpendicular and Radius in each Comparison; the Middle Part and Extreams in one Triangle shall be proportional to the Middle Part and Extreams in the other.

EXAMPLE.

In the Triangle $A Z P$, let there be given the Angle $A P Z = 24^{\circ} 19'$, $A P = 90^{\circ}$, and $P Z = 38^{\circ} 28'$, to find the Angle $Z A P$?

Solution.

Let fall the Perpendicular $Z B$, so that it may pass through the Pole of the Oblique Circle $P B S$, at G , and then it will be at right Angles to $A P$, and reduce the Oblique Triangle $A Z P$ into two right angled Triangles $A B Z$, and $P B Z$; Now in the Triangle $P B Z$, we have known $B P Z$, and $Z P$, as above, to find $B P$, the Segment of the Base $A P$, and Angle $B Z P$, of the three Parts, $B P$, P and $Z P$; P is the Middle Part, because it is an adjacent Extream, therefore $ct P + R = t B P + ct Z P$.

$Z 2$

Analogy,

Analogy, For BP.

As $\angle Z P$	=	$38^{\circ} 28'$	-	10.0999135
To Radius	=	$90^{\circ} 00'$	-	10.0000000
So $\angle P$	=	$24^{\circ} 19'$	-	9.9596535
To $\angle B P$	=	$35^{\circ} 54'$	-	9.8597400
From A P	=	$90^{\circ} 00'$		
Remain A B	=	$54^{\circ} 06'$		

Secondly, For the second Analogy we are to consider where the *Homogeneous* Parts lye in each Rect-angle Triangle, therefore I make Use of the Perpendicular ZB in both Triangles, then if ZB, BP and P be the three Terms, it is an adjacent Extream, and BP is the Middle Part, and consequently is a Sine, and the Angle P is $\angle$.

Again in the Triangle ABZ, I make Use of the Perpendicular ZB, AB and Angle A, here the three Things made Use of lye together, and AB is the Middle Part, Angle A (which I am seeking) is an Extream conjunct, and is $\angle$. Therefore I say the Homogeneous Parts are P, PB, BA and $\angle A$, now according to the Catholic Proposition, because AB and BP are Middle Parts, they are Sines, and the Angles at A and P are Co-Tangents, and consequently are Things of a like Nature. These Things being known it will hold,

As S. B P.	-	$35^{\circ} 54'$	Co Ar.	0.2318265
To $\angle P$	-	$24^{\circ} 19'$	-	10.3449888
So S. A B	-	$54^{\circ} 06'$	-	9.9085073
To $\angle A$	-	$18^{\circ} 07'$	-	10.4853226

And by the same way of Reasoning may the first Ten Cases, (as they are called) be resolved, whether the Perpendicular be let fall within, or without the Triangle.

A View of the first Ten Cases. See Fig. page 251.

Case 1. Given $\angle A$, AP and ZP, to find $\angle Z$. This is solved by Axiom 1.

Case 2. Given, $\angle A$, $\angle P$, and ZP. to find AZ. This is solved by Axiom 1.

Case

Case 3. Given Angle A, A Z and Z P, to find Angle Z. The Perpendicular is let fall from the obtuse Angle A Z P, within the Triangle, and is solved by the Homogeneous Parts

Case 4. Given Angle A, A Z and Z P, to find A P. The Perpendicular is let fall from Angle Z within, as in Case 3.

Case 5. Given, Angle A, A Z and A P, to find Angle P. The Perpendicular falls as in the 3d and 4th Cases.

Case 6. Given, Angle A, A Z and A P, to find Z P. The Perpendicular falls as in Case 3, 4, and 5.

Case 7. Given, Angle A, Angle P, and A Z, to find A P. The Perpendicular falls as in Case 3, 4, 5, 6.

Case 8. Given, Angle A, Angle P, and A Z, to find Angle Z. The Perpendicular falls, as in the 3d Case, &c.

Case 9. Given, Angle A, Angle P, and Z P, to find A P. The Perpendicular falls without the oblique Δ A Z P, viz. by continuing the Side A Z to D a Quadrant, then from D let fall the Perpendicular D P, to pass thro' G the Pole of the oblique Circle S A R. Therefore the first Proportion will be,

As R : $t < Z P D :: c s Z P : c t < P Z D$. Subtr. from $180 = < A Z P$. Then say by Axiom 1.

As $S < A : S Z P :: S < Z : S A P$.

Case 10. Given, Angle A, A Z, and Angle A Z P, to find P. Continue the Side P Z to $\mathcal{A}E$ and 'tis then a Quadrant, draw $\mathcal{A}E A$, therefore in the right angled Spherical Triangle A $\mathcal{A}E$ Z, there are known, $\mathcal{A}E Z A$, it being the Complement of the Angle A Z P to 180 , and A Z, to find $\mathcal{A}E A$, which is the Measure of the Angle A P Z, because $\mathcal{A}E P$ and A P are Quadrants: The Angle $\mathcal{A}E A Z$ is also known, it being the Complement of the given Angle Z A P to a Quadrant, then for $\mathcal{A}E A = \text{Angle } \mathcal{A}E P A$, say by the first Axiom,

As Radius	-	$90^{\circ} 0'$	-	10.0000000
To S A Z	-	55 28	-	9.9158200
So S. $\mathcal{A}E Z A =$		30 00	-	9.6989700
To S. $\mathcal{A}E A = \mathcal{A}E P A$		24 19	-	9.6147900

Case 11. Given, The Three Sides, to find an Angle.

Here the Perpendicular ZB falls within the given Triangle AZP , from the obtuse Angle at Z upon the longest Side AP , the Base as has been before directed. Then, in order to a Solution of this Case,

First, Take the half Sum of the Base AP .

Secondly, Take the half Sum of the other two Sides, *viz.* AZ and ZP .

Thirdly, Take the half Difference of the two shortest Sides, *viz.* of AZ and ZP .

Then say, As the Tangent of half the Base or longest Side, *viz.* AP , is to the Tangent of half the Sum of the other two Sides, so is the Tangent of half the Difference of the two shortest Sides, *viz.* AZ and ZP , to the Tangent of half the Difference of the Segment of the Base, *i. e.* The Distance between the Place where the Perpendicular cuts it, and the True Middle of the Base, which added to the half Base, gives the greater Segment $= AB$; but subtracted from half the Side or Base AP , gives the lesser Segment $= BP$; then if you would find the Angle included between the Base and the greater Side, say, as Radius to the Tangent of the greater Segment of the Base, so is Co-Tangent of the greater of the two lesser Sides, to the Co-Sine of the Angle included between the Base and the greater of the two lesser Sides.

But if you would find the Angle at the End of the Base adjacent to the lesser of the two other Sides, then say, in the Second Analogy,

As Radius, to the Tangent of the lesser Segment, so is the Co-Tangent of the least Side, to the Co-Sine of the Angle included between the Base and the lesser Side.

EXAMPLE.

In the Triangle AZP , Let $AP = 90$, $AZ = 55^\circ 28'$, $ZP = 38^\circ 28'$, what are the Angles?

Operation.

Operation.

$$\text{Base } A P = 90^\circ$$

$$\text{Half} = 45$$

$$A Z = 55^\circ 28'$$

$$Z P = 38 28$$

$$A Z = 55^\circ 28'$$

$$Z P = 38 28$$

$$\text{Sum} = 93 56$$

$$\frac{1}{2} = 46 58$$

$$\text{Diff.} = 17 00$$

$$\frac{1}{2} = 8 30$$

Now say,

$$\text{As } t. \frac{1}{2} \text{ Sum Base } 45^\circ 00' \text{ Co. Arith. } 9.9999999$$

$$\text{To } t. \frac{1}{2} \text{ Sum Sides } 46 58 \quad - \quad 10.0298376$$

$$\text{So } t. \frac{1}{2} \text{ Diff. Sides } 8 30 \quad - \quad 9.1744988$$

$$\text{To } t. \frac{1}{2} \text{ Diff. Seg. } 9 6 \quad - \quad 9.2043363$$

$$\frac{1}{2} \text{ Base} = 45 00$$

$$\text{Sum, greater Seg. } 54 06 = A B.$$

$$\text{Diff.} = \text{leffer Seg. } 35 54 = B P.$$

Secondly, For the Angle Z A P.

$$\text{As Radius} = 90^\circ 00' \quad 10.0000000$$

$$\text{To } t. A B = 54 06 \quad 10.1403339$$

$$\text{So } C t. A Z = 55 28 \quad 9.8376755$$

$$\text{To } C S. < A = 18 06 \quad 9.9780094$$

Lastly, For the Angle A P Z.

$$\text{As Radius} = 90 00 \quad 10.0000000$$

$$\text{To } t. B P = 35 54 \quad 9.8596661$$

$$\text{So } C t. Z P = 38 28 \quad 10.0999135$$

$$\text{To } C S. < P = 24 20 \quad 9.9595796$$

Another way to work this Case.

Take half the Difference of the two Sides, containing the required Angles, and add to it half the Side opposite to the Angle required, also subtract it from the same, Noting the Sum and Difference : Then set down the two Sides

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Sides containing the required Angle, and to them the Co-Arith. of their Artificial Sines, and under those the Artificial Sines of the aforesaid Sum and Difference, the half Sum of these Logarithmic Sines, shall be the Artificial Sine of half the Angle required.

Or in the first Part of the Work, which is better, from the half Sum of the three Sides, subtract the two Sides including the Angle required, these Differences will be the same with the Differences gained by the first Part of the Rule above. With which Work as has been taught, and the true Answer will come out, as will appear in working the last Example overagain.

The Work stands thus,

In the Triangle A Z P. Given, the three Sides, to find the Angle Z A P?

Sides including $\angle Z A P$	{	A P =	90° 00'
		A Z =	55 28
Difference	=		34 32
Half	=		17 16
Z P = 38° 28' Half	=		19 14
Sum	- - - - -		36 30
Difference	- - - - -		1 58

Or, *Secondly*, Thus,

A P	90° 00'	
A Z	55 28	
Z P	38 28	
Sum =	183 56	
Half =	91 58	Again
A P =	90 00	A Z =
Differ. =	1 58	Differ. =
		36 30

S. A P

S. A P	=	90° 00'	Co Arith.	9.9999999
S. A Z	=	55 28	Co-Arith.	0.0841799
S. Differ.	=	36 30	-	9.7743876
S. Differ.	=	1 58	-	8.5355228
Sum	=		-	18.3940902
Half S	=	9 3½	-	9.1970451
Doubled	=	Angle Z A P	=	18° 7'.

Secondly, For the Angle APZ the Work stands thus,

A P	=	90° 00'	
A Z	=	55 28	
Z P	=	38 28	
Sum	=	183 56	
Half	=	91 58	Again 91° 58'
A P	=	90 00	Z P = 38 28
Diff.	=	1 58	Diff. = 53 30

S. A P	=	90° 00'	Co-Ar.	9.9999999
S. Z P	=	38 28	Co-Ar.	0.2061683
S. Differ.	=	53 30		9.9051787
S. Differ.	=	1 58		8.5355228
Sum	=	- - - - -	-	18.6468697
Half	=	S. 12 9½	-	9.3233448
Double	=	24 19	=	< APZ. as before.

Case 12. Given, the three Angles to find a Side.

This is just the Converse of the 11th Case, and may be resolved in the same Manner, for if either of the Angles adjacent to the Side required be turn'd into a Side, (that is only taking it's Complement to 180) the Work is the very same as in the 11th Case, as I shall make more clear by Example. Let the three Angles of the $\triangle$ AZP be as before, viz. A 18° 7', Z 150°, and P 24° 19', I demand the Side A Z ? First I shall turn the Angles adjacent

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jacent to the required Side, viz. A and Z into Sides, by taking their Complements to 180° .

Now see the Work.

Angle	{	A = $161^\circ 53'$	Angle Z = $30^\circ 00'$
		Z = $150^\circ 00'$	Angle A = $18^\circ 07'$
X	=	<u>$11^\circ 53'$</u>	X = $11^\circ 53'$
Half	=	$5^\circ 56'$	Half = $5^\circ 56\frac{1}{2}'$
Half Angle opposite to A Z	=	-	= $12^\circ 9\frac{1}{2}'$
			Sum = $18^\circ 06'$
			Diff. = $6^\circ 13'$

Or find the Requisites thus, as in Case 11.

Angles	{	Z = $30^\circ 00'$	
		P = $24^\circ 19'$	
		A = $10^\circ 07'$	
Sum	=	<u>$72^\circ 26'$</u>	
Half	=	$36^\circ 13'$	Again $36^\circ 13'$
Angle Z	=	<u>$30^\circ 00'$</u>	Angle A = $18^\circ 07'$
X	=	$6^\circ 13'$	X = $18^\circ 06'$

Here you see the Requisites are the same with the other Method above.

S. Angle Z	=	$30^\circ 00'$	Co-Ar.	0.3010299
S. Angle A	=	$18^\circ 07'$	Co-Ar.	0.5073054
S. X	=	$18^\circ 06'$	-	9.4923083
S. X	=	$6^\circ 13'$	-	9.0345825
Sum	=	-	-	<u>19.3352261</u>
Half Sum = S.		$27^\circ 43\frac{1}{2}'$		9.6676130
Doubled	=	$55^\circ 27'$	= A Z,	as was required.

Lastly, Let the Angles be as before, and the Side A P required?

See

See the Work.

Angle Z turn'd into a Side = $30^{\circ} 00'$

Angle P = - - - $24 19$

Angle A = - - - $18 7$

Sum - - - $72 26$

Half - - - $36 13$ Again $36^{\circ} 13'$

A = - - - $18 7$ Angle P $24 19$

X = - - - $18 6$ X = $11 54$

S. Angle P = $24^{\circ} 19'$ Co-Ar. 0.3853353

S. Angle A = $18 7$ Co-Ar. 0.5073054

S. X = $11 54$ - 9.3142975

S. X = $18 6$ - 9.4923083

Sum - - - 19.6992465

Half Sum = S. $45 00$ 9.8496232

Doubled = $90 00$ = A P as was required.

Note, If to the Co-Arith. of the Sines of the two Angles adjacent to the Side required, you add the Sine of half the Sum of all the Angles, and the Sine of the Difference between half Sum of the Angles, and Angle opposite to the Side required, half the Sum of these Logarithmic Sines will be the Co-Sine of half the Side required.

I shall conclude Spherics with the Solution of two extraordinary Cases.

Case 1. *Given, Two Sides with the Angles opposite them, to find the other Side.*

The Celebrated Mr Edmund Gunter gives this Rule.

As the Sine of half the Difference of the given Angles, to the Sine of half the Sum of those Angles, so the Tangent of half the Difference of the Sides, to the Tangent of half the Side required.

EXAMPLE.

EXAMPLE.

In the Oblique Spherical Triangle AZP, page 251, let there be given $ZP = 38^\circ 28'$, $AP = 90^\circ$, Angle $A = 18^\circ 7'$ Angle $Z = 150^\circ$, to find the Side AZ ?

Operation.

$$\begin{array}{rcl}
 \angle Z = 150^\circ 00' & - & 150^\circ 00'. \text{ AP } 90^\circ 00' \\
 \angle A = 18^\circ 07' & - & 18^\circ 07'. \text{ ZP } 38^\circ 28' \\
 \hline
 \text{Sum} = 168^\circ 07' & X & = 131^\circ 53' \\
 \text{Half} = 84^\circ 03\frac{1}{2}' & \text{Half} & = 65^\circ 56\frac{1}{2}' \quad \frac{1}{2} \quad 25^\circ 46'
 \end{array}$$

Now say,

$$\begin{array}{rcl}
 \text{As S. } \frac{1}{2} X < & - & 65^\circ 56' 30'' \text{ Co-Ar. } 0.0394669 \\
 \text{To S. } \frac{1}{2} \text{ their Sum} & 84^\circ 3' 30'' & - 9.9976606 \\
 \text{So T. } \frac{1}{2} X \text{ Sides} & 25^\circ 46' 00'' & - 9.6836783 \\
 \text{To T. } \frac{1}{2} \text{ Side required} & 27^\circ 44' 4'' & - 9.7208063 \\
 \text{Doubled} = \text{AZ} & = 55^\circ 28' 8'' & \text{as was required.}
 \end{array}$$

Case 2. *Given, Two Sides with the Angles opposite to them, to find the other Angle.*

RULE.

As the Sine of half the Difference of the given Sides, to the Sine of half the Sum of those Sides, so the Tangent of half the Difference of the given Angles, to the Co-Tangent of half the Angles required.

EXAMPLE.

In the Oblique Triangle AZP, let AP be 90° , ZP $38^\circ 28'$, the $\angle AZP = 150^\circ$ and that A $18^\circ 7'$, I demand the Angle APZ?

Operation.

Operation.

AP = 90° 00'	90° 00'	Z = 150° 00'
ZP = 38 28	38 28	A = 18 07
X = 51 32	Sum 128 28	X 131 53
Half = 25 46	Half 64 14	Half 65 56½

Now say,

As S. $\frac{1}{2}$ X Sides	-	25° 46'	Co-Ar. 0.3618031
To S. $\frac{1}{2}$ their Sum	-	64 14	- 9.9545184
So t. $\frac{1}{2}$ X	-	65 56½	- 10.3502280
To ct. of $\frac{1}{2}$ < P	-	12 9 40"	10.6665495
Doubled = < P	-	24 19 20	

Case 3. In the Triangle A Z P. Let A Z = 55° 28' Z P = 38° 28' and the Angle included, viz. A Z P = 150°, I demand the Side A P ?

The Perpendicular here is Æ A , by which means the Oblique angled Triangle A Z P is reduced into two right angled Spherical Triangles, viz. A Æ Z , and A Æ P , both right angled at Æ ; now in the Triangle are given, A Z = 55° 28', and the Angle $\text{Æ Z A} = 30$; it being the Complement of the Angle A Z P to a Semicircle, to find Æ Z ? By Transposition I say,

As Radius	-	90° 00'	-	10.0000000
To t. A Z	-	55 28	-	10.1623245
So ct. Æ Z A	-	30 00	-	9.9375306
To t. Æ Z	-	51 32	-	10.0998551
Add Z P =		38 28		

$$\text{Sum} = \text{Æ P} = 90 00$$

Now 'tis plain from the Projection that A P = Æ P , and consequently A P is found without any further Calculation to be a Quadrant also.

But Secondly, If in this Case, we suppose A P < P, and Z P to be given, to find A Z, the Perpendicular Z B will fall within the Triangle, then proceed to find the

A a

Segment

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Segment of the Base P B. and then the Side A Z will come out as has been taught above.

The Versed Sine of an Arch is the Segment of the Diameter cut by a right Line falling perpendicular upon it, from equal Parts in the Circumference, and number'd in the Diameter with 10, 20, 30, 40, &c. to 180. Therefore the Natural Sine of any Arch taken from Radius will leave the Arithmetical Complement, or Versed Sine of that Arch's Complement; thus. the Natural Sine of 40° 6427876 subtracted from Radius = 10.000000 the Remainder = 3572124 is the versed Sine of 50, also the Natural Sine of 60° = 8660254 taken from Radius 10.000000 leaves 1339746 the versed Sine of 30° , &c. And for Arches above 90 we need no Natural versed Sines, because the Natural Sine of any Arch excess above 90, added to the Radius, is equal to the versed Sine of the said Arch; thus the Natural Sine of 40° = 6427876 added to Radius 10 000000 = 1642787 = to the versed Sine of 130° .

Also the Natural Sine of		13° = 2249511
Added to Radius	=	90 = 10000000
Versed Sine of	-	103 = 12249511

Again, The Natural Sine of		$1^{\circ} 32'$ = 267585
Add Radius	=	90 00 = 1000000
Versed Sine of		91 32 = 10.267585

The Excellent Use of these versed Sines will appear by the following Examples, for in all doubtful Cases where the Thing sought is more than a Quadrant. here it is seen at one View. for if the Sum of the Logarithms exceed Radius, then the Side or Angle enquired after, is greater than a Quadrant, but if the Sum of the Logarithms be less than Radius, the Thing sought is also less than 90, or a Quadrant.

The versed Sines are of great Use to calculate the Distance of Places on the Earth, according to the Arch of a great Circle. by having given their Latitudes, or Longitudes. Or the Distance of two Stars may be exactly found, or the Latitude of a Place may be had by the Altitude

tude of the two Stars, or Sun, by knowing the Azimuth, or Difference of right Ascensions.

EXAMPLE.

In the Triangle AZP , page 251, there are known, $AZ = 55^\circ 28'$, $ZP = 38^\circ 28'$ and the included Angle $AZP = 150$ to find the Side AP ?

RULE.

As the Cube of Radius, is to the Rect-angle of the Sines of the comprehended Sides, so is the Square of the Sine of half the contain'd Angle, to half the Difference of the versed Sines of the third Side, and of the Arch of the Difference between the two including Sides.

Which is thus, set down the Logarithm Sines of the two given Sides, and of the Double of half the given Angle, the Sum of all three Logarithms shall be the Logarithm Sine of an Arch, then take out of the Tables the Natural Sine of this Arch, and double it, to which add the Natural versed Sine of the Difference of the two given Sides, this Sum is the Natural versed Sine of the Side sought.

Operation.

The Sides.	{	$AZ = 55^\circ 28'$	L. Sine	-	9.9158200
		$ZP = 38^\circ 28'$	L. Sine	-	9.7938317
$\frac{1}{2}$ given <		double L. Line 75°		-	19.9698876

Log. Sine of	28	34		9.6795393
--------------	----	----	--	-----------

Natural Sine of	28	34	Double =	.9563620
X given Sides	17	00	V. Sine =	.0436952
The Side AP	90	00	V. Sine =	10.000572

EXAMPLE II.

Given $AP = 90$, $ZP = 38^\circ 28'$, and the Angle $APZ = 24^\circ 19'$, required AZ ?

A a 2

Operation,

Operation.

The $\angle A Z = 90^\circ 00$ Log. S. 10.000000
 Sides. $\angle Z P = 38 \ 28$ Log. S. 9.7938317
 Half Given $< 12^\circ 9' \frac{1}{2}$ doub. Log. S. = 18.6469740

Logarithm Sine of 1 35 = 8.4408057

Natural Sine of 1 35 = Doubl. = 552618

X given Sides 51 32 = V. Sine = 3779407

The Side A Z = 55 28 = V. Sine = 4332025

EXAMPLE III.

Given, $A P = 90^\circ$; $A Z = 55^\circ 28$, and the Angle
 $Z A P = 18^\circ 6'$, required $Z P$?

Operation.

Sides $\angle A P = 90^\circ 00$ Log. S. = 10.000000
 $\angle Z A = 55 \ 28$ Log. S. = 9.9158200
 Half given $< 9 \ 3\frac{1}{2}$ doubled = 18.3934392

Log. Sine of - 1 10 = 8.3092592

Natural Sine of 1 10 double = 407216

X given Sides = 34 32 V. Sine = 1762035

The Side Z P = 38 28 V. Sine = 2162035

If you would find the Verfed Sine of an Arch or Angle,
 See page 202.

C H A P VIII.

Of ASTRONOMY.

DEFINITION.

ASTRONOMY is deriv'd from two *Greek* Words, *viz.* *Aster*, a Star, and *Nomos* a Law, or Rule. It is a Science which by infalliable Demonstration, teaches us the Motions, Distances, and Magnitudes of the Heavenly Bodies; their Revolutions, *Anomalies*, *Aplelions*, *Eccenericilies*, *Elongations* and *Parallaxes* of the Planets, Eclipses of the *Luminaries*, *Occultations* of the Primary Planets and fixed Stars by the Moon. As also the Rising, Culminating, Setting, and Amplitudes, and in short whatever belongs to the right Understanding of the true System of the World; (*See my Uranoscopia.*) Of which there are Eight several Sorts, (but only one true), of which I shall first give a short View.

1. *Plato* the Divine *Athenian* Philosopher, travelled into *Egypt* and there Learned *Astronomy*, about 420 Years before *Christ*, invented a System, in which he makes the Earth the Center of the World immoveable, surrounded by the Air; next above that he imagines the Element of Fire; then the Sphere of the Moon; next above the Moon, the Sun; then *Mercury*; then the Orb of *Venus*; next above her, *Mars*; then *Jupiter*, then *Saturn*, and above *Saturn*, the Fixed Stars.

2. *Anno Dom* 135, Flourished the Great *Ptolomy*; there is a System goes by his Name, which has the Earth placed in the Center of the Universe immoveable: This he supposes is surrounded by Air, and the Air to be surrounded by the Element of Fire, next above this the Sphere of the Moon, next above the Moon is the Orb of *Mercury*, then *Venus*,

then the Sun ; next above the Sun the Orb of *Mars* ; then *Jupiter*, and the highest of all the Sphere of *Saturn* ; and he here supposes all the Orbs to be solid, and the Fixed Stars surrounding them all.

3. *Porphyrus* a Follower of *Plato*, Flourished 375 Years after *Christ*, agreeing with *Plato* in all Things, only in this, he placed *Venus* next to the Sun, and *Mercury* between the Orbs of *Venus* and *Mars*, these three Systems are all composed of Concentric Circles.

4. The *Egyptian* System was embraced by *Vitruvius*, *Martianus*, *Capella*, *Macrobius*, *Beda*, and *Argol*, about the Year 1638 ; this System is composed of Eccentric Orbs, for they make the Earth fixed, and the Center of the *Moon*, *Sun*, *Mars*, *Jupiter*, and *Saturn*, and the Fixed Stars ; but the Sun the Center of *Venus*, and *Mercury* intersecting the Sun's Orb twice, if this System were true, 'tis possible they would meet the Sun in his Way, what would be the Consequence of such a Salutation is well known to Astronomers.

5. In the Year 1572, Flourished *Tycho Brabe*, a Dean and Lord of *Knudsborg*, in the Island *Schonen* ; He contrived a System as a Mean between the *Ptolomaick*, and *Copernican*, in which he supposes the Earth fixed in the Center of the Universe. Concentric to which is first the Sphere, or Orb of the *Moon*, next that of the Sun's Annual Motion ; then the Sphere of the Fixed Stars, the Sun being the Center of the other Five Planets. The Orb of *Mars* intersecting the Sun's Orb twice, and in opposition is nearer the Earth than the Sun itself, and *Saturn* and *Jupiter* in Opposition, are nearer the Earth than *Venus* is when in *Apogee* : The Orbs of *Venus* and *Mercury* are drawn as it were two Epicycles to the Sun's Orb, both of them intersecting it twice ; now according to this it is possible for the Sun in his Way to meet with *Mars*, (like two Coaches in a narrow Lane, or Street of the City, when there is not Room to pass) the Consequence of such a Jostle, I leave to be discuss'd by Astronomers.

6. *John Baptist Ricciolus*, of *Ferrara* in *Italy*, about the Year 1651, broach'd a New System, in which he makes the Earth the Center of the *Moon*, *Sun*, *Jupiter*, *Saturn*, and Fixed Stars, all moving round her and she at rest ; and the Sun moving round the Earth, he makes the Center of
Mercury,

Mercury, Venus, and Mars all moving round him, in which the Orb of *Mars* cuts the Sun's Orb twice, he has *Jupiter's* four Satellites, and the two innermost of *Saturn's*.

These two last Systems suppose the Heavens, or *Ætherial* Region, to be pervius, fluid, and of a thin, liquid, and transparant Substance, like the Air we breath in, but more pure, and not consisting of solid Orbs, as the *Peripateticks*, and those of the *Ptolomaick* Schools affirm.

7. *Rosinus* published a System at *Nuremberg*, in the Year 1661, a strange Piece of hotch-potch Stuff, made up of an unaccountable Quantity of Eccentric Circles, bearing no Testimony of Truth, and not fit to be once named. These Systems are all exploded.

8. *Lastly*, We come now to the most celebrated, and at this Day, most generally received, *Mundane's* System, named from it's Reviver, *Nicholas Copernicus* a Native of *Thorn* in *Polish Prussia*: He was born Anno 1473, died Anno 1543, only see the first Sheet of his Works Printed, as saith *Fontinelle*. This System was first invented by *Pythagoras* 509 Years before *Christ*, (see *Mercus Menclius*), the Reason we don't give *Copernicus* place according to his time, is, because we would speak of that last which will be most lasting, or durable. For in this System we see as in a Glass, the Beauty of the Universe display'd; here are neither solid Orbs nor Epicycles, but every Thing appears in a demonstrable Order.

First, We find the Sun the Center of the World unmoveable, having no Circular Motion, but a Central only about it's Axis in the Space of $25\frac{1}{4}$ Days, which was discovered by the Telescope. This System is made up and adorned with Seventeen Bodies, which we shall speak of in the following Order, and first of the Sun.

All Philosophers and Astronomers now agree, that the Sun is a formal fiery Body, consisting of a true proper Elementary Fire; partly liquid, partly solid. The Liquid being an Ocean of Light, and moving with fiery Billows, and flaming Ebullitions, as is manifest to those who look at it thro' a Telescope.

The Sun is fix'd upon the lower *Foci*, or *Umbelici*, at 50, at that End towards B, in the Ellipses, page 153, and with this Law, that the Areas described by Lines
drawn

drawn from the Earth, (which moves in the Curve AC, BD, round the Sun) to the Sun, shall always be equal, and in equal and proportionable Times. And from this Figure, 'tis also plain, that the Sun's Diameter will appear greater when he is in *Pergeon* which is a little after the Winter *Solstice*, viz. December 18, and is then $32' 44''$, and less when she is in her *Apogee* which is a little after the Summer *Solstice*, June 18, for then the Sun's apparent Diameter is but $31' 38''$ as is found by Observation; and this difference in his apparent Diameter is a manifest Proof that the Earth moves in an Ellipsis and not in a Circle.

The Length of the Tropical Year, or the time the Sun apparently runs thro' the 12 Signs of the *Zodiac* is $365 d. 5 b. 49' 7''$; and if to this we add the Annual Anticipation of the Equinoxes $50''$ turn'd into Time = $20' 17'' 27'''$, we shall have for the Length of the Syderial Year $365 d. 6 b. 9' 24'' 27'''$.

The Angle of the Inclination of the Planes of the Ecliptic and Equator, or the Sun's greatest Declination, (commonly called the Obliquity of the Ecliptic,) hath always been invariably the same, viz. $23^{\circ} 29'$. And the Sun's *Horizontal Parallax*, (that is what the Earth's Semidiameter appears to be to an Eye at the Sun) is no more than $10''$; and from hence is proved the vast Distance of the Sun from our Earth, for as all Objects are beheld under a certain Angle, (See page 183), therefore it must necessarily follow that the lesser the Angle that any Object is seen under, the greater must be the Distance of that Object from the Eye of the Observer; from hence it is plain that the Earth's Semidiameter appearing under an Angle of so small a Quantity as $10''$, the Distance of the Sun from our Earth must be 82183140 English Miles; and this we fetch from the Moon's *Horizontal Parallax* $57' 30''$, of Sir *Isaac Newton's* Theory gives the Moon's mean Distance from the Earth 238212 , then by a Reciprocal Proportion say,

<i>Miles.</i>	<i>Miles.</i>
If $57' 30'' : 238212 :: 10'' : 82183140$.	

And he is 258309 times bigger than our Earth; his Apogee this present Year 1739 is $\pm 8^{\circ} 24' 19''$. Eccentricity

1692

One million times?

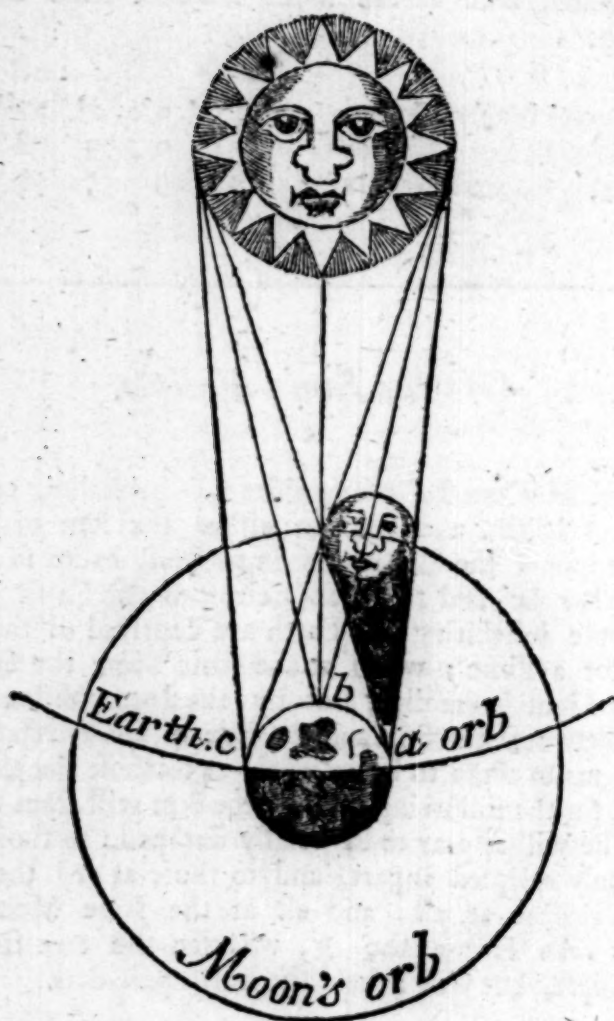
1692 Parts, such as the mean Distance from Earth is 1000000:

His ap-	Hourly Motion	0	0°	2'	27"	51'''
parent	Diurnal Motion	0	0	59	8	20
Mean,	Annual Motion	11	29	45	39	51

Of the Sun's Eclipse.

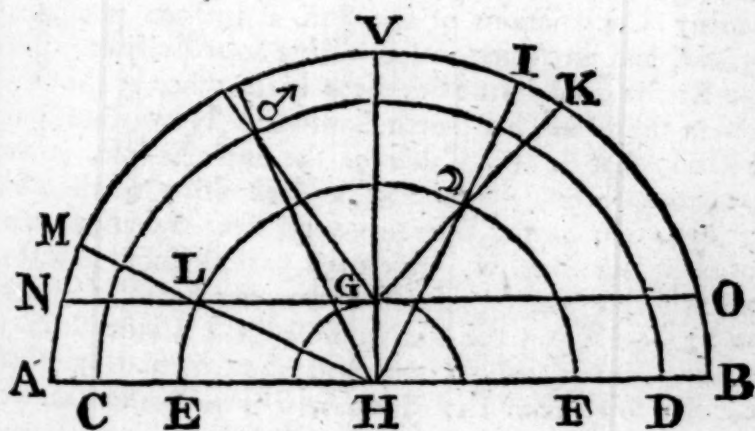
THE Word Eclipse signifies a Deprivation, or Want of Light, and that in either the Sun or Moon: The Eclipse of the Sun is very improperly called so; for it may rather be said to be an Eclipse of the Earth, or that the People inhabiting the Earth are deprived of the Sun's Light for a Time; when at the same Time the Sun loosing no Light in reality, but by the Interposition of the Moon between the Sun and the Eye of the Spectator, the Sun seems to them to be eclipsed, as to those People living at (a), (in the following Figure) the Sun will seem to them at (b) he will appear to be totally darkned, to those living to be only eclipsed in part, and to those at (c) there will be no Eclipse at all; and all at the same Moment of Time: An Eye at the D, will see the Sun free from any Eclipse, but will behold the Earth part dark.

This



This Diversity of *Aspect* is caused by the *Parallax* of the Moon. The Word *Parallax* signifies Variation or Diversity of *Aspect*, but in *Astronomy* it is the Angle that is made at the Moon by two Lines, one drawn from the Earth's Center, and the other from a Superficies, intersecting the Moon's Center, and being continued amongst the Fixed Stars, she will appear there considerably lower than she will to an Eye at the Earth's Center, and the nearer unto the Horizon that the Moon's is, the greater is her *Parallax* at that Time, and consequently her *Horizontal Parallax*

Parallax, is the greatest of all other. In this Scheme let N O be the visible *Horizon*, A B the True; let E D F represent the Moon's Orb, H the Earth's Center, G the Superficies: I say, an Eye at H will behold the Moon amongst the Fixed Stars at I; but to him that views her at G, the Earth's Surface, will see her at K; therefore the Angle $G \text{ } \text{D} \text{ } H = I \text{ } \text{D} \text{ } K$ is the Angle of *Parallax*: Of which there are several Sorts, but the *Parallaxes* in Longitude and Latitude are what are chiefly useful in the Sun's Eclipse, as now practised. The *Horizontal Parallax* is the greatest, being equal to the Angle G L H. The *Parallax* of *Mars*'s is = Angle H δ G, being less than the Moon's *Parallax* when they have equal Altitudes, because *Mars*'s distance from the Earth is greater than that of the Moon's; when a Planet is in the Vertex at V, there the *Parallaxes* vanish.

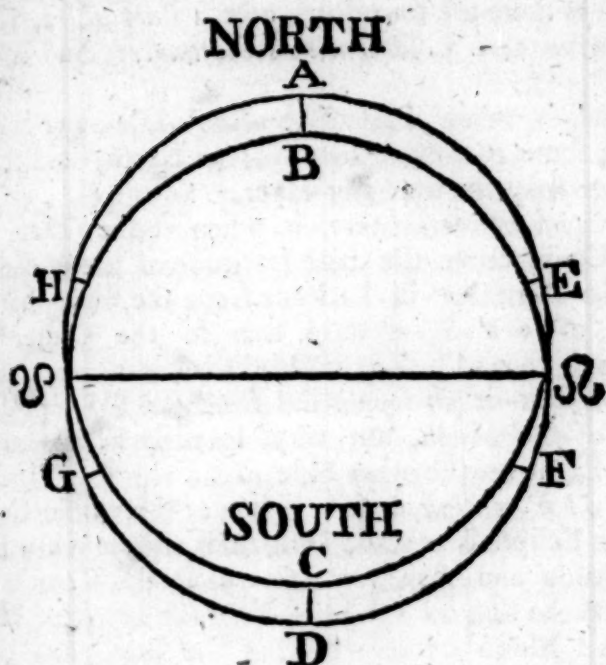


There are Four Sorts of *Parallaxes* in the Calculation of a Solar Eclipse, viz. the *Horizontal*, the *Parallax*, in Longitude, Latitude, and in Altitude; if the Luminaries at their Conjunction appear not in the Nonagesima Degree, but betwixt it and the *Eastern Horizon*, then the Time of the visible Conjunction will be accelerated, and will be before the Time of the true; and the Time of that Difference will always be proportional to the *Parallax* of Longitude at that Time: If the Luminaries be conjoin'd in the Nonagesima Degree, then the true and visible Conjunction will be Coincident: Lastly, if they be conjoin'd in

in the occidental Quadrant, that is between the Nonageffima Degree and the *Western Horizon*, then the visible Conjunction will be retarded proportionably to the *Parallax* of Longitude at that Time; when we speak of the Nonageffima Degree, you are not to understand it to be the Culminating Point, but that Degree of the Ecliptic which is 90° distant from the *Horizon*, except when *Cancer* and *Capricorn* are upon the *Meridian*, and then the Culminating Point, and Nonageffima Degree are Coincident.

Secondly, By the *Parallax* of Latitude is determined the Quantity of a Solar Eclipse; for to all those People that live in Northern Regions, if at any Conjunction of the Sun and Moon, her Southern Latitude exceeds the Sum of the Semidiameters of the Sun and Moon; I say by her *Parallax* in Latitude, she will then be depressed below the Sun's Limb, and consequently the Sun will be free from any Eclipse to us at all: And this is the Cause that so many Conjunctions of the Sun and Moon pass us in *England*, and we seeing so few visible Solar Eclipses. The same Reason hold in the Southern Parts, when at the Conjunction the Moon has North Latitude. Now since it is the Moon that is the Cause of the Sun's Eclipse, (as we have noted above), she being a dark Body of the same Species as our Earth, and also very near Spherical, as is proved by her Shadow, which is a perfect Cone, now this Conical Point reaching further than our Earth, therefore this Shadow upon the Earth, Disk is a Conic Section, viz. an Ellipsis, which we shall shew by and by when we come to speak of the *Penumbra*: The Reason that the Sun is not Eclipsed at every Conjunction or New Moon, is because the Moon moves not in the Plane of the Ecliptic $\oslash B \oslash D$, but in an Orbit $\oslash A \oslash C$ making an Angle therewith of $5^{\circ} 17' 20''$, and this Intersection of the Ecliptic and Moon's Orb, is always exactly in two oppo-

the Places, (see Diagram below,) or just six Sines assunder,



and these Intersections are called the Nodes of the Moon, these Nodes have a Motion of $3^{\circ} 11''$ every Day in *Antecedentia*, or contrary to the Succession of Signs, that is from *Aries* to *Pisces*, &c. and this is the Cause, why the Eclipses do not always happen in the same Place of the Zodiack, for the same Eclipses that happen now, will again happen 19 Years hence, but differ both in Time, Quantity, and Place, but the Period of Eclipses is something less than 19 Years, viz. it is only 18 y. 11 d. 7 h. $43' 15''$, in which Time the Nodes move 11 S. $18^{\circ} 43' 38''$, as above. This we shall demonstrate more plain from the *Diagram* above, for if at the Time of the true Conjunction of the Sun and Moon, they happen to be within $18^{\circ} : 10' : 41''$ of either Node, either before or after them, that is between F E, or G H, the Sun will then be Eclipsed, otherwise not, and this I call the Ecliptic Boundaries; otherwise at the visible Conjunction, the Moon's visible Latitude must be less than the Sum of Semidiameters of the Luminaries, or the Sun cannot be Eclipsed at that Time; these are the Laws which God Almighty gave to the

B b

Heavenly

Heavenly Bodies at the Creation, which Laws are constant, uniform, and regular without variation.

Of Eclipses there are four Sorts, viz. 1. *Partial*. 2. *Total without continuance*. 3. *Total with continuance*. And lastly, *Annular*.

1. *Partial*, is when Part of the Sun's Diameter is obscured from some particular Tract of the Earth, and these happen more frequent than any other.

2. *Total without continuance*, is when at the Time of the visible Conjunction the true Latitude of the Moon is equal to her *Parallax* in Latitude from the Sun, that so the Center of one is exactly seen in the Center of the other, and then also their visible Diameters are equal, that so the Sun is no sooner hid from our Sight by the dark Body of the Moon, but very speedily he is seen to recover his Light on the other Side.

3. *Total with continuance*, is when at the visible Conjunction the Eclipse is central, (which is always when the North Latitude and *Parallax* are equal) the Moon is in Perigeon, and the Sun in Apogee, then the apparent Diameter of the Moon exceeds that of the Sun, and this Excess can never amount to one Minute, so that the Total Darkness of any Solar Eclipse can never exceed 4' or 5' in Time, an Instance of this Nature we had in the Year of our Lord 1715, in which the Total Darkness at London was no more than 3' 20", the like having not happened at London for 575 Years before, which was in the fifth Year of the Reign of K. Stephen, Anno 1140, on Wednesday, March 20, it began at London, at 38' 37" past Noon, and continued 3 h. 1' 7", it's Total Darkness I make to be 3' 36" and Digits 12° 5'.

4. *Lastly, Annular*, is when the visible Central Conjunction, happeneth the Sun being in Perigeon, and the Moon in Apogee, here the Diameter of the Sun exceeds that of the Moon, and consequently there will then be a Ring of Light round the Moon. An instance of this Nature we read was observed at Rome, by Christopher Clavius, April 9, at Noon, Anno 1567.

That the Sun's Eclipse always begins on the West Side, and goes off, or ends on the East Side of his Body, this is a most manifest Truth, because the Moon (who is always the Cause of his Obscurity) moving in her Annual Motion always

always in *Consequentia*, or according to the Order of the Signs must of necessity first touch the Sun's Western Limb, and last leave his Eastern. Astronomers have divided the Sun's Diameter into 12 equal Parts, which they call Digits, so that if we speak of a Digits or Finger's Breadth, it is no more than $\frac{1}{12}$ Part of the Sun's Diameter. The Eclipse of the Sun does not agree to any particular Place on the Earth, but only to that Latitude and Longitude, for which it is calculated by Reason of the sudden Change of the Moon's *Parallax* above-mentioned. (See the Scheme page 275.)

*To calculate the Total Shadow of the Moon
on the Earth in the Sun's Eclipse.*

IN this Scheme let S be the Sun, B the Moon, and G the Earth, in which we are to find DEF, that is the Space that the Moon's Shadow falls upon in a Total Eclipse of the Sun.

B b z

h



In the Triangle HGF , we have given the Side GF , the Earth's Semidiameter, GH the Distance of the Earth from the Top of the Moon's Shadow, (which how to find will be shewn towards the End of this Chapter) = 1.019, and the Angle GHF , equal to the Sun's Semidiameter, to find the Angle $G FH$.

Supposing the Sun in Perigeon.

See the Work.

As $GF = 1$ Semidiameter	-	-	0.0000000
To $S. GHF$, Sun's Semidiameter	16° 22'	7.6774984	
So GH	-	-	1.019 0.0081742
To $S. GFH$	-	16 40	7.6856726
Now			

Now the Angle $E G F = G H F + G F H = 33^{\circ} 2''$ by Theorem 3, and such is the Arch $E F$, whose double $= D F 1^{\circ} 6' 4'' \times 69.5 = 74$ Miles, and so far on the Earth will the Sun's Eclipse appear Central and Total, when the Moon is in Perigeon.

1. To Exercise the young Tyro in these Matters, I shall here subjoin the Times of the Sun's Eclipse, *July* 14, 1748, from my System, by the New Equation, in my *Uranoscopia*, = Time true δ 13 d. 23 h. $28^{\circ} 25''$. $\odot$ and $\odot$ in Ω $2^{\circ} 42' 34''$.

Hence the Ap- parent Time at <i>London</i> , of the	Beg. 1748, <i>July</i> 13 d. 21 h. $4^{\circ} 55''$	} P M.
	Visible δ - 22 39 58	
	Greatest Obscuration 22 40 49	
	End - - 14 0 19 1	
	Total Duration - 3 14 6	
	Digits Eclipsed - 10 26 13	

on the Upper Side of the Sun.

The Use that may be made of Eclipse, is very great for determining the true Longitude of Places both at Sea and Land, for having an Eclipse truly calculated to any particular Place, as suppose *London*, having this Calculation with you at Sea, &c. at the Beginning of the Eclipse you must carefully observe the Time, as we will suppose it to be at 20 h. $29^{\circ} 47''$ PM, at *London* by our Calculation it begins at 21 h. $4^{\circ} 55''$, the Difference is $35^{\circ} 8''$ which turn'd into Time = $8^{\circ} 47' 00''$, by which I am assured that I am then so far to the *East* of the *Meridian* of *London*.

The Calculation of this General Eclipse stands thus, and in this we have nothing to do with *Parallaxes* at all, we have here given the Times and Places of the Passage of the *Penumbra* over the Earth's Disk, which are gain'd by Trigonometry, (See my *Uranoscopia*) only, in finding of which we are to observe, when we have gained the Time of the true Conjunction from Astronomical Tables: If then, I say, the Moon's true Latitude exceed the Sum of the Semidiameters of the *Penumbra* and Earth's Disk, the Sun at that Time will be Eclipsed no where upon Earth, but if less, then it will; the Semidiameter of the *Penumbra* is ever equal to the Sum of the Semidiameters of the Luminaries, and is nothing else but that faint Shadow which encompasseth the perfect Shadow of the Moon.

and is the *Penumbra*, so that we at *London* at the Time of this Eclipse are not immers'd into the perfect Shadow, but in the *Penumbra*: Now this diminish'd Light which encompasses the Shadow every Way is call'd the *Penumbra*. The Eclipse is seen to begin in the Supream Point of his vertical Diameter as follows.

The Times of the General Eclipse at London.

Apparent Time at London of the		D. h. ' "		
		Beg. at ☉ Rising, 1748, July 13	20	26 29
		Central Eclipse begins ☉ Rising	21	40 34
		In the Meridian -	23	3 9
		Nonageffimal Sun. -	23	19 49
		Middle - - -	23	25 7
		Central Eclipse ends at ☉ setting	1	9 40
		Ends at ☉ setting -	2	23 45
		Duration - - -	5	57 16

The Places where this General Eclipse happens is thus.

	Lat.		Long.	
Sun's begin Eclipse at Rising	35°	9' N.	51°	10' W.
Rises Centrally Eclipsed	45	23	76	17 W.
Centrally Eclipsed in Meridian	51	38	14	13 E.
Centrally Eclipsed in Nonageff.	48	47	20	8 E.
Sets Centrally Eclipsed	10	30	76	22 E.
Ends at ☉ Setting -	0	14 S.	53	59 E.

Sun's Lower Limb touched by ☽ Upper Limb beyond the Pole.

Sun's Upper Limb touched by ☽ Lower Limb 21° 19' N. 14° 13' E.

II. Of Mercury,

NEXT to the Sun in our System we find nimble hecl'd *Mercury*, he is called an inferiour Planet, because his Orb is circumscribed by the Earth's Orb, and by Reason

Reason of his Vicinity to the Sun, he is seldom seen with the naked Eye. For at his greatest Distance from the Sun when in Aphelion is no more than $28^{\circ} 21' 8''$; (See my Uranoscopia, page 65,) and when the Earth is in Aphelion and Mercury is in Perihelion, his greatest Elongation from the Sun, (or Angle at the Earth) is no more than $17^{\circ} 35' 42''$. His Eccentricity is 7964, such Parts as the Mean Distance of the Earth from the Sun is 10.00000, and his Mean Distance from the Sun 38262; his greatest Distance from the Sun in *English* Miles are 80353439. his least 52914839, his greatest Distance from the Earth is 97171952 and his least 37988985 of our Miles. He makes one Revolution round the Sun in 87d. 23h. $15' 53''$ his Heliocentric Diurnal Motion is $4^{\circ} 5' 32''$, and hourly Motion $10' 13'' 52''$, his Aphelion this present Year 1739, is $4^{\circ} 13' 17' 37''$, the Place of his Ascending Node $15^{\circ} 19' 50''$, and the descending Node, or $15^{\circ} 19' 50''$, so that if his Geometric Latitude be less than the Sun's apparent Semidiameter, (which at a Mean Distance from the Sun is $= 16' 5''$,) he may be seen with a good Telescope to make a black Spot on the Sun's Disk; the Times when he may be seen thus, I have calculated, as here you see.

1707 April	24d.	12 h.	0'	0''	
1710 October	25	12	0	0	
1720 April	26	21	25	19	very near but did not
1723 October	29	6	24	17	[touch.
1730 October	22	5	42	25	very near the Sun.
1736 October	31	0	4	8	
1740 April	21	12	54	5	
1743 October	25	0	14	8	
1753 April	24	20	37	23	
1756 October	26	18	0	13	
1769 October	19	11	41	53	
1776 October	22	11	41	1	
1782 November	1	5	20	34	
1786 April	22	20	46	51	
1789 October	25	5	32	37	
1799 April	26	4	34	15	
The Inclination of his Orbit $= 6^{\circ} 59' 20''$.					

When

When *Mercury* is in Conjunction with the Sun in the Upper Part of his Orbit, and near either Node as above-mentioned, he will pass behind the Sun, and in this Case is always direct in Motion. Every thirteen Years he is nearly in the same Place of the Heavens.

III. Of *Venus*.

NEXT above *Mercury*, is the glittering Planet *Venus*, who performs her Revolution round the Sun in the Space of 224 d. 16 h. 49' 24".

Her Eccentricity Parts	-	-	-	-	505
Mean Distance from the Sun ditto	-	-	-	-	72337
Aphelion this Year 1739	-	m	7°	8' 12"	
Inclination of her Orbit	-	-	3	23 20	
Heliocentric Diurnal Motion	-	-	1	36 8	
Hourly Motion	-	-	-	4 00 ¹	
Place of her North Node	II	-	14	18 2	

Her greatest Elongation from the Sun, is when the Earth is in Perihelion, and *Venus* in Aphelion, for then the greatest Side of the Triangle subtends the Angle at the Earth, and this Angle which is her Elongation, when at greatest is 47° 38' 35" (See my *Uranoscopia*, page 66,) but if she be in Perihelion her greatest Elongation can be no more than 44° 56' 14", and when she is in either of these Positions she is Direct. And every two Years, she is Stationary, and Retrograde as at the Earth, and when Retrograde, is always Occidental of the Sun. She strays further from the Ecliptic than any other Planet, for sometimes she will have 9° of Latitude to us at the Earth.

Her greatest Distance from the Earth is = 134017363, and least Distance 15165311 *English* Miles: She turns once round upon her Axis in 23 Hours, as has been discovered by the Telescope. She is supposed to have an Atmosphere, which reflects so strong and glaring a Light, that her Body is rarely seen clear and distinct, without applying

applying a proper Aperture to the Object Glass of your Telescope, and then you may see her increase and decrease in Light, Horn'd, and Gibbous as the Moon and Mercury are. This Planet also, as well as Mercury, when in Conjunction with the Sun in the Lower Part of her Orbit, and near one of her Nodes, (may be seen, by the Help of a Telescope) to make a black Spot on the Sun's Disk, like a Patch on a Lady's Face, as I have found by Calculation, and shall here set down.

1639 Novemb.	24 d.	3h.	19'	0"	observed by Horrox.
1761 May	25	22	44	53	} Retrograde.
1769 May	23	16	10	20	

When this Planet is conjoined with the Sun in the upper Part of her Orbit, she is always direct in Motion, and if her Geometric Latitude be then less than the Sun's apparent Semi-diameter, she will pass behind the Sun. This Planet, (as well as Mercury) can never be seen at Midnight, but always either in the Morning or Evening; and when she is seen in the Morning, she is called the Morning Star, and when in the Evening, the Evening Star.

Since these two inferior Planets, Venus and Mercury never recede farther from the Sun than has been above-mentioned; Astronomers well know from hence that the Earth's Orb, circumscribes their Orbits, and that they turn about the Sun, sometimes direct and sometimes Retrograde, shewing various Faces, and are never in opposition, no not so much as a Quadrant, or Sextile Aspect, which they would be if the Earth were at Rest in the Center of the Universe.

IV. Of the Earth.

THE Earth is one of the Seven Planets, and is plac'd in an Orb between Venus and Mars, she makes one entire Revolution round the Sun in 365 Days, 5 Hours, 4 Minutes, 7 Seconds, (see page 272) this is called the Tropical

294 Tropical Year, but the Siderial Year is longer by $20' 17'' 27'''$, being what is increased by the Proceſſion of the Equinox; beſides this Annual Motion, which is always direct, and according to the Order of the Signs, ſhe has a Second Motion upon her own Axis from Weſt to Eaſt, which in reſpect to the Fixed Stars, is performed in 23 h. 56'. The Earth's Axis is inclin'd to that of the Ecliptic, in an Angle of $66^{\circ} 31'$, and it's Figure is that of an Oblate Spheroid, ſwelling out towards the Equatorial, and flattened and contracted towards the Poles: So that the Diameter of it at the Equator, is longer than the Axis by about 34 Miles, as Sir *Iſaac Newton* has demonſtrated; for the Polar Diameter, or Axis, is to the Equatorial one, as 689 to 692. Experiments alſo made on Pendulums, which require different Lengths to ſwing Seconds, here and at the Equator do prove the ſame Thing. Our Countryman Mr *Rich. Norwood* found, by meaſuring from the Tower of *London* to the Middle of the City of *York*, in the Year 1635, that the Degree of the Arch of a great Circle upon the Earth's Surface, contain'd $69\frac{1}{2}$ Miles; which Experiment agrees very well with what the *French* Aſtronomers have found it to be. According to which Meaſure, we find the Earth's Circumference 25035.84, it's Diameter 7969.16, and the Heighth of the Atmosphere 47.12 *Engliſh* Miles. Eccentricity 1692, Horizontal Parallax $16' 51''$ diſtance from the Sun 82183140 as was ſaid in the Sun's Theory. For the Moon undoubtedly revolves about our Earth, and the Diſtance of the Sun is to that of the Moon at a mean Rate, as 20625, is to 60 in Earth's Semidiameters, and the Moon's Periodical Time, 27 d. 7 h. 43' 71'' the Periodical Time of the Sun, (if he moved) will be thus found, viz.

As the Cube of the Moon's Diſtance from the Earth, in Earth's Semidiameters = 216000, to the Cube of the Sun's diſtance 8773681640625, ſo is the Square of the Moon's Periodical Time 27, (omitting the Hours) = 729, to the Square of the Moon's Periodical Time, 29611175537 whole Square Root = 172079 Days = 471 Years, and ſo long would the Sun be in performing one Revolution, if he moved, and the Earth fixed in the Center of the World, as *Ptolomy*, *Tycho*, and their Followers vainly imagine. By which the Controverſy between our true Syſtem, and all

95 million miles

all the other false ones is absolutely determined in favour of the *Copernican*, and the Earth's Annual and Diurnal Motions for ever unquestionably established.

That the different Heats of our Seasons of the Year do not depend upon the nearness of the Sun to the Earth, but upon the Sun's Rays falling more direct, or more oblique upon us; for in the Distance of 82 Million of Miles, a little approach of the Earth to, or it's Recess from the Sun, will make no sensible Alteration as to Heat or Cold.

The Sun's Meridian Altitude at London,

$$\text{On. } \left\{ \begin{array}{l} \text{June} \quad 10 \\ \text{March} \quad 10 \\ \text{September} \quad 12 \\ \text{December} \quad 10 \end{array} \right\} \text{ is } = \left\{ \begin{array}{l} 61^{\circ} \quad 57' \\ 38 \quad 28 \\ 38 \quad 28 \\ 14 \quad 59 \end{array} \right.$$

From hence it is plain, that the Sun's Heat in Winter must be weakest, because then the Angle is more acute than it is in *June*, and the Rays falling most oblique on us, the same Quantity of Rays are scatter'd over a greater Space of Earth, and consequently it must be colder there than where the same Quantity of solar Rays possess a less Space or Earth; besides the Sun's Rays passing through a greater Part of the Atmosphere in the Winter than in the Summer, they must be more weak and faint in the first, than in the latter Case.

Of the Equation of Time.

THE Daily Revolutions of the Earth's Equator round it's Axis are exactly equal in Times to one another, and yet the Time from the Apparent Noon of one Day to that of the next, is unequal, and sometimes greater, and sometimes lesser.

And

And there is a double Cause of this Inequality ; First That the Plane of the Ecliptic doth not lye in the Plane of the Equinoctial, but makes an Angle with it of $23^{\circ} 29'$ the Difference between the Sun's Place in the Ecliptic, and his right Ascension in the Equinoctial turn'd into Time, by allowing 15° to an Hour, and 1° to $4'$ in Time, and so proportionably for a lesser or greater Quantity, which in the first and third Quadrants of the Ecliptic is to be added, but in the second and fourth is to be subtracted.

The second Cause is the Earth's annual Orbit, not being a Circle, but an Ellipsis, therefore with the Earth's Mean Anomaly, the Ecliptic Equation is taken out of it's proper Table in my System, and reduced into Time as before, by allowing 15° to an Hour, &c. gives you the Second Part of the Equation of Time, which if the Mean Anomaly be less then six Signs it addeth, if more subtracteth, to or from the equal Time gives the Apparent ; now if both these Parts add, or both subtract, their Sum, otherwise their Difference, is the absolute Equation of Time. Which applied to the equal Time, according to the Title of the greatest Part, gives the Apparent Time ; but to reduce the Apparent Time to the equal, use the contrary Titles.

EXAMPLE.

Anno 1739, June 5, at Noon equal Time, I demand the true Equation of Time ?

Operation.

Sun's Place at Noon is $11^{\circ} 24' 44'' 52'''$ and the Mean Anomaly from my Tables is $15^{\circ} 56' 21''$.

Sun's Place $11^{\circ} 24' 44'' 52'''$ gives	-	$1^{\circ} 54'' +$
Anomaly $15^{\circ} 56' 21''$ gives	-	$1^{\circ} 51' -$
Diff. is the true Equation of Time	=	$0^{\circ} 3' +$

So

So that this Day at Noon if your Clock or Watch be true it will be 3" slower than the Sun ; and the same is to be observed for any Day in the Year by the following Table, which (as a Mean) is calculated for Second past Leap Year, and may serve (for common Use) this Age, without any sensible Error. Here are four Days in the Year in which the Sun and Clock are together, viz. April 4, June 5, August 20, and December 13 ; there are four Days in the Year, in which the Sun and Pendulum Clocks differ most, which are these, with their Difference.

January	31,	Equation	14' 49"	Clock too fast.
May	4,		4 5	Clock too slow.
July	15,	-	5 57	Clock too fast.
October	22,	-	16 13	Clock too slow.

See the following Table.

*A Table of Equation of Time, for regulating of
Pendulum Clocks with the Sun.*

Deg.	Janua.	Febr.	March.	April.	May.	June.
18	58 14	48 10	80 50	4 20	50	Slow 37
29	21 14	47 9	51 0	34 4	40	Slow 25
39	43 14	45 9	34 0	18 4	50	Slow 12
410	6 14	42 9	17 0	+ 2 4	50	— 1
510	26 14	38 9	00	13 4	40	— 11
610	45 14	33 8	42 0	28 4	30	Clocks too 24
711	4 14	23 8	24 0	42 4	20	Clocks too 37
811	22 14	22 8	5 0	56 4	00	Clocks too 50
911	40 14	16 7	47 1	10 3	58 0	Clocks too 3
1011	57 14	9 7	29 1	23 3	56 1	fast, subtract 16
1112	14 14	4 7	11 1	36 3	53 1	fast, subtract 29
1212	30 13	52 5	52 1	48 3	49 1	fast, subtract 42
1312	44 13	43 5	33 4	0 3	44 1	fast, subtract 55
1412	58 13	34 5	15 2	11 3	38 1	fast, subtract 7
1513	11 13	24 5	56 2	21 3	32 2	from the 20
1613	23 13	13 5	37 2	31 3	25 2	from the 32
1713	35 13	2 5	19 2	41 3	18 2	from the 44
1813	46 12	50 5	12	51 3	11 2	from the 56
1913	55 12	37 4	42 3	0 3	4 2	from the 8
2014	05 12	28 4	23 3	8 2	56 3	equal 19
2114	13 12	12 4	4 3	15 2	47 3	equal 30
2214	20 11	58 3	45 3	22 2	38 3	equal 41
2314	26 11	43 3	27 3	29 2	29 3	equal 52
2414	32 11	28 3	9 3	35 2	19 3	equal 2
2514	37 11	13 2	51 3	41 2	9 4	Time. 12
2614	41 10	58 2	33 3	46 1	59 4	Time. 22
2714	44 10	42 2	15 3	50 1	48 4	Time. 31
2814	46 10	25 1	57 3	54 1	37 4	Time. 40
2914	47	1	40 3	57 1	26 4	Time. 48
3014	48	1	23 4	0 1	14 4	—
3114	48	1	6	1	3	—

A Table of Equation of Time, for regulating of Pendulum Clocks with the Sun.

Degr.	July.	August	Septem.	October	Novem.	Decem.
1	4 57	4 34	3 54	13 27	15 34	5 48
2	5 54	Clocks too 24	4 15	13 41	15 26	5 19
3	5 12	4 14	3 30	13 54	15 16	4 50
4	5 19	4 34	57	14 9	15 54	4 21
5	5 25	3 52	5 18	14 20	14 54	3 51
6	5 31	3 40	5 39	14 32	14 42	3 21
7	5 36	3 27	6 00	14 43	14 30	2 51
8	5 41	3 13	6 21	14 53	14 16	2 21
9	5 45	2 59	6 42	15 3	14 1	1 51
10	5 48	2 45	7 3	15 13	13 46	1 21
11	5 50	2 31	7 24	15 20	13 30	0 51
12	5 52	2 16	7 44	15 31	13 13	0 21
13	5 54	2 1	8 4	15 39	12 55	0 9
14	5 56	1 45	8 24	15 45	12 37	0 39
15	5 57	1 29	8 44	15 51	12 18	1 9
16	5 57	1 12	9 4	15 56	11 59	1 39
17	5 55	0 55	9 24	16 00	11 39	2 9
18	5 53	0 38	9 44	16 4	11 18	2 38
19	5 51	0 20	10 4	16 7	10 56	3 7
20	5 49	0 2	10 23	16 5	10 33	3 36
21	5 46	0 15	10 42	16 11	10 10	4 5
22	5 43	0 33	11 1	16 13	9 46	4 34
23	5 39	0 52	11 19	16 12	9 21	5 2
24	5 34	1 12	11 37	16 11	8 56	5 29
25	5 29	1 32	11 54	16 8	8 30	5 56
26	5 22	1 52	12 11	16 6	8 4	6 23
27	5 15	2 12	12 27	16 3	7 38	6 50
28	5 8	2 32	12 43	15 59	7 11	7 16
29	5 1	2 53	12 58	15 54	6 46	7 41
30	4 53	3 14	13 13	15 48	6 16	8 5
31	4 44	3 34	—	15 41	—	8 29

This Table shews you what the Sun gains or loses of the Pendulum Clock every Day in the Second past Leap Year, the Pendulum Clock keeps equal Time all the Year round, though the Sun doth not do so, but is very unequal in it's Apparent Motion, sometimes too fast at other Times too slow, as the Table plainly sheweth: So that if at any Time you want to set your Clock or Watch by the Sun, look into this Table and see what the Equation is on that Day, and set it accordingly so many Minutes and Seconds more or less, as you see by the Table, the Clock is too fast or too slow for the Sun.

EXAMPLE.

January 17, I see by this Table that the Clock is $13^{\text{h}} 35^{\text{m}}$ too fast for the Sun, then looking upon a good Sun Dial, I set my Watch or Clock so much faster than the Time is, by the Sun Dial, and then 'tis right. Again, on *August 29*, I see by the Table the Clock is too slow $21^{\text{h}} 53^{\text{m}}$, therefore set your Clock and Watch so much behind the Time shewn by the Sun Dial and 'tis right; more Examples in a Thing so plain are needless.

There is nothing more easy and simple than the Motion of the Earth, because it accounts for Appearances of Day and Night, in an easy and Natural Manner; for as the Earth revolves from West to East in 24 Hours Time nearly, it makes the Sun appear to do so from East to West in the same Time; and makes it Day to those Places of it's Surface, which are turned towards the Sun, and Night to such who are on the opposite Part of the Globe.

The Twilight is that dubious half Light, which we perceive before the Sun Rising, and after Sun Setting. 'Tis occasioned by the Earth's Atmosphere, and the Splendor of the *Æther* which environs the Sun; but because of many accidental Variations in both the Sun and Earth's Atmosphere, it cannot be always of one Degree of Duration or Brightness; yet it usually holds in the Evenings 'till the Sun is 18 Degrees below the *Horizon*, and appears so long before his Rising in the Morning. And where the Parallel of the Sun's Declination cuts the Parallel of 18° , there is shewn the Time in the Projection of the Sphere, when the Twilight begins and ends: For by a true Consideration
of

of the Sphere, it will be easy to determine in any Latitude, where the Parallel of Declination cuts the Parallel of 18° , and from thence to determine the Day when it ceases to be perfect Night, and also when perfect Night begins again.

EXAMPLE.

Suppose I would know in the Latitude of $51^{\circ} 32' N$. the Day when it ceases to be perfect Night, and also the Day when the Parallel of Twilight cuts the Parallel of 18° , for that is the Day when it begins to be perfect Night again?

First you are to consider that from the *Zenith* to the *Horizon* is $90^{\circ} + 18 = 108$ the Distance of the Parallel of Twilight from the *Zenith*, from this Sum $= 108$ subtract the Complement of the Latitude of the Place, and the Remainder is the Sun's Distance from the North Pole, or Complement of his Declination; seek this in the Tables of the Sun's Declination, in the Months between *March* and *June*, and where you find it is the Day that it ceases to be perfect Night, look again in the Months after *June*, and before *September*, and where you find the Sun's Declination, that is the Day that perfect Night begins again.

Operation.

Sun's Distance à <i>Zenith</i> , when Day breaks	108° 0'
Co-Lat. subtract	38 28
Sun's Distance à <i>North Pole</i>	69 32
Complement = Declination North	20 28

In the Tables of the Sun's Declination, I find this to answer to *May 11*, and *July 11*, between which two Days there is no perfect Night but Twilight, for all that Time the Parallel of the Sun's Declination never touches the Parallel of 18° below the *Horizon*.

EXAMPLE II,

For *Guernsey*, Latitude $49^{\circ} 36' N$ orth.

Sun's Distance from the <i>Zenith</i>	-	108° 0'
Co-Latitude subtract	-	40 24
Sun's Distance from North Pole	-	67 36
Complement = Declination North	-	22 24
C c z		The

The Days for answering this Declination $\odot$, are *May* 24, and *June* 28. The last and first Appearance of perfect Night in that Latitude.

EXAMPLE: III.

At *Madrid*, Latitude $40^{\circ} 10'$ North, which are the two Days when the perfect Night ceases, and also when perfect Night begins again?

Operation.

Sun's Distance from the Zenith	-	108° 00'
Co-Latitude subtract	- - -	49 50
Remains	- - -	58 10

This $58^{\circ} 10'$ being less than the Sun's least Distance from the North Pole $66^{\circ} 31'$ proves there is perfect Darkness at Midnight all the Month of *June*: For the Parallel of the Sun's Declination, never leaves the Parallel of the Twilight. But at *Petersbourg* in *Russia*, Latitude $60^{\circ} 4'$ North, there is no Night but Twilight, from the Tenth of *April*, to the Eleventh of *August*.

V. Of the Moon.

THE Moon is a Secondary Planet, and respects our Earth for her Center; she always accompanies our Earth, and therefore is properly called the Earth's Satellite: She performs one Revolution round the Earth in 27 d. 7 h 43' 7", and in the same Space of Time by a strange Correspondence and Harmony of the two Motions, she revolves the same Way about her own Axis, whereby one Motion as much concurring it to, as the other turns it from the Earth, the same Side is always exposed to our Sight.

The Figure of the Moon's Orbit is always changing, so that at one Time it is a long Ellipsis, at another time more short, and nearer a Circle, by which her Eccentricity at

at one Time is 66782 in Apogee, and in Perigee 43319 which are the Two Extrems, between which the Moon's Eccentricity is always found. See my *Satellite Astronomy*, page 16. Her mean Diurnal Motion is $13^{\circ} 10' 35''$, and her mean hourly Motion $32' 56''$; and her

Greatest	{	Distance from the Earth.	{	62.518	{	Earth's
Mean.				59.784		Semidia-
Least				55.985		meters.

Which in { 249107.97 }
English { 238241.13 } *mean 236808.60*
Miles are. { 223076.71 } *say 240 miles*

We have shewn before that all Objects are seen under a certain Angle; therefore the same Body, at different Distances will appear to have very different Magnitudes: It also follows that a small Body, when near us, will appear to be equal, or even to exceed another at a great Distance, tho' immensely bigger. And this is the Reason that the Moon, which is the least of all the Planets, appears to us vastly bigger than any of them, nay, even to equal the Sun himself, which is many thousand times greater in Magnitude.

By the Will of the great Creator, which Philosophers call the Law of Gravity, or Gravitation, whereby all heavy Bodies have a Tendency towards the Center of the Earth, &c. and by which, that vast Machine, the Planetary System is governed, all Bodies gravitating towards each other; and this Gravity is Proportional to the Quantity of Matter at unequal Distances, it is universally as the Square of the Distance: All Bodies mutually attract, or tend towards each other.

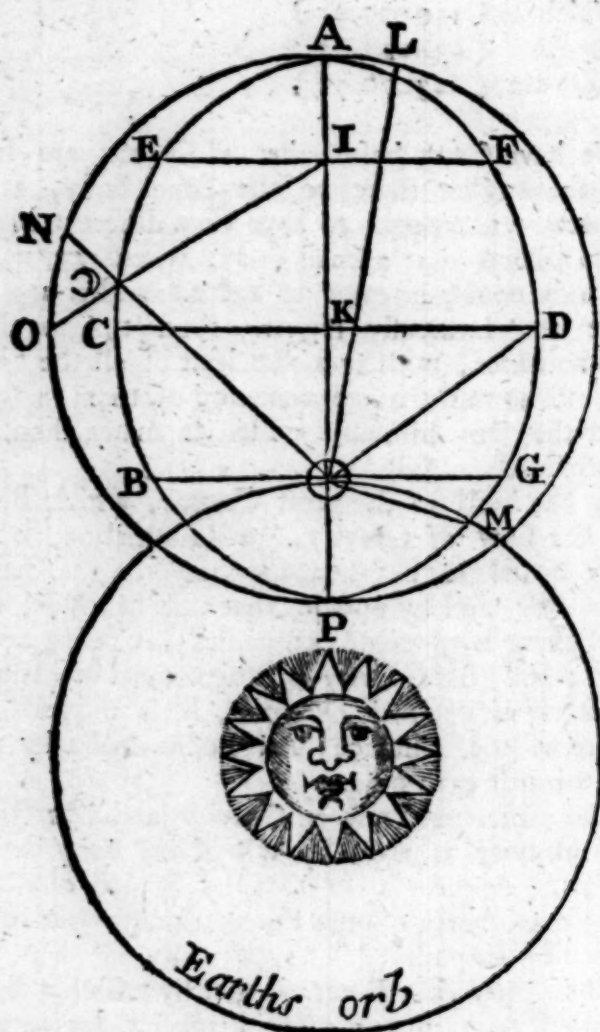
The Difference between Gravity and Attraction is this, viz. Gravity is that by which one Body tends towards another. Attraction is that by which one Body tends towards another by some Force, though perhaps it may happen by Impulse.

The *Centripetal* Force, is that by which a Body, (suppose the Moon, or any other Planet) is drawn or impell'd towards the Center.

Centrifugal

Centrifugal Force, is that by which a Body endeavours to fly off in a Tangent Line. Now these two Forces being equal, the Planets are all kept in their Orbits, they neither fly off nor drop down to their Centers: Which they would do if either of these Laws were taken away.

The Moon moves in an Ellipsis, ACPD, whose lower *Focus* is the Center of the Earth, round about which she describes Areas Proportional to the Times, wherein they are described, and the Earth carries the Moon round the Sun in her Annual Motion:



Let the Moon be at L, from whence in a certain Time she moves to A, the Space or Ray $\ominus L$, describes is $L \ominus A$; the Moon moving on 'till she comes to M, and from the Center of the $\ominus$, draw the Line $\ominus M$, so that the Elliptic Space or Area $\ominus P M$ may be equal to the Area $L \ominus A$, then in this Case she will move thro' the Arch P M in the same Compass of time, that she did thro' the Arch L A. Which Arches are unequal, and nearly in a Reciprocal Proportion to their Distances from the Earth's Center. For because of equal Areas, the Arch P M must be so much in Proportion greater than the Arch L A, as $\ominus A$ is greater than $\ominus P$: This Law was first discovered by *Kepler*, and now demonstrated by *Sir Isaac Newton*. And for a farther Vindication of this Universal Law of the Heavenly Bodies, they have demonstrated that the Squares of the Periodical Times of the Planets, are as the Cubes of their Distances from the Center of their Orbits, about which they perform their Motions regularly.

Of the many inequalities in the Moon's Motion, that which is called her Elliptic Equation is greatest, which arises from a two-fold Motion, one perform'd on the Upper Focus at I, and the other on the Lower at $\ominus$; for suppose the Moon in her Orbit at D, to which if we draw the Line $\ominus D$ and I D, her mean Place will be at O, and her true Place at N, so that the Elliptic Equation is no more than the Angle $\ominus D I = N D O$, which how to find, see my *Satellite Astronomy*, page 16. And this Equation is to be subtracted all the Time the Moon moves from the Apogee at A, to the Perigee at P, that is the first Six Signs of Anomaly. But in the other Semicircle of her Orbit, while she moves from her Perigee at P, to her Apogee at A, that is when her mean Anomaly is 6, 7, 8, 9, 10, or 11 Signs, this Elliptic Equation is to be added to her mean Place to gain her true. And the same Reason holds good in all the Planets.

Of the Moon's Phases.

NEW Moon, or which is all one, the Change of the Moon, is when the Sun and Moon are both together upon one, and the same Circle of Longitude, that is, when they are in the same Sign and Degree of the *Zodiack*, and now sets with the Sun; and by Reason the Moon is always direct in Motion to us at the Earth, soon after the Change she is seen in the Evening after the Sun is set, and puts on then a falcated Face, and her Horns point towards the East; when she is two Signs, or 60° distant from the Sun, that is one Sixth Part of the *Zodiack*; this Distance is called a *Sextile Aspect*; she moving one Sign more in *Consequentia*, 'till she is 3 Signs or 90° from the Sun, this is called a *Quadrate* or *Square Aspect*, or the first Quarter of the Moon, and she is now Dechotomized. When by her Menstrual Motion she is four Signs distant from the Sun towards the East; the Face she puts on now is *Gibbous*, and so continues 'till she comes to the Full, that is Six Signs distant or Diametrically opposite to the Sun, she now shews a full Face to us at the Earth, because that Side which is towards the Sun, is also turned towards the Earth. (*See my Lunar Instrument*). The Earth being partly between the Sun and Moon; she now riseth as the Sun apparently sets, (if the Time of the Full Moon be at Sun setting, else not so exactly,) we have above shewn, that the Moon performs one Revolution round the Earth in 27 d. 7 h. 43' 7'', which is called her Menstrual Month, but in that Time the Sun has apparently moved $26^{\circ} 55' 46''$ from the Place of the last Conjunction with the Sun, therefore the Moon has yet 2 d. 5 h. 0' 59'' farther to travel before she can come up with the Sun to make the next New Moon, which make in all 29 d. 12 h. 44' 6'', as I thus prove.

Moon's

i.e. half-enlightened

Moon's Revolution.

Sun moves in that Time.

27 d.	7 h.	43'	7"
2	1	3	2
	3	40	9
		16	29
		1	14
			5
29	12	44	6

26°	55'	46"
2	00	52
	9	3
		41
		3
29	06	25

This 29d. 12h. 44' 6" the Time between one New Moon and the next, is called the Synodical Month. And now from the Full to the last Quarter, the Moon puts on a *Gibbous* Face again, but on the contrary Side to what it was from the first Quarter to the Full; because that Side of her Face that was then towards the Earth, is now turned from it (by Reason of the Moon's Rotation upon her Axis). The Moon moving on her Menstrual Motion, 'till she is Eight Signs distant from the Sun in *Consequencia*, she makes then what is called a *Trine Aspect*, which she made when she was Four Signs from him, which takes it's Name from being an Equilateral Triangle, or one third Part of the *Zodiack*: One Sign farther towards the East brings her to a Quadrant again which is called the last Quarter, and is now again Dechotomized, or just half enlighten'd to us at the Earth, as she was at the first Quarter, but now turns the contrary Side towards us. From the last Quarter to the Change or New Moon, she is again falcated, but as the Horns in the first Quarter were turned towards the East, they now are towards the West. This Law the Moon observes continually, only with a little Variation caused by her Libration, which is more Philosophical than Astronomical, and so I shall pass it by at this Time.

Astronomers divide the Diameter of the Sun and Moon into 12 equal Parts called Digits, which according to every Day of her Age is shewn by this Table, her Increase from Change to Full, and her Decrease from Full to Change.

A

*Syn &odos viz. similar way
or corresponding motion &c*

A Table of the Digits and Decimal Parts of a Digit, to every Day of the Moon's Age. According to her Mean Motion.

Moon's Age.		Digit's Light.	Moon's Age.		Digit's Light.
0	New 0		29	12	
1		0.81	29		0.44
2		1.63	28	Falcated.	1.22
3		2.44	27		2.03
4	Falcated.	3.25	26		2.85
5		4.07	25	Falcated.	3.66
6		4.88	24		4.47
7		5.69	23		5.29
7	First 9	6.	22	Last 3	6.
8	Quarter.	6.5	22	Quarter.	6.10
9		7.32	21		6.92
10		8.14	20		7.73
11	Gibbous.	8.95	19		8.51
12		9.76	18	Gibbous.	9.36
13		10.58	17		10.17
14		11.39	16		10.98
14	Full 18	12.	15	0	11.8

This Table is so plain, little needs be said in Explanation, only this, look in the first, or third Column for the Moon's Age, and in the next on the Right Hand, is the Quantity of Light that the Moon has at that Time. Examples are needless.

A Table of the Moon's Place, 1739.

Days	January.			February.			March.			April.		
	S.	°	'	S.	°	'	S.	°	'	S.	°	'
1	11	0	30	18	55	0	26	29	2	11	38	
2	11	14	01	1	42	1	9	42	2	24	6	
3	11	27	30	1	14	7	21	33	3	5	56	
4	0	10	33	1	26	16	2	3	45	3	17	43
5	0	23	15	2	8	14	2	15	44	3	29	32
6	1	5	26	2	20	2	2	27	35	4	11	29
7	1	17	34	3	1	51	3	9	24	4	23	33
8	1	29	33	3	13	41	3	21	17	5	6	16
9	2	11	25	3	25	38	4	3	14	5	18	56
10	2	23	15	4	7	43	4	15	24	6	2	12
11	3	5	8	4	20	0	4	27	52	5	15	51
12	3	17	45	2	31	5	5	10	38	6	29	54
13	3	29	9	5	15	17	5	23	45	7	14	12
14	4	11	18	5	28	19	6	7	13	7	28	16
15	4	23	35	6	11	35	6	20	58	8	13	15
16	5	6	46	6	25	1	7	4	57	8	27	43
17	5	18	43	7	8	35	7	19	5	9	12	2
18	6	1	35	7	22	28	8	3	19	9	26	13
19	6	14	42	8	6	24	8	17	31	10	10	10
20	6	28	26	8	20	45	9	1	36	10	23	56
21	7	11	47	9	4	56	9	15	47	11	7	33
22	7	25	45	9	19	12	9	29	48	11	21	2
23	8	10	11	10	3	30	10	13	44	0	4	19
24	8	24	32	10	17	46	10	27	35	0	17	27
25	9	9	18	11	1	57	11	11	18	1	0	25
26	9	24	10	11	15	57	11	24	54	1	13	11
27	10	9	1	11	29	43	0	8	18	1	25	43
28	10	23	43	0	13	11	0	21	28	2	8	4
29	11	1	16	—	—	—	1	4	24	2	20	12
30	11	22	7	—	—	—	1	17	7	3	2	10
31	0	5	4	—	—	—	1	29	29	—	—	—

A Table of the Moon's Place 1739.

Days	May.			June.			July.			Augst.		
	S.	°	S.	S.	°	S.	S.	°	S.	S.	°	
1	3	13	58	4	27	55	6	1	35	7	20	34
2	3	25	46	5	10	4	6	14	10	8	4	24
3	4	7	34	5	22	29	6	27	29	8	18	34
4	4	19	30	6	5	16	7	11	2	9	3	5
5	5	1	38	6	18	29	7	25	1	9	17	54
6	5	14	4	7	2	11	8	9	29	10	2	56
7	5	26	54	7	16	22	8	24	19	10	18	1
8	6	10	11	8	0	51	9	9	27	11	3	2
9	6	23	56	8	16	0	9	24	42	11	17	44
10	7	8	10	9	1	10	10	9	53	0	2	5
11	7	22	46	9	16	22	10	24	50	0	15	58
12	8	7	30	10	1	23	11	9	28	0	29	24
13	8	22	36	10	16	9	11	23	40	1	12	22
14	9	7	28	11	0	29	0	7	24	1	24	57
15	9	22	10	11	14	2	0	23	42	2	7	13
16	10	6	34	11	28	2	1	3	35	2	19	14
17	10	20	40	0	11	13	1	16	7	3	1	4
18	11	4	7	0	24	8	1	28	24	3	12	53
19	11	17	56	1	6	46	2	10	28	3	24	40
20	0	1	12	1	19	10	2	22	24	4	6	31
21	0	14	13	2	1	24	3	4	15	4	16	27
22	0	27	1	2	13	30	3	16	4	5	0	33
23	1	9	40	2	25	27	3	27	53	5	12	51
24	1	22	7	3	7	21	4	9	45	5	21	22
25	2	4	24	2	19	12	4	21	43	6	8	5
26	2	16	33	4	1	2	5	3	49	6	20	50
27	2	28	32	4	12	55	5	16	4	7	4	7
28	3	10	28	4	24	5	5	28	28	7	17	2
29	3	22	8	5	6	54	6	11	7	8	0	5
30	4	47	7	5	19	8	6	23	58	8	14	4
31	4	15	5			17	7	7	7	8	28	4

A Table of the Moon's Place 1739.

Days.	September.			October.			November.			December.		
	°	'	"	°	'	"	°	'	"	°	'	"
1	9	12	53	10	21	50	0	13	46	1	19	48
2	9	27	21	11	6	40	0	27	11	2	2	27
3	10	11	54	11	20	27	1	10	32	2	14	54
4	10	26	37	0	4	36	1	23	33	2	27	10
5	11	11	19	0	18	22	2	6	23	3	9	18
6	11	25	51	1	2	4	2	19	23	3	21	19
7	0	10	7	1	15	25	3	1	11	4	3	13
8	0	24	1	1	28	26	3	13	15	4	15	4
9	1	7	30	2	11	5	3	25	11	4	26	53
10	1	20	32	2	23	23	4	6	59	5	8	49
11	2	3	13	3	5	26	4	18	46	5	20	50
12	2	15	28	3	17	19	5	0	40	6	3	8
13	2	27	30	3	29	4	5	12	43	6	15	43
14	3	9	21	4	10	50	5	25	00	6	28	43
15	3	21	5	4	22	42	6	7	40	7	12	6
16	4	2	5	5	4	45	0	20	43	7	26	10
17	4	14	44	5	17	4	7	4	13	8	10	18
18	4	26	43	5	29	41	7	18	5	8	25	1
19	5	8	59	6	12	39	8	2	18	9	9	56
20	5	21	28	6	26	1	8	16	40	9	25	1
21	6	4	15	7	9	40	9	1	28	10	10	2
22	6	17	19	7	23	37	9	16	10	10	24	54
23	7	0	37	8	7	44	10	00	46	11	9	27
24	7	14	10	8	21	53	10	15	12	11	23	39
25	7	27	49	9	6	10	10	29	25	0	7	29
26	8	11	38	9	20	22	11	13	22	0	21	9
27	8	25	31	10	4	31	11	27	6	1	4	15
28	9	9	27	10	18	31	0	10	33	1	17	5
29	9	23	30	11	2	22	0	23	52	1	29	38
30	10	7	38	11	16	19	1	6	56	2	11	58
31			0	0	0	7			2		23	59

The Use of the foregoing Tables of the Moon's Place.

This Table of the Moon's Place, is that in Longitude, which is sufficient for my Reader's Purpose in this Place.

The Epacts.	The Yearly mean Motion of the Moon to be added to her true Place, any Day in the Year 1739.			S.	°	'
12	4	12	41			
23	8	25	23			
41		8	4			
15	5	20	46			
26	10	3	27			
7	2	16	9			
18	6	28	50			
29	11	11	31			
11	3	24	13			
22	8	6	54			
3	0	19	36			
14	5	2	17			
25	9	14	58			
6	1	27	40			
17	6	10	21			
28	10	23	3			
9	3	5	4			
20	7	18	46			
1	0	1	7			

Look in this Table for the Epact for the given Year, and the Number right against it added to the Moon's true Place any Day in the Year 1739, gives her Mean Place the same Day at Noon. Remembering in Leap Year to add the Motion of the Moon in a Day, $13^{\circ} 11'$ more to the rest.

EXAMPLE I.

Anno 1740, April 9, I would know the Moon's Place at Noon?

1739, April 9, D Place
Epact 12 gives
Leap Year 1 Day

Moon's Place

S.	°	'
5	18	56
4	12	41
	13	11

10	14	48
----	----	----

EXAMPLE

EXAMPLE II.

Anno 1740, August 10, I would know the Moon's Place at Noon.

Operation.

	S.	°	'
1739, Aug. 10, D Place	0	2	5
Epaft 12 gives	4	12	41
Leap Year, 1 Day		13	11
Moon's Place	4	28	7

EXAMPLE III.

I would know the Moon's Place in the Zodiack, Anno 1734, June 12?

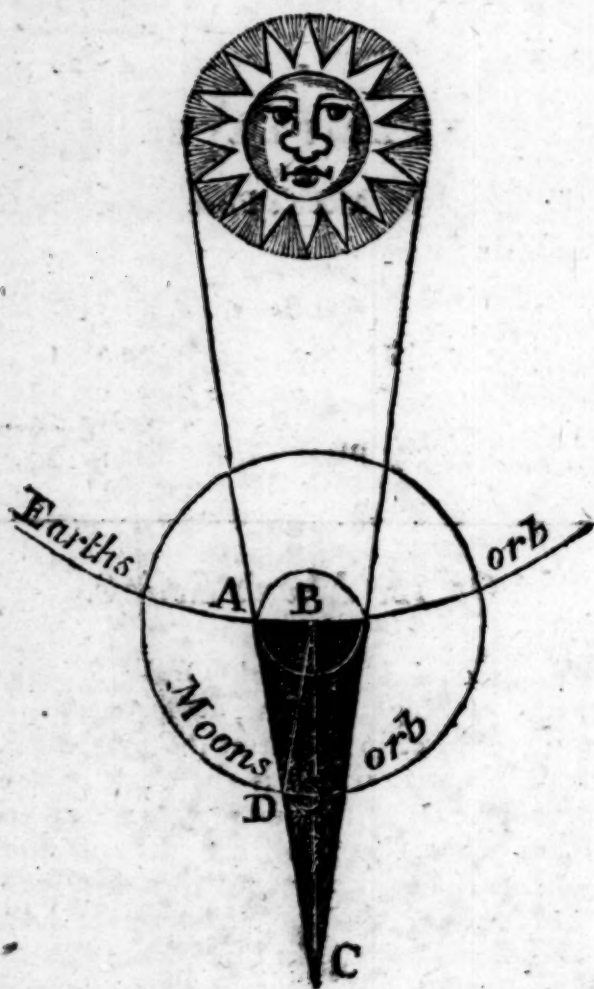
Operation.

1739, June 12, D Place	10	1	23
1734 Epaft 6,	1	27	40
1734, June 12, D Place	11	29	3

Of the Moon's Eclipse.

THE Moon being a dark Opake Spherical Body, having no Light but what she receives from the Sun, can never be eclipsed but at the Full Moon, because the Earth's Shadow, which is the Cause of the Eclipse, is always opposite to the Sun; but not at every Full Moon; but when she is within 12° of either Node. See the Scheme, in page 277, and then her Latitude will be less than the Sum of the Semidiameters of the Moon and Earth's Shadow; but all the Full Moons that happen between EAH and GDF will pass without any Eclipse at all: The Latitude of the Moon is measured on a Circle of Longitude, from the Moon to the Ecliptic, and passing thro' it's Poles, and consequently cutting the Ecliptic at right Angles: The Quantity of an Eclipse depends on

the Moon's Latitude, for the less the Latitude, the greater will the Eclipse be, and the greater the Latitude the less will the Eclipse be: So that if the Full Moon happen at the very Node, the Eclipse will be Central and Total with continuance, I say continuance, because the Moon's Diameter is but a small Matter more than $\frac{1}{3}$ of the Earth's Shadow through which the Moon then passes, as you may the better perceive by this Figure.



If the Moon's Latitude at the Time of the Full Moon be more than the Sum of the Semidiameters of the Moon and

and Earth's Shadow, there will be no Eclipse at that Full Moon, for if the Latitude be North, she will pass over the Shadow, if South under it. There are three Sorts of Lunar Eclipses, viz. 1. *Partial*. 2. *Total without continuance*. 3. *Total with continuance*.

1. *Partial*, is when some Part of the Moon's Diameter or Body falls within the Earth's Shadow, and this always happens when the Sum of the Moon's Latitude and her Semidiameter is greater than the Semidiameter of the Earth's Shadow, in which the Digits eclipsed are always less than 12, and if her Latitude be then North, she looſeth Light on the South Side of her Body, but if her Latitude be South, then she is Eclipsed on the North, or Upper Side.

2. *Total without continuance*, is when the Moon's Latitude is equal to the Difference between the Semidiameter of the Moon and Earth's Shadow, for in this Case the Moon is no ſooner involved in the Shadow, but she is out again; and therefore the Total Darkneſs is of no Duration, the Digits are juſt 12.

3. *Total with continuance*, is when the Sum of the Moon's Latitude and her Semidiameter is less than the Semidiameter of her Shadow: or, which is the ſame Thing, when her Latitude is less than the Difference of her Semidiameter and Shadow, here the Digits are more than 12, and less than $22^{\circ} 28' 12''$, as I thus prove from the Table, page 62, of my *Compleat System*.

Moon's greateſt Horizontal Parallax	61	24
Sun's add	-	10
Sum	61	24
Sun's leaſt Semidiameter ſubtract	15	49
Rem. greateſt Semidiameter $\ominus$ Shadow	45	45
Moon's greateſt Semidiameter, add	16	40
Sum	62	25
Latitude Moon, ſubtract	0	00
Reſt Parts deficient	62	25

Now

Now for the Digits eclipsed, say,

As Semidiameter D	-	16'	40"	L. L.	5563	add
To Six Digits 6°	-	00	00	-	10000	a
So Parts deficient	-	62	25	-	171	to
Sum	-	-	-	-	5734	sub.
To the Digits eclipsed		22°	28'	12"	4266	

This is the greatest Quantity that can be eclipsed.

4. The Eclipses do not always happen in the same Place of the *Zodiack* by Reason of the Retrograde Motion of the Nodes, and Sun's apparent Motion, *see page 299*, which is 3' 11" a Day, and they perform one Revolution in 18 Years 224 d. 4 h. 38'. So that that Eclipse which falls this Year in $\varnothing$ 13°, will the next Year fall in $\varnothing$ 2° and 11 Days sooner, by which the Period of Eclipses called the *Chaldean Saros* is = 18 y. 10 d. 7 h. 43' 15" when the same Eclipse will return again. The greatest Duration of Eclipses happen about, or near the Apogee of the Moon, she moving then slowest, tho' passes through a less Portion of the Earth's Shadow than when in Perigee: The Moon's Apogee moves forwards according to the Order of the Signs 6' 41" in one Day, in a Year 1 S. 10° 39' 50", therefore is one Revolution perform'd in about 8 Years, 309 Days, 8 Hours, and 32 Minutes. The Moon's Eclipse always begins on the East Side and ends on the West, contrary to that of the Sun; for the Moon's Eclipse being real by reason of her direct Motion in Longitude always Eastward, the East Side of her Body must necessarily first touch the Earth's Shadow, and the West Side last leave it. In Total Eclipse of the Moon, even when she is near the Center of the Shadow, her Body is frequently to be seen of a languid Colour, as I have often observed, and particularly, *August 29, 1718*, at 54' past 7 at Night, when she was within 50' of her southern Node, and Total for the Space of 1 h. 37' fall that Time she did not look Black at all, but of a Kind of a yellow swarthy Colour, which could not be without being illuminated with some Light; (for of herself she

she has none,) and some will wonder from whence arise this Light, some suspected that it was the Native and proper Light of the Moon herself, others derived it from the Planets and Stars; for the Interposition of the Earth intercepts all the Light of the Sun, and seems to bring a thick Darknes upon the Whole Space taken up by the Conical Shadow. But we must consider that the Earth is surrounded with a Space of Air 49.66 Miles high, which is of a considerable Density, and has a Refracting Power whereby it turns the Rays of the Sun out of their Way, when they fall upon it, and makes them enter the Conical Shadow; which will therefore be illuminated by that small Quantity of Light which falls obliquely on our Atmosphere, and imparts to all the Bodies within it, a faint Light, the which will illuminate the Moon, even when she is in the midst of the Shadow, and will make her visible to us.

5. The Length of the Earth's Shadow may be found at any Time by Trigonometry, thus, for the Semiangle of the Cone ACB, See Fig. p. 306, is always equal to the Sun's apparent Semidiameter, which when the Sun is in Apogeeon it is $15^{\circ} 49''$, at a Mean Distance $16^{\circ} 5''$, and in Perigeon $16^{\circ} 22''$, so that in the Triangle ABC, we have given the Angle A C B equal to the Sun's Apparent Semidiameter, and A B, one Semidiameter Earth to find B C, the Length of the Axis of the Cone of the Shadow.

Operation.

As T. $\angle$ A C A	-	$0^{\circ} 15' 49''$	7.6627105
To one Semidiameter = A B	=		0.0000000
So Radius	-	-	10.0000000
To B C. $\ominus$ Semidiameter		217.41	2.3372895

$$217.41 \times 3984.58 = 866287.5378 \text{ Miles.}$$

At a Mean Distance from Earth, the Height of the Shadow is 213.775 Semidiameter of the Earth = 851803.5895 Miles. And at the least Distance 210.014 Semidiameter of the Earth = 836817.158412 Miles.

6. By the *Horizontal Parallax* of the Moon we also find her Distance from the Earth, for in the Scheme, page 275 we have given, the Angle H L G the Moon's *Horizontal*

Horizontal Parallax, and H G Earth's Semidiameter, to find H L her Distance from the Earth's Surface.

Let it be required then to find her Distance from the Earth's Surface when she is in Apogee? Look into the Table page 62, of Vol. II. of my *System* and you'll see her Horizontal Parallax is = $54^{\circ} 59''$, and $57^{\circ} 50''$, at a Mean Distance from the Earth, and $61^{\circ} 24''$ in Perigee.

Now for her greatest Distance, say,

As S. < HLG Horiz. Par.	$54^{\circ} 59''$	8.2039376
To H G 1 Semidiameter	-	0.0000000
So C S. < HLG	$54^{\circ} 59'$	9.9999444
To H L $\ominus$ Semidiameters	62.518	1.7960068

$62.518 \times 3984.58 = 249107.97244$ Miles, but from the $\ominus$ Center 249315.1706.

Secondly, For her Mean Distance $\triangleright$ from the Earth.

As S. HLG Horiz. Paral.	$57^{\circ} 50''$	Co-At. 1.7741252
To 1 Semidiameter $\ominus$	-	0.0000000
So C S. H L G	$57^{\circ} 50'$	9.9999386
To H L in $\ominus$ Semidiameter	59.43	1.7740638

$59.43 \times 3984.58 = 236803.5894$ Miles, but from the $\ominus$ Center 238246.00736 Miles.

Lastly, For the least Distance.

As S. HLG Horiz. Paral.	$1^{\circ} 1' 24''$	8.2518576
To H G 1 Semidiameter $\ominus$	-	0.0000000
So C S. H L G	$1^{\circ} 1' 24''$	9.9999306
To H L in $\ominus$ Semidiameters	55.985	1.7480730

$55.985 \times 3984.58 = 223076.7113$ from $\ominus$ Surface.
But from the Earth's Center $55.994 = 223112.57252$ Miles.

VII. To calculate the Diameter of the Earth's Shadow at the Distance of the Moon.

IN the Scheme, page 306, let B be the Earth's Center, D the Moon's Center, BC in the Cone of the Earth's Shadow, and at the Earth's greatest Distance = 217.41 Semidiameters, therefore DC the Cone of the Earth's Shadow beyond the Moon, is found in the three several Stations thus,

$$\begin{array}{r} BC = \text{Cone } \odot \text{ Shad. } 217.41 \text{ and } 213.775 \text{ and } 210.014 \\ \text{Dist } D \text{ of } \odot = B D \quad 62.518 \quad 59.784 \quad 55.985 \end{array}$$

$$\text{Shad. beyond } D = 154.892 = 153.991 = 154.029$$

As Radius	-	90° 0' 0"	10.0000000
To D C =	-	154.892	2.1900236
So T. D C D	-	15' 49"	7.6627105
To D D = Semidiam Shad. $\odot$		.71241	9.8527441

$2 D D = \text{Diameter Shadow } 1.42482 \times 3984.58 = 5669.4198$, which is the Diameter of the Earth's Shadow at the Moon, in Apogee of the Sun; but when the Sun is at a Mean Distance from the Earth, it is 5740.4250228 the Sun in Perigee, the Diameter $\odot$ Shadow at the Moon is 5840.995822.

VIII. To calculate the Diameter of the Moon, and Height of her Shadow.

IN the Scheme, page 306, Let B be the Earth's Center, D the Moon's Center, B D the Moon's Distance from the Earth, and the Angle D B D, the Moon's Apparent Semidiameter, to find D D.

1. The Moon in Apogee.

As C S. D B D	0° 14' 52"	Co-Ar.	0.0000039
To B D	62.518	-	1.7960068
So S. D B D	0 14 52		7.6358290
To D D	.27029 = 1076.9921282		9.4318316

Hence because the Cone of the Earth's and Moon's Shadow are Similar Bodies, their Heights will be in proportion to the Semidiameter of their Bases Base: For having the Base and Height of the Earth's Cone, and the Base of the Moon's, the Height of the Cone of the Moon's Shadow will be obtain'd thus,

As Semidiameter $\odot$	.1	-	0.0000000
To Height of it's Shadow	217.41		2.3372895
So Semidiam. Base D Shadow	.27029		9.4318316
To Height of it's Shadow	58.766		1.7691211

And after this Manner will the Height of the Moon's Shadow at a Mean Distance from the Earth be found to be 58.279 Semidiameter of the Earth. And when she is in Perigee or nearest to the Earth, the Height of the Cone of her Shadow is 57.013 Semidiameter of the Earth, which multiply'd by 3984.58 = 2271.7285954 Miles.

How

How to Calculate the Times and Quantities of Eclipses, particular and general. See my Compleat System of Astronomy, and my Uranoscopia, where the Reader will find Satisfaction to his Desire.

The Use that may be made of Eclipses is for discovering the Longitude of Places at Sea or Land, for having an Eclipse truly calculated to any particular *Meridian*, as suppose *London*, the Eclipse is found by Calculation to begin at 5 h. 39' 52" P M, and being at Sea, the same Eclipse is observed to begin at 6 h. 15' P M. the Difference is 55' 8", which turn'd into Time = $8^{\circ} 47' 00''$ to the East of the *Meridian* of *London*. But if you observe the Time sooner than it is by the Calculation, you are then so much to the West of *London*.

VI. Of Mars.

MARS is a Superiour Planet, and moves in an Orbit round the Sun, between the Earth and *Jupiter*. And make one Revolution in 321 d. 23 h. 27' 30". His Mean Annual Motion is 6 S. $11^{\circ} 17' 10''$, Diurnal Motion is 31' 27", and in an Hour he moves 1' 19". His Eccentricity 14169.5 and his Mean Distance from the Earth 112022636 *English* Miles. There are two Years and fifty Days between every Conjunction with the Sun. The Inclination of his Orbit $1^{\circ} 51'$; *Anno* 1739, the Place of his Ascending Node 1 S. $17^{\circ} 49' 24''$, and the Place of his Aphelion π $1^{\circ} 18' 51''$, it's Annual Motion 1' 12". This Planet has an Atmosphere like our Earth: He is Direct, Stationary, and Retrograde to us at the Earth, and has a Rotation about his own Axis from West to East in 24 h. 40'. He has no Satellite or Moon to attend him, but has Fasciæ or Spots, as appears thro' the Telescope.

VII. Of the Planet Jupiter.

JUPITER is a Primary and Superiour Planet, moving round the Sun in an Orb between *Mars* and *Saturn*; and appears thro' the Telescope to have always a round full Face; he is liable to be eclipsed to an Inhabitant of the Earth by all the Planets except *Saturn*: Receives all his Light from the Sun, and by the Telescope he is known to have Belts which surround his Body, with several Spots by which is discovered his Rotation upon his Axis, from West to East in 9 h. 56': He makes one Revolution round the Sun in 11 y. 317 d. 12 h. 20' 25",

His Mean Annual Motion is	1S: 0° 20' 38"
Diurnal Motion - - - -	4 59
Hourly Motion - - - -	12
Place of his Ascending Node } this Year 1739 - - - }	♊ 8 6 40
Aphelion - - - -	♈ 10 20 36
Eccentricity - - - -	25074.5

Mean Distance from the Earth 337681693 *English Miles*. This Planet as well as *Mars*, and *Saturn* is much nearer the Earth in Opposition to the Sun, than in any other Position. *Jupiter* is attended with four Satellites, and were first discovered by *Galileus*, Anno Domin. 1610. These Moons move round *Jupiter* in a constant direct Order, as here is set down.

Periodical Times.

Diurnal Motition

	d.	h.	'	"	S.	°	'	"	Magnitudes.
1. in	1	18	28	36	6	23	29	20	2
2. in	3	13	17	54	3	11	22	29	3
3. in	7	3	59	36	1	20	19	3	1
4. in	16	8	5	12	0	21	34	16	4

Their

Their Distances from Jupiter in Semidiameter of Jupiter are,

1.	2.	3.	4.
5.91	9.4	15.	26.38.

By these Magnitudes and Distances from *Jupiter*, they may be easily distinguished one from another, when you look at them through a Refracting Telescope, they will appear on the contrary Side of *Jupiter*, but a Reflecting Telescope will shew them really as they are in the Heavens. The Apis of the Moons this Year 1739, is $\propto 21^{\circ} 12'$, with a Yearly Motion of $36'$. They always appear amongst the Fixed Stars to lie nearly in a straight Line, except their Latitudes cause a little Variation, (as I have often observed). And the Position of this Line has Respect unto the Altitude of the Nonagesime Degree in *Jupiter's* Orb, as being at one Time Parallel to the *Horizon*, at other Times more oblique, making Angles therewith of different Quantities. The Fourth, or outmost Satellite passes wide of the Shadow two Years in every six, as it did in the Year 1734, and will again in 1740, &c. but the other three pass through *Jupiter's* Shadow once in every Revolution, and are then eclipsed; and they also eclipse one another. Besides the Immersions, they become twice invisible to us in every Revolution except when their Latitude is too great, that is, once when they are between the Eye and *Jupiter*, and again when they are behind his Body. When *Jupiter* is our Morning Star, and Rising before the Sun, the Spectator sees only the Immersions, or the Entrance of the Satellite into *Jupiter's* Shadow; but when he is our Evening Star and Setting after the Sun, we then see the Emerisions out of the Shadow. Four and thirty Days every Year *Jupiter* lies hid under the Sun Beams, and consequently can't be seen with the naked Eye. By observing the Eclipses of *Jupiter's* Moons from opposite Parts of the *Magnus Orbis*, it is found that Light is not instantaneous, but takes up 9 or 10' Minutes Time to travel from the Sun to us, which distance is more than 82 Millions of Miles: Which Space the Light passes thro', in so small a Time that so

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prodigious

95 millions

prodigious a Velocity cannot easily be conceived by us, which so much exceeds the Velocity of the swiftest Bodies we know.

By the Eclipses of these Satellites, the Longitude of Places upon the Terraqueous Globe may be known after the same Manner as has been shewn in the Eclipses of the Sun and Moon.

VIII. Of Saturn.

SATURN is a Primary and Superior Planet, and is the highest in our System; he is always observed with a round full Face, is of a heavy dull Lead Colour, and is liable to be eclipsed from our Sight, by all the other Planets, if their Latitudes correspond; he makes one Revolution round the Sun in 29 y. 174 d. 6 h. 36' 26"; his mean annual Motion is $12^{\circ} 13' 21''$, his Diurnal Motion $2' 1''$ and hourly Motion $5''$. Every Year and thirteen Days is conjoin'd with the Sun; he is direct Stationary and Retrograde to us at the Earth once every Year, and makes all his Aspects with the Sun. It is uncertain whether this Planet moves round his Axis or not, by Reason of his great Distance from us, which is 575426579 *English Miles*. His Eccentricity 54376, such Parts as the Mean Distance of the Earth from the Sun is 1000000. The Place of his Aphelion this Year 1739, is in $\uparrow 29^{\circ} 25' 20''$, and the Place of his Ascending Node $\overline{\text{sc}} 21^{\circ} 16' 48''$, the annual Motion of the Aphelion $1' 20''$, and of the Node $18''$, the Inclination of his Orbit $2^{\circ} 30' 10''$, and by the Telescope's Observations is found to have five Satellites or Moons moving round him, as he moves round the Sun, and according to the latest and best Observations, do make their Revolution in Times, as is here set down.

i. in

200,000 miles per second

	d.	h.	'	"	Distance.	
1. in	1	21	18	26	4.893	} Semidiameters of of Saturn's Globe.
2. in	2	17	41	10	6.268	
3. in	4	12	25	10	2.754	
4. in	15	22	41	28	20.295	
5. in	79	7	46	00	59 154	

Besides these five Moons this Planet is known to be encompassed on every Side with a Ring, but doth no where touch his Body, the Diameter of *Saturn* to that of the Ring is as 4 to 9. *Cassini* supposes the four interior Satellites to be moved according to the Plane of the Ring, or their Orbits to be inclined to the Orbit of *Saturn* by an Angle of 30° , but *Hugens* of 31° . When *Saturn* is in the Middle of the Sign *Gemini* and *Sagittari*, then the greatest Axis of his broad spreading Ring, is found to be precisely double to the lesser, and lieth most open to our View, as it was in the Year 1737, and will be so again in 1751; but on the contrary, when this Planet is in *Virgo* or *Pisces*, the Ring seems to us to be quite shut, and only as it were a right Line, and no more to be seen and this will be seen in the Year 1744, and again in 1758, and 1759. But this is spoken to *Astronomers* who are furnished with long Glasses wherewith to make Observations of this Kind. Those that would see the Reason of the *Retrogradations* of the Planets, let them consult my Planetary Instruments. And here in the Satellites as well as in the Planets themselves, this Universal Law of Nature also holds good, *viz.* that the Squares of the Periodic Times are as the Cubes of their Distances from the Centers of their Primary.

IX. The Characters and Poetical Names of the Planets.

- ♄ *Saturn, Chronus, Phœon, Falcifer.*
- ♃ *Jupiter, Phaëton, Zeus, Jove.*
- ♂ *Mars, Ari, Pyrois, Mavors, Gradivus.*

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☉ *Sol,*

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- ☉ } *Sol, Titon, Itios, Phæbus, Apollo, Phaon, Osyris*
 Deiſpiter.
 ♀ *Venus, Cytheria, Aphrodite, Erycina.*
 ☿ *Mercury, Hermes, Stilbone, Cyllenius, Arctus.*
 ☾ } *Moon, Lucina, Cynthia, Diana, Phæbe, Proſepina,*
 Noctiluca, Latona.

The Reason of which Names may be ſeen in Myſtagogus Poeticus.

- ⊕ *Terra, the Earth,*
 ♂ *Dragon's Head.*
 ♂ *Dragon's Tail.*

X. The Characters and Names of the Twelve Signs, with the Numbers of Stars they contain.

♈	<i>Aries</i>	21.
♉	<i>Taurus</i>	43.
♊	<i>Gemini</i>	30.
♋	<i>Cancer</i>	15.
♌	<i>Leo</i>	40.
♍	<i>Virgo</i>	39.
♎	<i>Libra</i>	18.
♏	<i>Scorpio</i>	10.
♐	<i>Saggitarius</i>	14.
♑	<i>Capricorn</i>	28.
♒	<i>Aquarius</i>	41.
♓	<i>Piſces</i>	36.

XI. The Characters and Names of the Five Aspects.

- ♂ *Conjunction, is the ſame Sign and Degree.*
 ♄ *Sextile, two Signs, or 60 Degrees aſunder.*
 ☐ *Square, three Signs, or 90 Degrees aſunder.*

△ *Trine,*

△ *Trine*, four Signs, or 120 Degrees assunder.

⊗ *Opposition*, six Signs, or 180 Degrees assunder.

XII. Of the Fixed Stars.

FI X E D Stars are such as do not, like the Planets or Erratic Stars, change their Positions or Distances in Respect of one another. And because they have no sensible Parallax arising from the annual Motion of the Earth, they are justly esteemed to be of such an immense Distance, that the Earth's annual Orbit is but as a Point in comparison of it.

Light takes up more Time in Travelling from the Stars to us, than we in making a *West India Voyage*. That Sound would not arrive to us from thence in 50000 Years, nor a Cannon Ball in a much longer Time, this is easily computed by allowing 10^l for the Journey of Light from the Sun hither, and that it moves 968 Feet in a Second of Time.

Where - ever the Spectator is placed in our System, whether in the Sun, in the Earth, or even in *Saturn*, will see the same Face of the Heavens, and the same Stars, with the same Magnitudes, and the same Figure of the Constellations, and the Heavens which surrounds and involves them all, will have the same Face.

The Antients divided the Visible Firmament into 48 Images, 12 of which filled the *Zodiack*, the Names of which we have given in page 318. In the Northern Region there are 21 Images, viz. the *Lesser Bear*, the *Great Bear*, the *Dragon*, *Cephus*, *Botes*, the *Northern Crown*, *Hercules*, the *Harp*, the *Swan*, *Cassiopeia*, *Perseus*, *Andromeda*, the *Triangle*, *Ariga*, *Pegasus*, *Equilus*, the *Dolphin*, the *Arrow*, the *Eagle*, *Serpentarius*, and the *Serpent*. Afterwards they added to them two others, viz. that of *Antinous*, which was made of the Stars that are not included within any Image, and are near the *Eagle*; and the Constellation called *Berenice's Hair*, these Stars lie near the *Lyon's Tail*.

On

*Light travels
200,000 feet per second*

On the South Side there are 15 *Asterisms*, which were known to the Antients, *viz.* the *Whale*, the *River Eridanus*, the *Hair*, *Orion*, the *Great Dog*, the *Lesser Dog*, the *Ship Argo*, *Hydra*, the *Cup*, the *Crow*, the *Centaur*, the *Wolf*, the *Altar*, the *Southern Crown*, and the *Southern Fish*. To these are added 12 more Constellations, which are not seen by us who inhabit the Northern Regions, because of the Convexity of our Earth, but in the Southern Parts they are very conspicuous. These are the *Phoenix*, the *Crane*, the *Peacock*, the *Indian*, the Bird of *Paradise*, the *Southern Triangle*, the *Fly*, the *Chameleon*, the *Flying Fish*, the *Toucan*, or *American Goose*, *Hydrus*, or *Water Serpent*, *Xiphas*, or the *Sword Fish*.

Astronomers have added two in the Northern Hemisphere, *viz.* *Charles's Heart*, and *Sobieski's Shield*.

The *Galaxy*, *Via Lactea*; Or *Milkey Way*, is a broad white Tract encompassing the whole Heavens, and extending itself in the Sign of *Capricorn* from the Equinoctial to the Tropic of *Cancer* with a double Path, and the rest of it is a single one. Some of the Antients, as *Aristotle*, imagined that this Path consisted only of a certain Exhalation hanging in the Air, but by the Telescope's Observations of this Age it has been discovered to consist of an innumerable Quantity of Fixed Stars, different in Situation and Magnitude; from the confused Mixture of whose Light it's white Colour is supposed to be occasioned. (With the Telescope I have counted 23 Stars in the Compass of the *Pleiades*.) It's greatest Declination North is about 65° , and South 69° , it crosses the Equinoctial from South to North in $18^{\circ} 5'$, and from North to South in 25° . It's breadth, where broadest, which is about *Aquila*, is 25° , in other Places it doth not exceed 10 Degrees.

Astronomers have divided the Stars into six several Sizes or Magnitudes, of which the greatest or brightest of them are called Stars of the first Magnitude, as *Capella*, *Alburnus*, *Regulus*, *Sirius*, &c. and the next to them in brightness are called the Stars of the second Magnitude, next to them in brightness are Stars of the third Magnitude and so forth, 'till we come to Stars of the sixth Magnitude, which comprehend the smallest Stars that can be discern'd with the naked Eye.

The Number of Fixed Stars that their Places have been Rectified by several Astronomers are these.

<i>Hypparchus</i> , 140 before Christ had	-	1022
<i>Pliny</i>	-	1600
<i>Ptolomy</i> 135 Years after Christ	-	1026
1437 <i>Uligb Beighi</i>	-	1017
1460 <i>Regiomontanus</i> .	Ditto	
1536 <i>Nich. Copernicus</i>	Ditto	
1572 <i>Tycho Brahe</i>	-	777
1620 <i>John Kepler</i> .	-	1163
1603 <i>Dr John Bayerus</i>	-	1725
Prince of <i>Hess</i>	-	400
1635 <i>Brachius</i>	-	1672
1651 <i>John Ricciolus</i>	-	1468
1670 <i>John Hevelius</i>	-	1888
1676 <i>Dr Edmund Halley</i>	-	373
1690 <i>John Flamsteed</i>	-	3000
1726 <i>This Author</i>	-	777

The Fixed Stars move upon the Poles of the Ecliptic, by which they have a progressive Motion of 50" a Year, that is one Degree in 72 Years, and one Revolution in 25920 Years ; by which Motion they are always kept parallel to the Ecliptic, and so never alter their Latitudes ; but *Dr Halley* tells us the *Philosophical Transactions*, Numb 355, that four of the Fixed Stars have altered their Latitudes since *Ptolomy's* Time, viz. the *Bull's Eye* 20' more South, *Sinius* 22, *Arcturus* 33', and the Bright Shoulder of *Orion*, almost a Degree more North.

But these may be look'd upon as bad Observations made in *Ptolomy's* Time, for want of good Instruments, rather than attributing the Difference of the Latitude to any inequality in the Stars Motions.

The Fixed Stars are known from the Planets by their Scintillation, or Sparkling ; for the Planets have no such Vibration, twinkling or glimmering of Light : But generally all the Fixed Stars more or less, and at some Times more than others : The Cause of their Scintillation is variously discoursed of, both by Philosophers and Astronomers. *Aristotle*, among the Antients, assigns the Cause

Cause thereof to their remoteness from our Sight, by which they are weakly, and as it were by a trembling weariness reached. Some again will have the Cause of this Twinkling to proceed from Refraction; others assign the Cause to arise from the unequal Superficies of the Fluctuating Air or Medium, as Stones in the Bottom of a River by the Rapid Motion of the Water, seem to have a kind of termulus Motion, which is only in the crisped and uneven Undulation of the Stream. *Gassendus* more probably conceives this *Scintillation* of the Fixed Stars to proceed from that Native and Primigenial Light they are indued with, like that of the Sun's sparkling, casting forth such quick darted Rays as our weaker Light cannot behold without that trembling Passion.

XIII. Of Comets,

COMETS are what are commonly called Blazing Stars. The Antients, especially *Aristotle*, and his Followers supposed them to be Meteors or *Exhalations*, set on Fire in the Middle Region of the Air. The Modern Astronomers have found that they are within our Planetary System, and that their Orbits are very Eccentric, being sometimes at a moderate Distance from the Sun, at other Times they ascend to vast Distances, far beyond the Orbit of *Saturn*, and so become invisible. The great Comet that appeared here in 1680, 1681, was seen before in our Hemisphere, *Anno Domin.* 1106, once before, about the Year 532, and also 44 Years before our Saviour's Birth: And therefore Astronomers conclude the Time of it's Periodical Revolution to be 575 Years. The Time of the Revolution of another Comet which appeared in the Year 1682, will appear to us again in 1758, is $75\frac{1}{2}$ Years. in making one Revolution, another probably may be seen here again *An. Dom.* 1789, makes it's Ellipsis round the Sun in 129 Years. And that these Periods are all that are known to Astronomers, is what we are to be satisfied with 'till further Discoveries are made.

I might

I might here give the Projection of the Sphere, and apply Spherical Trigonometry in the Solution of all the Practical Problems of the Sun, Moon, and Stars, but this being already handled in my *Compleat System of Astronomy*, I shall forbear it here, and proceed to the Art of Dialling.

CHAP IX.

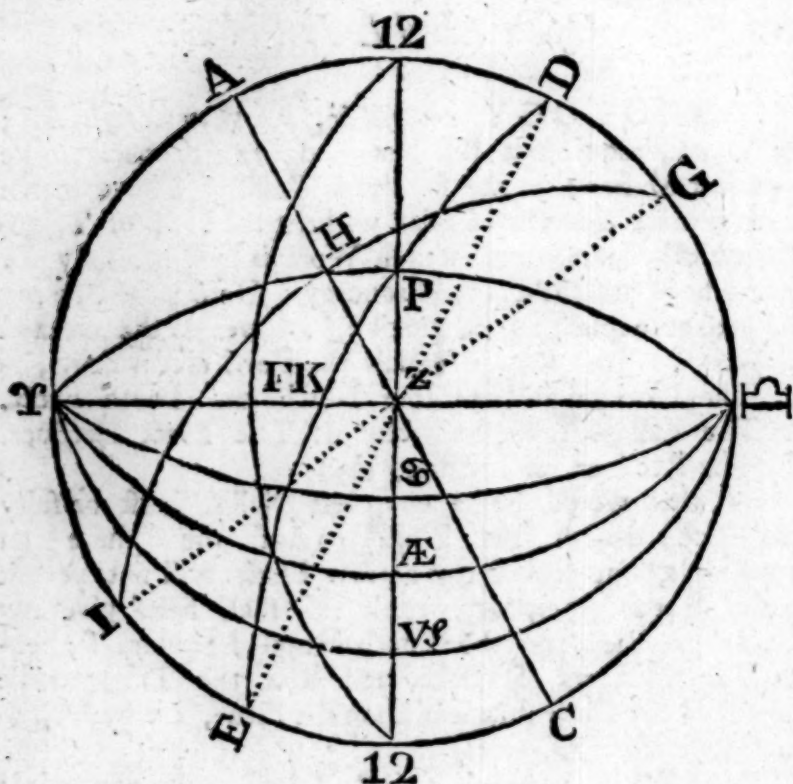
Of DIALLING.

HOROLOGY, or Dialling is a very curious Art; and requires a full Knowledge in Astronomy to be a perfect Master; in my *Mechanic Dialling* I have shewn how to make all Sorts of Dials without the Help of Trigonometrical Calculation, which Work is intended only for him who is unskill'd in Astronomy: Here it shall be my Business to explain the calculative Part even to the meanest Capacity. First then of Dials there are divers Sorts, as
 1. The Horizontal. 2. The North and South Erect.
 3. The East and West Erect. 4. The Erect Decliner.
 5. The Recliner, or Incliner, &c.

He that would be a compleat Diallist must be fully acquainted with the Projection of the Sphere on any Circle, but because my narrow Limits will not permit me to treat so largely as I would; I shall here give my Reader a View of the Fundamental Diagram, for expressing all Sorts of Planes, and how the Triangles lie in which the Hour distances upon the Plane, are by Trigonometry calculated.

First, With a Sweep of 60° opened to any convenient Radius, draw the Primitive Circle $\cap 12 \simeq 12$, which shall represent the Horizon of the Place; and Z the Zenith. 2. Quarter it, and draw $\cap Z \simeq$ for the Prime Vertical, and $12 Z 12$ for the Solstitial Colure. 3. Take the Tangent of $19^\circ 14'$, it being $\frac{1}{2}$ the Co-Latitude of *London*, and set it from Z to P, so shall P represent the North Pole, thro' the three Points $\cap P \simeq$ (or with the Secant of $51^\circ 32'$) draw $\cap P \simeq$ for the Equinoctial Colure, or Six o'-Clock

Clock Hour. 4. With the Secant of $61^{\circ} 57'$, the Sun's greatest Meridian Altitude at *London* draw $\gamma \text{ } \omega \text{ } \omega$. 5. With the Secant of $38^{\circ} 28'$, being the Height of the Equinoctial at *London*, draw $\gamma \text{ } \text{Æ} \text{ } \omega$. 6. With the Secant of $14^{\circ} 59'$ the Sun's least Meridian Altitude at *London*, draw $\gamma \text{ } \text{vs} \text{ } \omega$: A Z C, 12 F 12, D P E, and G H I are Planes of different Positions drawn at Pleasure: Thus is the Scheme finished sufficient for our present Purpose. See p. 250.



Then doth the Primitive Circle represent a Horizontal Plane. 2. A Vertical Plane, parallel to the Prime Vertical Circle passing through the Zenith and East and West Points of the Horizon, is the Erect, Direct North and South Plane, and is represented by $\gamma \text{ } Z \text{ } \omega$ the Prime Vertical Circle. 3. A Meridian Plane, parallel to the Meridian, and passing through the Zenith and Pole, such are the East and West

West direct Planes, and represented by 12 P Z 12, the Solstitial Colure. 4. A Polar Plane, parallel to the Earth's Axis or Hour of Six is here represented by γ P $\triangle$, it passes through the Poles, and through the East and West Points, cuts the Equinoctial and Meridian at right Angles, but inclines to the Horizon with an Angle $= 12 \gamma$ P $= 51^{\circ} 32'$, reclining from the Zenith $38^{\circ} 28' = \angle$ P γ Z. This is called a direct Incliner. 5. The Equinoctial Plane, is parallel to the Equinoctial passing through the East and West Points being right to the Meridian, but Inclining to the South Horizon (in North Latitudes) with an $\angle = 38^{\circ} 28'$ the Co-Latitude $= \angle$ γ $\triangle$ 12 represented by γ $\triangle$ $\triangle$ the Equinoctial, and consequently and properly is called a Direct Incliner, because the upper Face looks to the North, and the under to the South. 6. A Direct inclining Plane, parallel to any great Circle which passes through the East and West Points of the Horizon, being right to the Meridian, but not passing thro' the Poles of the World, nor parallel to the Equator, is represented by γ $\triangle$ $\triangle$, or by γ γ $\triangle$ being the two Positions of the Ecliptic, when the very Beginning of *Cancer* and *Capricorn* are upon the Meridian. 7. A Vertical declining Plane, parallel to any great Circle which passeth through the Zenith, being right to the Horizon, but inclining to the Meridian with an Angle of 30 Degrees. 8. A Polar declining Plane, parallel to some great Circle which passeth through the Pole, being right angled with the Equinoctial, but inclining to the Meridian with an Angle of $24^{\circ} = 12$ P D, and this Plane is represented by D P E. 9. A Meridian Plane inclining to the Horizon, parallel to any great Circle which passeth through the Points of North and South, and cuts the Prime Vertical at right Angles, this Plane is represented by 12 F 12. 10. A Declining inclining Plane, parallel to any great Circle, which passeth neither through the North, South, East, or West, nor through the Pole, nor Zenith, but lieth oblique to them all, it is represented by G H I making an Angle with the Horizon of 45° , and Declining from the Meridian $51\frac{1}{2}^{\circ} = 12$ G. Each of these Planes except the Horizontal have two Faces, whereon Hour Lines may be drawn.

I. To make an Horizontal Dial.

THIS Plane is represented in the Fundamental Diagram by the Primitive Circle $A \cap C \cap$, which is best made of Brass to stand the Weather. Now if we project the Sphere Stereographically upon the Plane of the Horizon for the given Latitude, and where the projected Meridians or Hour Lines cut the Plane of the Horizon, draw Lines from the Center or Zenith to the Border of the Dial, and they shall be the true Hour Lines, and the Stile or Gnomon must stand upon the Meridian, or 12 o'Clock Hour Line, and to make an Angle at the Center of the Hour Lines, equal to the Latitude of the Place for which the Dial is made.

But to do this by Trigonometry, let $A B C D$ represent a Horizontal Plane, on which I am to draw a Dial; upon which draw as many Concentric Circles for the Boundaries of the Dial as you fancy will serve your Turn, and draw the Diameter $A B$ for the Meridian, or 12 o'Clock Hour Line: Take the Radius of the Circle in your Compass and set the Sector on the Line of Tangent, and take off the Tangent $19^{\circ} 14'$ that being half the Complement of the given Latitude, and set it from Z the Zenith, to P the North Pole. Through the Pole P , at right Angles to the Meridian $A B$ draw the Line $6 P 6$ for the Six o'Clock Hour Line. Now for the other Hour Lines, their Distance from the Twelve o'Clock or Meridian, we must have recourse to the Fundamental Diagram in page 324, then suppose the oblique Circle $D P E$ to represent the One or 11 o'Clock Hour Circle, then the Angle $12 P D = 15^{\circ}$, in which Triangle right Angle at 12 , there are given $P 12 = 51^{\circ} 32'$ the Latitude of the Place = Elevation of the Pole above the Horizon, with the Angle $12 P D = 15$, to find the Side $12 D$ the distance of One and Eleven o'Clock from the Meridian upon the Horizon.

Equation. $S P 12 + R = C T < P + T 12 D.$

Analogy.

As $C T < P$	-	-	$15^{\circ} 0'$	10.5719475
To Radius	-	-	$90^{\circ} 0'$	10.0000000
So $S. P 12 =$ Pole's Elevation	51	32		9.8937452
To $12 D$, 1 Hour on the Plane	11	51		9.3217977
Or				

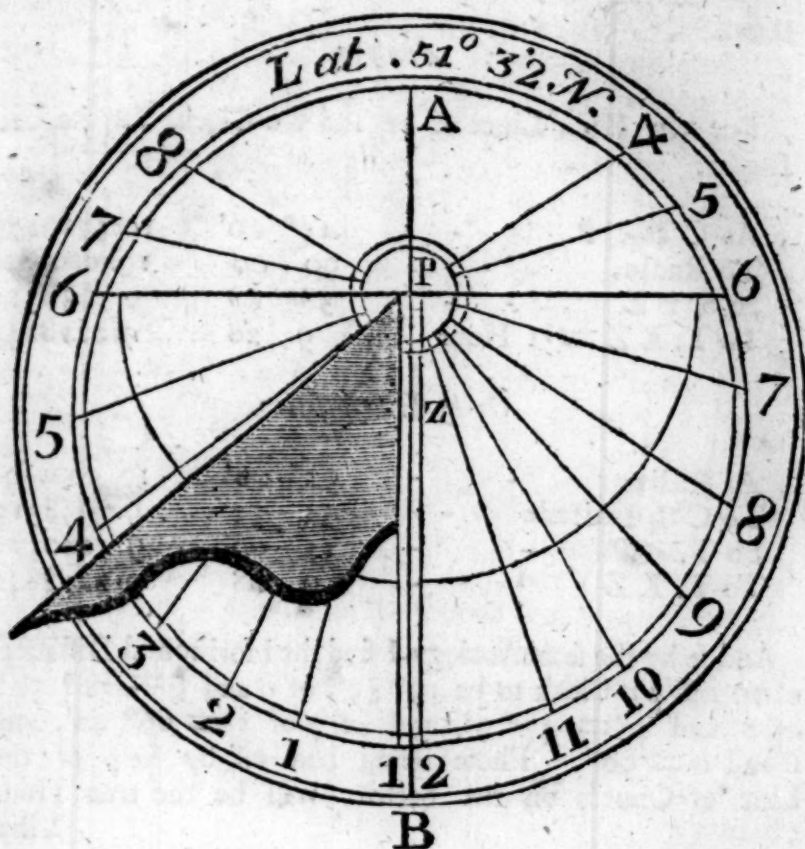
Of DIALLING.

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Or by Transposition, see p. 245.

As Radius	-	90° 0'	10.0000000
To T < P		15 00	9.4280525
So S. P 12 = Latitude		51 32	9.8937452
To T. 12 D		11 51	9.3217977

And thus by making the Angle at the Pole 30, 45, 60, 75°, and working as above you will have the distance of every Hour upon the Plane, from the Meridian Line A B. Then by allowing for the thickness of the Stile, draw two Quadrants on each Side the Twelve o'Clock Hour Line, and set the Sector to that Radius, and take the Chords of the several Hour Distances, and set them off in each Quadrant, and those Marks draw Lines from the Center at P to the Margin of the Dial, and they shall be the true Hour Lines required. The Stile or Gnomon must Make an Angle at P = the Latitude of the Place for which the Dial is made. which being set at right Angles over the Twelve o'Clock Hour Line, the Dial is finished.



H. Of an Erect Direct North and South Dial.

THE Hour Lines of these Dials are calculated by the Triangle KPZ, right angled at Z. In the Scheme, page 324, there are given, the Angle at P 15, 30, 45, 60, 75° the Equinoctial Distances, and ZP the Distance between the Zenith and Pole, equal to the Complement of the Latitude of the Place, to find KZ the Hour Distances upon the Plane.

Equation.

$$S. PZ + R = TKZ + CT < P.$$

Analogy.

For the Hour Lines of 11 and 1 o'Clock, for the Lat. London.

As CT < P	15° 00'	10.5719475
To Radius	90 00	10.0000000
So S. P Z	38 28	9.7938317
To T. KZ = 1 Hour	9 28	9.2218842

Or by Transposition.

As Radius	90° 00'	10.0000000
To C S. Latitude	51 32	9.7938317
So T. < P	15 00	9.4280525
To T. KZ	9 28	9.2218842

And so by the same Analogy I find the Horizontal Distances of 10 and 2 o'Clock to be 19° 45', of 9 and 3 = 31° 54', of 8 and 4 = 47° 9', and of 7 of 5 = 66° 42', and 6 and 6 = 90°. These being laid off by help of the Line of Chords on the Sector, will be the true Hour Lines

Lines on a South Erect Direct Plane for the Latitude of $51^{\circ} 32'$ North. And for the North Dial you may either continue the Hour Lines of 4 and 5 thro' the Center to serve for 7 and 8, and 7 and 8 shall be 4 and 5 on the North Plane, or else lay them off by help of the Chords, as above has been shewn. The Gnomon or Stile must hang at right Angles over the 12 o'Clock Hour Line, and because the top Edge of all Dial Clocks represents the Earth's Axis, and consequently must lye parallel to it; and in these Dials it must make an Angle with the Plane equal to the Complement of the Latitude of the Place: And in the South Dial it must point downward to the South Pole, and the Cock of the North Dial must point upwards to the North Pole. In all the Cases of Dialling we suppose the Earth no more than a Point, in respect of the great Distance that it is from the Sun, for if it were reckoned otherwise, the Stile of the Dial could not be parallel to the Earth's Axis, and at the same time point to the true Poles of the World.

When the Sun has North Declination, (which he has from the Ninth of *March* to the Twelfth of *September*), he comes upon the South Erect direct Plane in the Morning at the same time that he is due East, or upon the Prime Vertical, and goes off in the Afternoon when he is due West.

III. Of an Erect Declining Plane.

THE first Thing to be done, is carefully to take the Plane's Declination, which is no more than an Arch of the Horizon comprehended between the Axis of the Plane, and the Meridian of the Place; and that you may the better distinguish Planes, you are to consider and understand three several Lines. The First is the Horizontal Line; the Second, the Line perpendicular to the Plane, commonly called the Axis of the Plane. Lastly, The Meridian Line. There are four several Ways to find the Declination of a Plane, as (1) by the Magnetic Needle,

which is of all the most uncertain by Reason of the Variation. (2) By two Observations, the one made in the Forenoon, the other in the Afternoon. *See my Mechanic Dialling.* (3) By applying a Dial made for the same Latitude to a Circle divided into 360° upon a Board placed Horizontally to the Plane, move the Dial to the true Hour of the Day and then the Twelve o'Clock Line on the Dial will point to the Degree of the Plane's Declination on the divided Circle. Lastly; By the Sun's Azimuth and Horizontal Distance, which is thus, with a good *Astronomical Quadrant*, (such as is now made by *John Barston* and *Jos. Turner* in *Hatton Garden, London*) take the Sun's Altitude, then in the Triangle AZP , page 211, we have given AZ the Co-Altitude, ZP the Co-Latitude, and AP the Sun's Distance from the North Pole, or Co-Declination, to find the Angle AZP , work as is taught in page 259, and you will have the Sun's Azimuth from the North, whose Complement to 180 is the Angle AZ the Sun's Azimuth from the South.

At the same Moment of Time that you take the Sun's Altitude, let an Assistant fix a Semicircle divided into an 180 Degrees to the Horizontal Line of the Plane, so that the Limb, or graduated Edge be towards the Sun, hold up a Thread or Plummets near the Limb, so that the Shadow thereof may pass through the Center of the Semicircle, and observe the Degrees cut by the Shadow of the Thread, and Number them from that Side of the Semicircle which is perpendicular to the Plane, for those Degrees are the Horizontal Distance sought. This being had with the Sun's Azimuth from the South, observe these two Rules

RULE I.

If the Shadow of the Thread fall between the South and that Side of the Quadrant, which is perpendicular to the Plane, then add the Sun's Azimuth from the South to the Horizontal Distance, and that Sum is the Declination; and it is to the same Coast with the Sun's Azimuth, that is, East, if the Observation be made in the Forenoon, but Westward in the Afternoon.

II. If the Shadow of the Thread fall not between the South and that Side of the Quadrant that is perpendicular

lar to the Plane, then the Difference between the Sun's Azimuth from the South, and the Horizontal Distance is the Declination of the Plane; and if the Azimuth be the greater of the Two, then the Plane declines to the same Coast whereon the Sun is; but if the Horizontal Distance be the greater, then the Declination is to the contrary Coast.

EXAMPLE.

Anno 1725, March 17, at London, in the Afternoon I observed the Sun's Azimuth from the South $38^{\circ} 52'$ and the Horizontal Distance fell not between the South and the Axis of the Plane, consequently their difference $29^{\circ} 8'$ is the Plane's Declination to the East. These Things being found, there are four Things to be considered.

1. The Inclination of the Meridian of the Plane, with the Meridian of the Place.

2. The Height of the Pole or Stile above the Plane.

3. The Distance of the Subtile from the Meridian.

4. The Distance of the Hour Lines from the Subtile.

All these I shall demonstrate in the following Diagram.

Geometrical Construction.

With the Chord of 60 draw the Primitive Circle which shall represent the Plane of the Dial, draw ZN and HO at right Angles, take the Chord of $29^{\circ} 8'$ the Plane's Declination, and set it from N to D , lay a Ruler from Z to D and it will cut the Horizon in A , through which the Meridian of the Place must pass; or set the Semi-tangent of the Declination from C will give A as before. Take the Co-Secant of the Plane's Declination $= 60^{\circ} 52'$ and draw the Meridian ZAN , it's Pole will be found at E ; take the Latitude of the given Place $51^{\circ} 32'$ from the Chord and set it from O to L , and from N to G , a Ruler laid from E to L cuts the Meridian in P the Pole of the World, and a Ruler laid from E to G cuts the Meridian in $\mathcal{A}$, that is, where the Equinoctial cuts it. Lastly, Lay a Ruler from P to C , and draw FI for the Substile Line; now have we the Right angled Spherical Triangle ZFP , in which

ZF

II. In the same Triangle there are the same Things given, to find Z F the Substile's Distance from the Meridian.

Equation.

$$CS < Z + R = CT. PZ + T. ZF.$$

Analogy.

As CT. PZ. Co-Lat.	38° 28'	10.0999135
To Radius	90 00	10.0000000
So CS. < Z. Co-Declinat.	60 52	9.6873895
To T. ZF. the Substile	21 9	9.5874760

Or by Transposition.

As Radius. to T. P Z, So CS < Z to T. ZF.

III. There are the same Things given to find P F, the Height of the Pole Artic above the Plane equal to the Stile's Height.

Equation.

$$S. PF + R = S. PZ. + SZ.$$

Analogy.

As Radius	90° 00'	10.0000000
To S. PZ. Co-Latitude	38 28	9.7938317
So S < Z. Co-Declinat.	60 52	9.9412575
To S. P F. Stile's Height	32 55	9.7350892

These three Requisites being found, the next Thing to be done is to make the First and Second Columns of the following Table, which are done thus, the Plane's Difference of Longitude being 35° 27', that is more than 30° = to two Hours in the Equinoctial in Time, shews that the Substile will fall between the Hours of Nine and Ten in the Forenoon, place this 35° 27' right against the Hour of Twelve, and 21° 9' against it in the next Column, because

cause $3^{\circ} 45'$ answers to one Quarter of an Hour, and 15 to an Hour in the Equinoctial, therefore subtract $3^{\circ} 45'$ from $35^{\circ} 27'$ continually as long as it will bear, 'till I have only $1^{\circ} 42'$ remaining, which answers to three Quarters past 9 in the Morning; hence, because this brings me to the Meridian, and Place of the Substile in the Dial, I subtract $1^{\circ} 42'$ from $3^{\circ} 45'$, and the Remainder is $2^{\circ} 3'$ the Equinoctial Degree answering $9\frac{1}{2}$, to which add $3^{\circ} 45'$ continually, 'till I come to $88^{\circ} 18'$ the Equinoctial Degrees answering $2\frac{3}{4}$ in the Morning, which Angles at the Pole, or Equinoctial Distances are more than we have Occasion for in this Dial, as you will see when you come to lay down the Hour Lines upon the Plane by Help of the Line of Chords, which I leave for you to practice at your leisure.

Hours	Equin. Dist.	Dist. & Substile.	Hours	Equin. Dist.	Dist. & Substile.
2	77 3	67 4	10 0	5 27	2 58
3	73 18	61 6	1	9 12	5 2
5 0	69 33	55 32	2	12 57	7 8
1	65 48	50 24	3	16 42	9 16
2	62 3	45 41	11 0	20 27	11 27
3	58 18	41 21	1	24 12	13 43
6 0	54 33	37 21	2	27 57	15 5
1	50 48	33 40	3	31 42	18 33
2	47 3	30 17	12 0	35 27	21 9
3	43 18	27 7	1	39 12	23 54
7 0	39 33	24 10	2	42 57	26 50
1	35 48	21 24	3	46 42	29 58
2	32 3	18 47	1 0	50 27	33 21
3	28 18	16 19	1	54 12	37 0
8 0	24 33	33 56	2	57 57	40 57
1	20 48	11 40	3	61 42	45 15
2	17 3	9 28	2 0	65 27	49 57
3	13 18	7 19	1	69 12	55 3
9 0	9 33	5 13	2	72 57	60 33
1	5 48	3 10	3	76 42	66 58
2	2 3	1 7	3 0	80 27	72 48
	Merid.	Substile	1	84 12	78 17
3	1 42	0 55	2	87 57	86 14

Lastly,

Lastly, To find the Distance of each Hour Line from the Substile, and the Analogy for this, is the very same as in the Horizontal Dial, See *the Triangle*, P 12 D, page 234, with this difference, that instead of the Elevation of the North Pole above the Horizon of the Place, commonly called the Latitude, here you must make Use of the Height of the Pole or Stile above the Plane.

One Example will be sufficient to illustrate the whole Process to the Meanest Capacity, and that shall be to find the Distance of the Hour, or Quarter of an Hour before Ten in the Forenoon, the Angle at the Pole, or Equinoctial Distance, you see in the Second Column, of the Table is $1^{\circ} 42'$.

Operation, By Transposition.

As Radius	-	90° 00'	10.0000000
To S. Stile's Height		32 55	9.7351345
To T < at the Pole		1 42	8.4724538
To T. Hour on the Plane		0 55	8.2075883

Now all the Work of this Dial is finished, only transferring the Hour Lines to the Plane, and by help of your Line of Chords it is done the very same Ways as has been taught in the Horizontal Dial, therefore need no Repetition; only in fixing the Gnomon, observe to place it on the Substiler Line, at right Angles to the Plane, and to make an Angle therewith of $32^{\circ} 55'$, and it may be either a Plate of Brass, Copper, &c. or an Iron Rod just the thickness of the Hour Lines.

A General Rule.

If the Plane decline East, the Substile falls among the Morning Hours, but if it declines West, among the Afternoon Hours.

IV. Of the East and West Erect Direct Dials.

THESE Dial Planes lying in the Plane of the Meridian, the Earth's Axis being parallel thereto the Pole itself has no Elevation, and consequently the Hour Lines

Lines can have no Centers, but are parallel to each other, the Six o'Clock Hour Line being the Substile, the rest of the Hour Lines are drawn by the Tangent of 15, 30, 45, 60, 75°, each Way as far as you have Occasion for Hour Lines, and let them make an Angle with the Horizon equal to the Latitude of you Habitation: The Height of the Stile must be equal to the Tangent of 45°, or the Distance of the Nine and Three o'Clock Hour Line.

V. Of an Equinoctial Dial.

WHAT I call here an Equinoctial Dial, is not so in Fact, but an Horizontal Dial, or a Dial as is made under the Equinoctial Circle, for there the Pole has no Elevation; therefore the Hour Lines and Stile are all parallel to the Plane, and is made as has been just now shewn in the East and West Dials, with this difference only, that this faces the Zenith, as the other do the East and West Points of the Horizon.

VI. Of a Polar Dial.

THIS and the last mentioned Dial have different Names by different Writers; some call this an Equinoctial Dial, some a Polar Dial, but I shall not in this Place dispute Words, for they are of no more Use than to convey the Idea of Things from one Person to another. Therefore if you would make ~~one~~ for the Latitude of 90° either North or South, it is no more than to divide a Circle into 24 equal Parts, and from those Parts draw Lines to the Center, and they shall be the true Hour Lines, and the Stile is no more than a Wire set perpendicular to the Plane in the Center of the Hour Lines.

VII. To make an Equinoctial Dial.

THIS Plane has two Faces, the Upper in North Latitudes serves all the Time the Sun is in Northern Signs, ♈, ♉, ♊, ♋, ♌, ♍: The under Face of the Plane serves the other Part of the Year, viz. while the Sun passes through ♎, ♏, ♐, ♑, ♒, ♓. and the making of it and it's Stile is all one and the same with the last mentioned Plane; and this Dial has an Advantage above many others, viz. it is Universal, for in what Latitude soever you be, place it so that the Plane may lye parallel to the true Equinoctial in the Heavens, and the Stile parallel to the Meridian of the Place, and then it will shew the true Hour of the Day.

VIII. Of a South Reclining, or North Inclining Dial.

THESE Planes must first be reduced to a new Latitude; in which they may become Horizontal Planes, and is thus performed: If the Reclination be less than the Complement of your Latitude, then subtract the Reclination from the Complement of the Latitude, and the Remainder is the new Latitude, in which it will become an Horizontal Plane.

2. But if the Reclination exceeds the Co-Latitude, subtract the Co-Latitude from the Reclination and this difference in the New Latitude, that to these new Latitudes thus found, draw Hour Lines, as has been taught in the Horizontal Dial, and they will be the true Hour Lines upon the South Reclining Plane.

3. But if the Reclining be equal to the Complement of your Latitude, and Recline from the Zenith towards the North Pole, then doth such a Plane lye directly in the Earth's Axis and have no Elevation; a Dial in such a Plane

as this must be made in all Respects as has been taught in the Equinoctial Dial, in the Fifth hereof. That is, An Horizontal Dial under the Equinoctial Circle.

IX. *Of the East and West Reclining and Inclining Dials.*

TH E S E Dials must also be refer'd to a new Latitude, that so they may become upright declining Planes, which is done thus: The new Latitude is always the same with the Complement of the Latitude in which the Plane Reclineth, and the Declination in that new Latitude will be always the same with the Complement of the Plane's Reclination: So that in the Latitude of $51^{\circ} 32'$ if an East or West Plane should recline from the Zenith 35° , and you would know in what Latitude that Dial shall become an upright declining Plane. Then Note, That the new Latitude is $38^{\circ} 28'$, the Complement of the given Latitude, and the Declination on the Plane is 55° , the Complement of the Reclination: So if you make a Vertical Dial (as has been taught in page 328), for the Latitude of $38^{\circ} 28'$, and the Declining 55° , that Dial will be a true Dial, for an East or West Plane Reclining from the Zenith 35° in the Latitude of $51^{\circ} 32'$.

X. *To make a Declining Reclining Dial.*

TH E S E Planes must be refer'd to a new Latitude, and to a new Declination before Hour Lines can be drawn upon them.

EXAMPLE.

In the Latitude of $51^{\circ} 32'$, a Plane declines to the East or West 34° , and Reclines 50° from the Zenith, what's the new Latitude and new Declination?

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For the new Latitude always say,

As Radius	-	90° 0'	10.0000000
To C S. Declination	-	34 0	9.9185742
So C T Reclination	-	50 0	9.9238135
To T. of	-	34 55	9.8423877

Now observe these Rules.

I. This fourth Tangent must be compared with the old Latitude, and the Complement of the Difference is the new Latitude.

Operation.

Old Latitude	-	51° 32'	
Tangent subtract	-	34 55	
Difference	-	16 37	Complement 73° 23'

is the new Latitude.

II. To find the new Declination.

As Radius	-	90° 00'	10.0000000
To C S. Reclination	-	50 00	9.8080675
So S. Old Declination	-	34 00	9.7475616
To S. new Declination	-	21 4	9.5556291

Secondly, Suppose a Plane Recline just so far as to pass thro' the Pole; then will the Hour Lines be all parallel to one another, as on a direct East or West Equinoctial, Horizontal, &c. Then the Analogy for finding the Reclination is this.

As C S Declination, to Radius, so T. Latitude, To C T of the Reclination.

For the Declination.

As Radius, to C S Reclination found in the last, so is S. old Declination, to S new Declination.

Lastly, Suppose a Plane Recline from the Zenith 20, and Declination 34 as before; *Query* the new Latitude and new Declination? Say as in the first, and you will find the new Latitude 75° 14', and new Declination 31° 42'. And these are all the Varieties that can happen. I might here have spoke of Reflective, Refractive, Concave, and Convex Dials, with the Globe and Cross Dials, but these I have already explained in my *Mechanic Dialling*, to which I refer my Reader for further Satisfaction.

CHAP X.

Of Surveying of Land.

SURVEYING of Land is a very useful Branch of the Mathematicks; and requires that the Practitioner be well skilled in Numbers, both Whole and Broken, with Extraction of Roots, Geometry, and in Plain Trigonometry, which this Treatise will make him a compleat Master of, without any other Book or Tutor. Therefore I would have the Young Student to perfect himself in one Chapter before he proceeds in another; and first you are to Note, that by a Statute made in the 33^d Year of the Reign of King *Edward I*, Anno 1305; an Acre of Land was to contain 160 Square Perches, and the Perch $5\frac{1}{2}$ Yards; 40 of which Poles or Perches makes one Furlong, and 8 Furlongs makes one Mile, as is better and more at large expressed in this Table.

Inches.	Feet.	Yard.	Pole.	Furl.	Miles.
12	1				
36	3	1			
198	$16\frac{1}{2}$	$5\frac{1}{2}$	1		
7920	660	220	40	1	
63360	5280	1760	320	8	1

All Land is now measured by Mr *Gunter's* Chain, which contains four Poles, each Pole is $16\frac{1}{2}$ Feet, the Whole Chain being 22 Yards, or 100 Links, but more largely and better expressed in this Table.

Inches.	Link	Yard.	Pole.	Chain.	Mile.
7.92	1				
36	4.55 <i>ferè.</i>	1			
198	25	$5\frac{1}{2}$	1		
792	100	22	4	1	
63360	8000	1760	320	80	1

And for the Learner's better Information I shall here present him with a new Table of Square Measure.

OF SURVEYING.

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A Table of Square Measure, which is to be read thus, viz. in 1 Chain there are 16 Square Perches, 174.24 Square Paces, 484 Square Yards, 4356 Square Feet, 10000 Square Links, and 627264 Square Inches, &c.

Inches.	Links.	Feet.	Yards.	Paces.	Perches.	Chains.	Acres.	Mile.
627264	1	1	1	1	1	1	1	1
144	2.295	9	27	10.89	16	1	1	
1296	20.75	25	30.25	174.24	160	10	1	
3600	57.38	272.25	484	1742.4	1600	100	1	
39264	625	4356	4840	1115136	102400	6400	640	
6272640	100000	435600	3097600					
4014489600	64000000	27878400	3097600					

A Table shewing how Square Links are made in any Piece of Land.

Square Links.	Perches.	Square Links.	Acr.	Rods	Perches.
625	1	14375			23
1250	2	15000			24
1875	3	15625			25
2500	4	16250			26
3125	5	16875			27
3750	6	17500			28
4375	7	18125			29
5000	8	18750			30
5625	9	19375			31
6250	10	20000			32
6875	11	20925			33
7500	12	21250			34
8125	13	21875			35
8750	14	22500			36
9375	15	23125			37
10000	16	23750			38
10625	17	24375			39
11250	18	25000		1	
11875	19	50000		2	
12500	20	75000		3	
13125	21	100000	1		
13750	22				

By the foregoing Table you may cast up the Content of any Piece of Land by Inspection, for when you have multiply'd your Dimensions in Chains and Links together, that Product is square Links, as suppose you had 35127 Square Links, what is the Content?

Look for the nearest less in the Table, (if you cannot find the exact Number) which is here 25000 answering to one Rood of Land, and the overplus 10127, answers to 16 Perches, and 127 Square Links over, which being less than a Perch is not regarded.

Now

Now because *Gunter's Chain* is the only Chain now made Use of in Surveying of Land, you are to observe that 10 Chains long, and 1 Chain broad doth make a Statute Acre of 160 Square Perches, as by this Table is more fully shewn.

Length in Chains.	Breadth in Chains, Links, and Parts of a Link.
1	10.00
2	5.00
3	3.33 Parts 33
4	2.50
5	2.00
6	1.66 Parts 66
7	1.42 Parts 85
8	1.25
9	1.11 Parts 11.

Likewise if you would set out an Acre in Yards, you may see by the Table, page 340, that the Chain is 22 Yards long, which multiply'd by 10 makes 220 Yards long, and 22 Yards broad makes one Acre. Because $220 \times 22 = 4840$ Square Yards; and if you would set out an Acre by Perches you must know that 40 Perches long, and 4 broad, is an Acre $= 160$ Square Perches.

II. Of the Instruments that the Surveyor must be furnished with.

- 1 **A** *Gunter's Chain*, as above described.
- 2 A Case of Instruments.
3. A Parallel Ruler.
- 4 A Plain Table.
5. A Theodolite.
6. A Plotting Scale, which has on it two Diagonal Scales, commonly one of 10, and the other of 20 in an Inch on one Side, and on the other Scales of 11, 12, 16, 20, 24,

20, 24, and 30 in an Inch, &c. As the Protractor and Line of Chords are used in laying down Angles, so are these Plotting Scales used in laying down of straight Lines, *i. e.* the Sides of Inclosures, the Diagonals, Perpendiculars and Off-sets which were actually taken in the Field.

Now a Scale of any Number of equal Parts whatsoever, is no more than to divide the Inch into as many equal Parts as you design the Scale shall contain Parts. As if I would have a Scale of 10 equal Parts in an Inch, (which Scale will make the greatest Plot of all), divide the Inch into 10 equal Parts and 'tis done. If you would have a Scale of 16 in an Inch, divide the Inch into 16 equal Parts, then 10 of those Parts will be a Scale of 16 in an Inch, &c.

N. B. I here recommend the Use of the Theodolite before that of the Plain Table, because the Paper upon the Table in the Field will be subject to contract, or expand according as the Air is dry or moist, and consequently will make your Plot too much, or too little, and err from the Truth considerably. These Instruments, with all others for Practical Mathematics, are made and sold by Mr *John Fowler*, at the *Globe* in *Switbin's-Alley*, by the *Royal Exchange, London*, fitted up at the best Manner, and at the lowest Prices.

In Measuring with the Chain, be careful to get the shortest Distance between any two Objects, otherwise you will make more of the Land than it really is. In Order hereunto you must provide Ten small Sticks like Arrows, sharp at the End to stick in the Ground; having set up White's at the Angles of the Field, and at other Places where you see Occasion, let him, (your Assistant) that leads the Chain, carry the Arrows in his Left Hand, all, save one in the Right, with the Chain, and always when he sticks down one of his Arrows, be sure that it be in a right Line with the Eye of him that follows the Chain, who is to gather up the Sticks, go on thus to the End of the Distance you are to measure, whether it be a Diagonal, Perpendicular, or the Boundary, Off-set, &c. by this Method he that follows the Chain and gathers up the Sticks has a sure Method of counting the Number of Chains, without any Trouble, or fear of Error: As suppose I find

find a Distance to be 8 Chains and 30 Links, you must set it down in your Field Book, ruled in proper Columns for that purpose, and it will stand thus 8.30, and 4 Chains 7 Links will stand thus, 4.07, always minding to set a Cypher in the Place of Primes in the Links, if they be under Ten. Now the Reader being well acquainted with Decimal Arithmetic, and with the Mensuration of Superficies, as taught in this Treatise, he is qualified to go into the Field, in Order to measure the same.

III. *How to take the Plot of a Field, by the Chain only, and to cast up the Content thereof.*

IN Order hereunto, you must provide yourself with a small Instrument call'd a Cross, which you may make of Wood thus, provide a Piece of Box Wood, and let it be turn'd round, and about two or three Inches Diameter, with a fine Saw cut two Slits cross ways, that so they may be exactly at right Angles with each other, and on the under Side of it let there be a Hole for Convenience of placing it upon a Staff about your own Height in Time of Practice. Being thus prepared, go into a Field, and upon a Piece of Paper draw the rough Draught of it, as near the Shape as possible you can, while you are doing this, let your Assistant place Pieces of white Paper at A, B, C, D, E, F, G, O, H and I. Then your Assistant taking the Chain and Arrow, standing at B, direct him to I, and as you go along, you must by help of your Cross take the Perpendicular K C, that is done by looking through one of the Slits in the Cross to I, then looking thro' the other Slit at C and find the Point K, put down the Chains and Links B K 150, leave a Mark at K, and measure on to L; here by help of your Cross looking at B and A, you will find out the Point L, that is where the Perpendicular A L doth fall, set down the Number of Chains either from B to L, or from K to L,
it

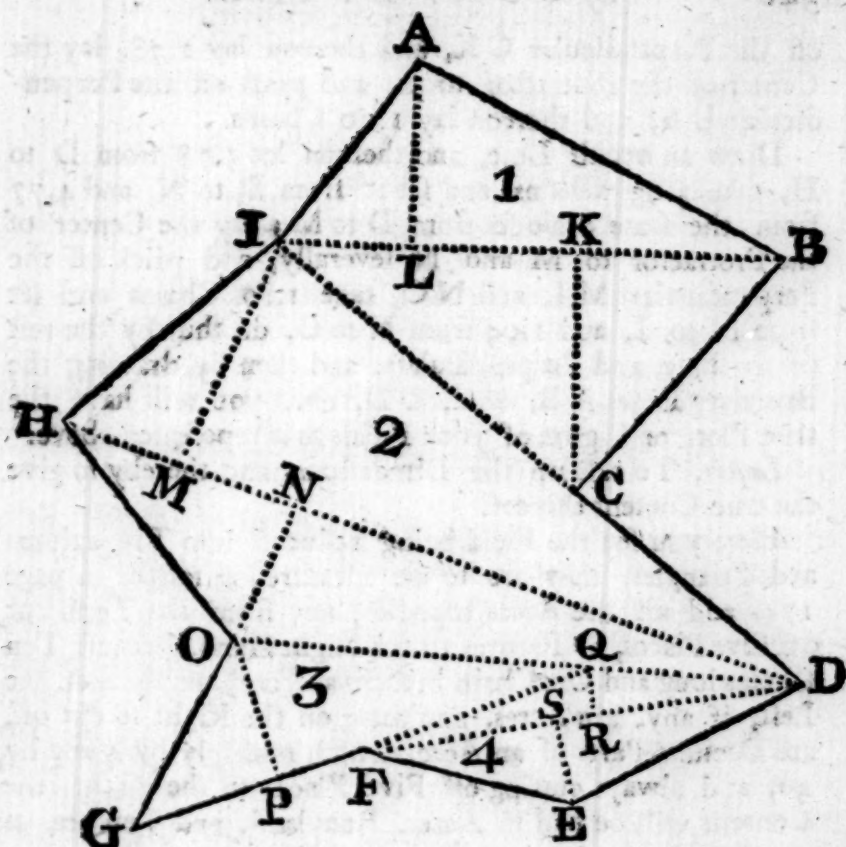
it matters not which, leave a Mark at L, and measure on to I, and set down the Whole Diagonal BI 3.75. Go back to L and measure the Perpendicular LA 1.40, and CK 1.78, which set down upon your rough Draught. And thus is the Trapezium ABCI finished, which I Note with the Figure 1.

Secondly, For the Trapezium IDOH, which I mark with the Figure 2. Then going to D, direct the Assistant towards H, and having measured 3.93 Chains, I observe the Point N, and the Point M fall at 4.77, where leave Marks, and measuring on to H, I find the Whole Base DH to be 5.78 Chains, which I insert in my rough Draught as before, then measuring the Perpendicular MI, I find it to be 1.80, and NO 1.09.

Thirdly, I go into the Triangle GOQ, which I mark with the Figure 3, and beginning at G, I measure towards Q, and as I go along, I observe with my Cross that the Perpendicular OP falls at P. 1.11 from G at P, I leave a Mark, and measure to Q, which I find to be 3.54 Chains. Then going back to P, I measure PO and find it to be 1.08 Chains.

Lastly, Here still remains the Trapezium FQDE unmeasured, which I Note with the Figure 4. Then coming to F, I measure toward D, and I find the Perpendicular SE to fall at 1.43 Chains from F, which I Note down in my rough Draught as before; measuring on I find, the Perpendicular RQ to fall at 1.72 Chains from F, which I also Note down: And measuring to D, I find FD to be 3.23 Chains, which I also Note down. Then coming back to the Marks which I left at R and S, I measure SE and find it 0.60 Link, and QR 0.35 Link, which I set down in my Field Draught, and so is the measuring all finished. And for the Reader's better Information, I shall here insert each particular Measurement in Chains and Links.

N. B. You must remember, when in the Field, to measure ID and DO, they being of Use in the plotting, but not in casting up of the Dimensions.



BK 1.50
 KL 1.22
 BI 3.75
 AL 1.40
 CK 1.78
 DN 3.93
 DM 4.77
 DH 5.78
 MI 1.80

NO 1.09 ID 4.94
 GP 1.11 DO 4.15
 GQ 3.54
 PO 1.08
 FS 1.43
 FR 1.72
 FD 3.23
 SE 0.60
 QR 0.35

Secondly, Shewing how to Plot the same.

When you have finished the Work in the Field, you may plot and cast it up at your leasure thus.

Take a clean Sheet of Paper, and with a Black Lead Pencil draw a Line at Pleasure, take one Chain 50 Links from your Scale and lay it from B to K, take 1.22 and lay it from K to L, and 3.75 Chains from B to I. This done, take your Protractor and lay the Center to K, prick off

off the Perpendicular C K, and thereon lay 1.78, lay the Center of the Protractor to L, and prick off the Perpendicular L A, and thereon lay 1.40 Chains.

Draw an occult Line, and thereon let 5.78 from D to H, take 3.93 Chains and set it from D to N, and 4.77 from the same Scale set from D to M; lay the Center of the Protractor to M and N severally, and prick off the Perpendiculars M I, and N O, take 1.80 Chains and set from M to I, and 1.09 from N to O, do thus by the rest of the Bases and Perpendiculars, and then by drawing the Boundary Lines A B, B C, C D, &c. you will have the true Plot, or Figure of your Fields as is represented above.

Lastly, To cast up the Dimensions, and thereby to give the true Content thereof.

Here you see the Field being reduced into Trapeziums and Triangles, they are to be measured as taught in page 174, add all the Areas together, and from the Total cut off Five Places, or Figures to the Right Hand, (because Ten Chains long and One Chain broad is an Acre) and those on the Left, if any, are Acres, and those on the Right so cut off, are Decimal Parts of an Acre, which multiply by 4 and by 40, and always cutting off Five Places to the Right, the Content will be had in Acres, Roodland, and Perches, as you may the better perceive by the Work at large.

A L =	1.40	IM =	1.80
C K =	1.78	NO =	1.09
	<u>3.18</u>	Sum -	2.89
Half =	1.59	Half DH =	2.89
1 B =	3.75		2601
	<u>795</u>		2312
	1113		<u>578</u>
	<u>477</u>		.83521
1 =	.59625		
2 =	.83521		
3 =	.19116	G Q =	3.54
4 =	.15181	Half O P =	.54
Acres.	1.77443		<u>1416</u>
	<u>4</u>		1770
Roods =	2.99772		.19116
	<u>40</u>		
Poles =	39.90880		

$ \begin{array}{r} F D = 3.23 \\ \text{Half S E \& Q R} = .47 \\ \hline 2261 \\ 1292 \\ \hline .15181 \end{array} $	$ \begin{array}{r} S E = 0.60 \\ Q R = .35 \\ \hline \text{Sum} = .95 \\ \text{Half} = .47 \end{array} $
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II. *How to take an Angle in the Field either with the Plain Table or Theodolite.*

SUPPOSE in the Field above, I would take the Angle at A, set up two Marks at B and I, place your Instrument at A, bring the Index to 360 on the Limb, and turn the Instrument about till through the Sights you see the white at B, here fix the Instrument, and move the Index till you see the Mark at I, the Degrees cut by Index on the Limb are the Quantity of the Angle B A I, and after the same manner may any Angle be taken: And for Proof of your Work, when you have taken all the Field, before you begin to Plot, if you would know whether it will Close, multiply 180° by a Number always less by 2 than the Number of Angles within the Field, and if that Product be equal to the Sum of all the Angles, then is the Work right, other ways nor.

III. *How to take the Plot of a Field by going round the same, or any large Champion Ground, as Salisbury-Plain, Hounslow-Heath, Forrest of Dalameer, &c.*

IN the 312 Page of the Abridgments of Acts of Parliament, we are told that *Hounslow-Heath* contains 4293 Acres, and one Rood Land of Ground. Now let us
H h
suppose

Suppose the Figure in page 347 to represent one of the above-mentioned Champion Grounds.

With your Theodolite, Chain and Assistant, beginning at A, take that Angle, and at every turning of its Bounds as at B, C, D, &c. take the Angle as has been taught above, and as you go along measure the Distances A B, B C, C D, &c. with your Chain, and find them as is here set down.

Sides		Angles	
A B	3.10	A	98
B C	2.31	B	76
C D	2.15	C	93 30 external.
D E	1.80	D	68 00
E F	1.64	E	140 00
F G	1.73	F	145 00 external.
G O	1.53	G	45 00
O H	2.00	O	114 30 external.
H I	2.04	H	91 00
I A	1.70	I	168 00 external.

The Sum of the internal Angles is 1440 which is equal to 180 multiplied by 8, that is less by 2 the Number of Angles, which is a Proof of the Work.

IV. How to take the Horizontal Line of a Hill.

IT is best to Measure Hilly-Ground by the Chain, by setting up Beacons upon several of the highest Hills, from which you can see the Angles in the Hedges, and direct your Sights from Station to Station, for the Chain being drawn over the Hills and Dales, must necessarily make a larger Plot, than it would do if you went all round the Hedges, all the way upon level or even Ground; now because you cannot make a Convex Superficies upon the Paper, you must only Plot the Horizontal Line or Base thereof.

Place

Place your Theodolite at one side of the Hill, and cause a Mark to be set on the Top of the Hill, just as high as your Instrument is from the Ground, and take the Angle of Altitude, and measure with your Chain from the Instrument to the Mark on the Top: Then by Trigonometry say, As Radius, to the Distance from the Instrument to the Top of the Hill; So is C S, Altitude, to the part of the Base from the Instrument to the Perpendicular Line. Go to the opposite Side of the Hill and make the like Observations, find by the above Analogy the Distance between the Instrument and the Perpendicular Line, which added to the other just now found, and the Sum is the Horizontal Line sought.

Or it may be found at one Operation thus, As the Sine of the Angle at the Instrument is to the opposite Side of the Hill, so is the Sine of the Angle at the Top of the Hill, to the Length of the Base.

V. How to take the Plot of a Field at one Station, when all the Angles are seen.

PLANT your Instrument any where in the Field where all the Angles can be seen, set up Marks, take all the Angles and note them down in your Field-Book, then measure the Distance with your Chain from your Instrument to every Angle, and set them down in your Field-Book, and the Work is done.

N. B. If it happens that you cannot see all the Angles at one Station, you must make two or three as you see most convenient for your purpose, measuring your Stationary Distances.

VI. To take the Plot of a Field, or the Map of a Country.

MAKE choice of two Stations where you can see all the Angles of the Field, place your Theodolite at one of the Stations, and take the Angle of the Field,

H h 2

measure

measure to your second Station the Distance in Chains and Links, Plant your Theodolite at your second Station, and take all the Angles of the Field, and Note them down in your Field-Book: Now these Angles and Stationary Distance being Protracted upon Paper, the Intersection of the Lines from each Station will determin the just Figure of the Field, or the true Situation of a Country, and make a perfect Map of places in your View, which Distances are gain'd either by Trigonometry, or by measuring them by the same Scale by which the Stationary Distance was laid down.

VII. *How to Survey Woods, or great Pools of Water.*

THIS must be done by going round the same on the out Side, and at every Bending take the Angle and measure the Distance from one Bending to another, Note all down in your Field-Book, and when you come round to the Place where you began, if your Plot close, you may be assur'd your Work is right, other ways there is some Error. Then when you have plotted your Work, take off the Bases and Perpendiculars from the same Scale you laid down the Plot, and from thence cast up the Contents in Acres as has been taught in page 348.

VIII. *An Expeditious Method, to cast up the Superficial Content of Land in Acres.*

WHAT we have already said on this Head, is to reduce the Field into Trapezias and Triangles, and those Bases and Perpendiculars measured by the Scale, may possibly produce some Error if great care is not taken in drawing the Lines fine; therefore in Geometry we have shewn how to reduce a Figure of many Sides to a Triangle, &c. by which means many Operations are saved in casting it up, and consequently liable to less Error.

IX *How*

IX. *How to take the Plot of a Gentlemans Estate, Mannor or Lordship.*

IT will be best first of all to take a View of the Land, and consider where it is best to begin ; then provide a Station Staff about the same Height with your Instrument, and slit it at the Top to put in a quarter of a Sheet of white Paper, beginning at any convenient place ; fix your Instrument and take the Bearing or Situation of your first Station, and send your Station Staff by an Assistant as far as you can see, measure to every Off set as you go along, and the Off-set themselves, entering all in your Field-Book, remembering where you begin to make that your first Meridian, and then as you go on in the Work, take Notice how such or such a remarkable Place bears from your first Meridian, by this means the Mannor will be kept in its true Position ; proceeding thus, through all the Work, always being careful to take Notice of all Things that are remarkable, and incert them in your Field-Book, that so when you come Home to Plot your Days work at Night, you may readier discern any Error if any such Thing should be. When the Map is made, you must incert the Name of every Inclosure, with its Content in Acres, and distinguish each with their proper Colours, the Gentleman's Seat, Coat of Arms, and Points of the Compass, truly representing the Situation of the Estate ; and also a Scale of Miles adapted to the Map. The North-side of the Map is always placed upwards, and the East on the right, as the Compass more plainly expresses it. If the Lordship is large it will be best to draw Upright, or North and South, and East and West Lines, denoted by Letters at the top and bottom, and also on the sides ; to be refer'd to by the Table of References, for the ready finding any Field or Parcel of Land therein contain'd.

X. How to find the Diameter of a Circle that shall contain any Number of Acres.

R U L E.

AS 11 is to 14, so is the Number of the Acres given, to the Square of the Diameter of the Circle required.

E X A M P L E

What's the Diameter of a Circle whose Area is one Acre?

Turn the given Acre into square Links thus 1.00000, then,

As 11 : 14 :: 100000 : 12.727270 (3.57 $\square$ V ferè.

XI. To Set out an Acre, three Rood Land, half an Acre, one Rood, or any Number of Perches under 40 Perches, or a Rood Land.

Say, As the Length, or Breadth,

Chains			
Is to	10.00	So is	1.00 Acre.
	7.50		1.00 three Rood.
	5.0		1.00 half Acre.
	2.50		1.00 Rood.
	.625		.01 one Perch.
	.625		.02 two Perch.
	.625		.07 three Perch.

E X A M P L E

Suppose, I had 1.32 Chains in length, how much must I have in Breadth to make 10 Perches.

As 1.32 : .625 :: 10 : .48 Links, ferè.

XII. How

XII. *How to reduce a large Plot of Land or Map into a lesser Compass, according to any given Proportion, or on the contrary how to enlarge it.*

TH E best way to do this, if it be a Field or two, &c. is to Plot it over again by a greater or lesser Scale as need requireth, but if it be large as the Map of a Country or Mannor, &c. the only way is to Circumscribe it with a Geometric Square, and Divide that great Square into several other lesser Squares, and by this means, every Close, House, Mill, Tree, &c. in one will fall in the same Square in the other.

XIII. *To find how many Inches makes a Mile, half a Mile, or a quarter of a Mile, by any Scale whatsoever.*

FIRST, you are to consider that the least Number of parts any Scale is Divided into are 10, and this Scale makes the largest Plot of any other. Secondly, You see by the Table in page 340, that 80 Chains of all Scales whatsoever make one Mile, so that by the Scale 10 in an Inch, 80 Inches in one Mile, but to find how many Inches is a Mile by any other Scale, observe these inverted Proportions.

		Scales		
As 10 : 80 ::	{	11	:	72.72
		12	:	66.66
		16	:	50.
		20	:	40.
		24	:	33.33
		25	:	32.
		30	:	26.26
		40	:	20
		50	:	16
				Inches in a Mile.

XIV. To

XIV. To reduce Statute Measure to Customary, and the Contrary.

THAT which is called Statute Measure, has $5\frac{1}{2}$ yards to the Pole or Perch, = $16\frac{1}{2}$ feet, and the Acre hath 160 of those Square Perches: But in *Lancashire* they have 8 yards to the Pole = 24 Feet, which they call a Rood, and 160 Square Poles to the Acre. In *Cornwal* (I am informed) their Perch is 3 yards, and the Acre 640 Poles or Perches. The *Irish* Perch is 7 yards = 21 Feet, and 160 Square Perches to the Acre. Now how many Acres Statute Measure will 20 Acres in *Lancashire* make? See the Work.

As 30.25 : 20 :: 64 : 42.3 Statute.

Again, Suppose 42.3 Statute Acre, how many Acres will they make Customary in *Lancashire*?

As 64 : 42.3 :: 30.25 : 20 Acres.

That is, As the Square of the Customary Perch,
Is to the given Number of Acres,
So is the Square of the Statute Perch,
To the Number of Acres sought.

F I N I S.



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	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
2	26	28	30	32	34	36	38	40	42	44	46	48
3	39	42	45	48	51	54	57	60	63	66	69	72
4	52	56	60	64	68	72	76	80	84	88	92	96
5	65	70	75	80	85	90	95	100	105	110	115	120
6	78	84	90	96	102	108	114	120	126	132	138	144
7	91	98	105	112	119	126	133	140	147	154	161	168
8	104	112	120	128	136	144	152	160	168	176	184	192
9	117	126	135	144	153	162	171	180	189	198	207	216
10	130	140	150	160	170	180	190	200	210	220	230	240
11	143	154	165	176	187	198	209	220	231	242	253	264
12	156	168	180	192	204	216	228	240	252	264	276	288
13	169	182	195	208	221	234	247	260	273	286	299	312